

MATHEMATICAL INSTITUTIONS.

In Three P A R T S.

I. *CLAVIS* }
II. *JANUA* } The { KEY,
III. *ANCILLA* } { G A T E,
 { HAND-MAID,

TO THE
Mathematical Sciences.

WHEREIN,
The Doctrine of *Plain* and *Spherical* **TRIANGLES**,
is Succinctly Handled, Geometrically Demonstrated, Arith-
metically, Geometrically, Instrumentally Performed;

And Practically Apply'd to

GEOMETRY, } SCIOGRAPHIA,
COSMOGRAPHY, } NAVIGATION,
GEOGRAPHY, } And THEORIES
ASTRONOMY, } of the PLANETS.

By **WILL LEBBOURN**, Philom.

Ingredere ut Proficias.

L O N D O N.

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INSTITUTION

ST. GEORGE'S

MEDICAL SCHOOL

Mathematical Sciences

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TO THE
Young STUDENT
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MATHEMATICKS.

IN the *Study* (as well as in the *Teaching*) of any *Art* or *Science*, a Gradual Proceeding therein, ought First, and Principally, to be considered: For, It is one thing to *Teach*; and another *How to Teach*: And, according to the Adage, *Qui bene distinguit, bene Docet*.

Now, the Design of this *Book*, being for the *Directing* and *Instructing* of such as would apply themselves to the Study of any *Mathematical Science*, so as to become such a *Proficient* therein, as to give a *Demonstrable Account* of what he does; must not attempt, at the first Onset, to fall directly upon that *Part* he principally aims at, but first acquaint himself well with such *Elements* as concern, and are subservient to, them all.

P R E F A C E.

And to that end, The several *Parts* and *Sections* of this *Book*, are disposed in such *Order*; That, Beginning with the *First Part* (which is as a *KEY* to give you entrance into the *GATE*; which is the *Second Part*; Through which, having perfectly passed, you may confidently proceed to fall upon the *Study* (or *Practice*) of any *Mathematical Science*; And not only on those that I have here (by my *HAND-MAID*, in the *Third Part*) directed you to; but to any other you shall attempt: *TRIGONOMETRIA* being the *Basis*, *Foundation*; nay, the very *Primum Mobile*, of all the rest.

For Illustration whereof, I have applyed the *Doctrine* of *Triangles* to *Practice* (in the several *Sections* of the *Third Part*) in the Solution of such Problems, as are of frequent use in several *Parts* of the *Mathematicks*. So in the *First*, Which teacheth how to take all manner of *Heights* and *Distances*, accessible or inaccessible: And in the *Sixth*, which treateth of *Navigation* both by the *Plain Sea-Chart*, and that called *Mercators*; in both which, I have first shewed how *Geometrically*, by *Scale* and *Compasses*, to lay down (upon Paper) a *Figure* answerable to the *Question* Propounded: In which *Figure*, so laid down, you will have constituted a *Right-Lined Triangle* or *Triangles*, of some kind or other, *i. e.* either

P R E F A C E.

either *Right* or *Oblique-Angled*; And in it discovered what *Parts* thereof are *Given*, by the demand in the *Question*; and then the *Part* or *Parts Unknown* will be such (as being found) will answer the *Question* demanded: And herein the Excellency of the *Doctrine* of *Plain Triangles* is in part made manifest.

Likewise, In the 1st. 2^d. 3^d. 4th. and 5th. Sections, which treateth of *Cosmography*; and therein of *Geography*, *Astronomy* and *Dialling*: I do first declare, and shew how the Operation of the *Question* or *Proportion*, is to be performed upon the *Terrestrial* or *Celestial Globe* (according as the thing proposed does require.) Which *Question* being resolved upon the *Globe*; and the *Globe* resting in the same *Position* it was when the *Question* was resolved thereupon: You will upon the *Body* of the *Globe*, (by the great *Circles* thereon described, and those appendant to it) apparently discover a *Spherical Triangle* or *Triangles* Constituted; in which (according to the Tenor of the *Question*) you will plainly perceive what *Parts* thereof are *Given*; and then, the other *Parts* being found, must necessarily Answer the *Question* Propounded: And herein, in some measure, is the *Doctrine* of *Spherical Triangles* made applicable to all the above-named Practices. And Moreover, by Projecting of the Sphere,
as

P R E F A C E.

as is done for the *Geographical Problems* in *Part III. Section the Second.* And for *Astronomical Problems*, as is shewed in *PART II. Section II. Of Projection of the Sphere in Plano, &c.*

Also in the *VIIth. Section*, Which treats of the *Theories*, and finding the *Places* of the *Planets, &c.* there the *Doctrine of Triangles* both *Plain* and *Spherical*, are joyntly concerned : So that, let your *Mathematical Practice* or *Study* be, *Opticks, Perspective, Fortification, Gunnery*, or any other *Mathematical Art* : The most *Problems* relating to any of them, must be beholding to *Trigonometry* for their *Solutions*.

And now I have but one thing more to advertise the Reader of : That, Whereas in the performances of all the *Operations* in the several *Parts* of this Book, the First (which treats of *Practical Geometry* only) excepted, there is continual Use to be made of a *CANON, TRIGONOMETRICAL*; or *TABLES* of *Artificial SINES, TANGENTS* and *LOGARITHMS*; So that the Reader cannot expect but that such *TABLES* should have been *Added* unto these *Precepts*. And, indeed, it was so intended, but that almost in every Book that hath any thing of the *Mensuration* of *Triangles* in it, there are such *Tables Printed*; and that one or other of them may be in every Man's *Hands*; and because all the *Problems* in this Book
may

P R E F A C E.

may be solved by (almost) any of those *Canons* or *Tables*, was the occasion of the omission of such *Tables* here at present. But chiefly, because there is intended (in some short time) to be Printed,) A *CANON* of Artificial *SINES* and *TANGENTS*, both **Seragenary** and **Centesimal**; differing from any yet Extant; it supplying both ways of *Numeration* at one view, and yet in the same *Room*: And to this *CANON* shall be Added Ten *CHILIADS* of *LOGARITHMS* commodiously Contracted; and the Use thereof in Logarithmical Arithmetick.

Thus having given the Reader a *General Account* in this *Preface*, I refer him to a more *Particular* in the following *Contents*, and so leave him to the Practice of them: In which I wish him good success; and till he hears farther from me, bid him Farewell.

WILL. LEYBOURN.

E R R A-

E R R A T A.

Page	Line	For	Read
9	20	the given Point	the given Point P.
11	16	same	same
13	28	from N to O,	from M to O.
20	19	<i>Means</i> 8 is	<i>Means</i> 8 in 8 is
21	5	to D E.	to A E.
52	14	E C, intercepted	E F, intercepted
52	16	E and D;	E and F;
52	17	Arch E D is;	Arch E F is;
59	30	The Angle B P O	The Angle P B G
63	1	<i>Disjunct</i>	<i>Disjunct</i> .
69	17	contrived :	continued :
95	29	$\frac{1}{2}$ Sun $\angle$ s :	$\frac{1}{2}$ Sum $\angle$ s :
110	37	Measure is $\odot$ O,	Measure is Q α .
111	15	T α ,	Q α
111	16	and T,	and Q
142	23	of R, S, P, T.)	of R, A, P, T.)
144	29	draw a Line L,	draw a Line L b,
<i>ibid</i>	<i>ib.</i>	also b, an	also an
148	21	Angle at F,	Angle at D,
151	14	From A G to	From G to
164	1	of the	The
196	<i>ult</i>	Plain Triangles	Spherical Triangles
279	23	Point B :	Point in B :
281	22	Hour	Hours
282	2	26 de.	36 de.
282	16	23.25	23.35
287	2	Declination	Reclination
<i>ibid</i>	21	Reclination 120	Reclination 20.
297	8	is to be most	is most
308	37	<i>Problems</i> and	Problems in
327	<i>ult.</i>	6.50 deg.	5.50 deg.
328	32	M P O of A B,	M P of A B,
330	2	6.50 de.	5.50 de.
347	8	B T E	B F E.

Figures in the Margins Omitted, or Misplaced.

Page 7. Against *Probl. I.* put *Fig. VIII.* and against *Prob. II.* put *Fig. IX.* in the Margin.
 Page 9. Against *Probl. V.* put *Fig. XIII.* in the Margin.
 Page 20. For *Fig. XXXIV.* put *Fig. XXXVII.* in the Margin.
 Page 50. For *Fig. VI.* put *Fig. XII.* in the Margin.
 Page 89, 90. 91. *Fig. XXXI.* in the Margin, must be *Fig. XXXII.*
 Page 274. *Fig. LIII.* in the Margin, is omitted.
 Page 285, and 286. for *Fig. LIX.* read *Fig. LVIII.* in the Margin.
 Page 53. The three last Lines are thus to be read :

The Triangle $\left\{ \begin{array}{l} B A C \\ B D C \\ C D E \end{array} \right\}$ hath $\left\{ \begin{array}{l} \text{One Right Angle, and Two Acute.} \\ \text{Two Obtuse, and One Right.} \\ \text{One Obtuse, and Two Acute.} \end{array} \right.$

C O N.

CONTENTS.

Part I. The KEY, Of GEOMETRY.

And therein of

Definitions.
Practical Problems
Theorems

From Page 1 to Page 7
7 to 17
18 to 27

Part II. The GATE, Of TRIGONOMETRY.

And therein of

1. **D E F I N I T I O N S**

From Page 29 to Page 32

- | | | |
|--|---|----------|
| 2. Right-Lines applied
to a Circle, | { Sines,
Tangents,
Secants,
Their Construction } | 32 to 38 |
| 3. The Affections of Right-lined, or Plain Triangles | | 38 to 43 |
| 4. The Mensuration (or Solution) of Plain Triangles | | 43 to 50 |
| 5. S P H E R I C A L T R I A N G L E S, | | |
| { Definitions,
Theorems, } | | 50 to 56 |
| { Affections of Great Circles of the Sphere, in order to
the Demonstrating of Spherical Triangles } | | 56 to 60 |
| 6. The Solution of Right-Angled Spherical Triangles | | 60 to 68 |
| 7. Oblique-Angled Spherical Triangles, and Propositions concerning them | | 68 to 70 |
| 8. The Solution of them | | 70 to 83 |
| 9. Oblique-Angled Spherical Triangles, Resolved without regard
had to a Perpendicular | | 84 to 96 |
| (a) | | 10. Tri- |

The CONTENTS.

10. Trigonometical Problems <i>Extraordinary</i>	96 to 105
11. Spherical Triangles Geometrically performed, <i>by Projecting of the Sphere in Plano</i>	106 to 114
12. Spherical Trigonometry, Instrumentally Performed <i>several ways: As, by the Steriographical, Orthographical</i>	114 to 138
<i>} Planispheres</i>	

Part III. *ANCILLA: Vel, Trigonometria Practica.*

In Seven SECTIONS.

SECT. I. OF GEOMETRY.

And therein of

1. **A**LTIMETRIA; Or the Taking of Heights, (as of Towers, Trees, Steeples, &c. *Whether Accessible or Inaccessible; several Ways* From Page 141 to Page 147
2. LONGIMETRIA; Or the Measuring of Distances: *As of Remarkable Places upon the Earth, Ships upon the Sea, &c. Either Accessible, or Inaccessible* 147 to 152
3. PLANOMETRIA; Or the Measuring of Plains: *As of Board, Glass, Pavements, Hangings, Wainſcot, &c.* 152 to 155
4. Geodæcia; Or Measuring of Land 155 to 158
5. STERIOMETRIA; Or the Mensuration of Solids: *As Timber, Stone, &c. And of Regulars: As Cubes, Globes, Prisms, Parallelipedons, Pyramids, Cones, &c. whether Whole or Dissected* 158 to 160

SECT. II. OF COSMOGRAPHY.

And therein of

1. The Material Sphere, or Globe, Celestial and Terrestrial: *And of such Circles, Lines and Points, as are described thereon, or Appendant unto it:* 161 to 167
2. COS-

The CONTENTS.

2. *COSMOGRAPHICAL ELEMENTS*, both
Geographical and Astronomical 167 to 175

S E C T. III. OF G E O G R A P H Y.

And therein of

The Use of the Terrestrial Globe, and thereupon to find the Situation of Places both in Longitude and Latitude : And to find their Distances : And also by Trigonometrical Calculation

176 to 185.

S E C T. IV. OF A S T R O N O M Y.

And therein of

1. *ASTRONOMICAL TABLES* 185 to 197
2. Astronomical Problems, relating to the Sun and Fixed Stars :
To Work them upon the Globes ; And by Trigonometrical Calculation also 197 to 256
3. Astronomical Problems, relating to Astrology 257 to 266

S E C T. V. OF S C I O G G R A P H I A, or D I A L L I N G.

And therein of

1. *D I A L L I N G* in General; with the Situation of all Plains
on which Dials may be made : And to find out any such Situation. 267 to 272
2. *The Making of all sorts of Dials, by finding out the places of Substile, Stile and Hour-Distances, by the Globe : And also, to find the same Requisites, and Hour-Distances, by Trigonometrical Calculation, after a new Method* 272 to 296
3. *The Inscription of the Greater and Lesser Circles of the Sphere upon all sorts of Dial Plains*

And such are

- | | | |
|--|---|------------|
| <p>{ Parallels of Suns Course throughout the Zodiack,
Parall. of the Days length, Suns Rising, Setting, &c.
Azimuth or Vertical Circles,
Almicanters, or Circles of the Suns Altitude,
Jewish, Italian, and Babylonish Hours</p> | } | 296 to 303 |
|--|---|------------|

S E C T.

The CONTENTS.

SECT -VI. Of NAVIGATION.

And therein of

1. *PLAIN SAILING*; Or *Sailing by the Plain Sea-Chart*: And therein, (1) *By laying down upon a Blank Chart, Places according to their Longitudes and Latitudes*: (2) *To find their Distances, Difference in Longitude, Latitude, Rumb and Distance upon the Rumb.* (3) *To Work a Travers, consisting of many Courses. With variety of Problems, useful in that Art.--- All being performed Geometrically by Scale and Compasses; and by Trigonometrical Calculation* 304 to 324
2. *Sailing by Mercators, or the True Sea-Chart* 324 to 329
3. *Sailing by the Middle Latitude* 329 to 331
4. *The several Ways compared* 331 to 332

SECT. VII. Of ASTRONOMY Theorical.

And therein of

1. *The Planetary System* 333 to 335
 2. *The Theory of the Sun, and other Primary Planets* 335 to 336
 3. *Of the Motions of the Planets in their Elliptical Orbs* 336 to 339
 4. *Finding the true Place of a Planet in its Orb* 339 to 340
 5. *To Calculate the Place of a Planet Trigonometrically* 340 to 346
 6. *Of the Magnitude of the Sun, Earth, Moon, and other Planets* 346 to 349
- Geometrical Astronomy* 349 to 366

C L A-

CLAVIS MATHEMATICÆ.

THE
KEY

TO THE
Mathematical Sciences.

PART I.

OF
Practical Geometry.

THIS First Part consists only of such DEFINITIONS, PROBLEMS and THEOREMS, GEOMETRICAL, which of Necessity ought to be understood and practised, before farther Progress be made in any other Mathematical Science: And so I will begin It with these following

Geometrical Definitions.

I. *A Point is that which hath no Part.*

That is, it hath no Parts into which it may be divided; It being the least thing that by Mind and Understanding can be imagined or conceived; than which there can be nothing less: As the *Point* in the Margin, noted with the Letter A over it: It being neither *Quantity*, nor any Part of *Quantity*; but only the *Term* or *End* of *Quantity*.

B

II. A

II. A Line is a Length without Breadth; as the Line A B:

A ————— B

Unto Quantity there appertain Three Dimensions; viz. Length, Breadth and Depth (or Thickness;) of which, a Line is the first, and hath Length only, without Breadth or Thickness; as the Line C D,

C | ————— | ————— | ————— | D
 E F

which may be divided into Parts; either Equally, in the Point E, or Unequally, in the Point F.

III. The Ends, or Limits, of a Line, are Points G ————— H.

For a Line hath its beginning from a Point, and likewise endeth in a Point: So the Points G and H are the Ends of the Line G H, and are no Parts of it.

IV. A Right Line is that which lyeth Equally between its Points.

Or, it is the Shortest Distance that can be drawn between Point and Point; so the Right Line G H, is the Shortest Distance between the Points G and H.

V. Parallel (or Equidistant) Right Lines are such, which being drawn upon the same Plain, and infinitely produced, would never meet.

And such are these Two Lines, A B and C D.

A ————— B
C ————— D

VI. A Plain Angle is the Inclination (or Bowing) of Two Right Lines, one to the other, and the one touching the other; and not being directly joined together.

Fig. I.

So the Two Lines A B and B C incline one to the other, and touch each other in the Point B; in which Point (by reason of the Inclination of the said Two Lines) is made the Angle A B C; But if the Two Lines which incline one to the other, do (when they meet) make one Streight Line, then do they make no Angle at all: As the Lines D E and E F incline one to the other, and meet each other in the Point E, and yet they make no Angle.

And

Geometrical Definitions.

3

And here Note, That an *Angle* (generally) is noted with *Three Letters*, of which, the middlemost Letter represents the *Angular Point*; so, in this *Angle A B C*, the Letter *B* denotes the *Angular Point*. —And of *Angles* there are *Three Kinds*; viz. *Right*, *Acute* and *Obtuse*.

VII. When a *Right Line* standing upon a *Right Line* maketh the *Angles* on either *Side* thereof *Equal*, then either of those *Angles* is a *Right Angle*; and the *Right Line* which standeth erected is called a *Perpendicular Line* to that *Line* upon which it standeth.

So upon the *Right Line C D*, suppose there do stand another *Right Line A B*, in such sort, that it maketh the *Angles A B C* and *A B D* (on either side of the *Line A B*) equal; then are either of those *Angles*, *A B C* and *A B D*, *Right Angles*; and the *Line A B*, which standeth erected upon the *Line C D*, (without inclining on either side) is a *Perpendicular* to the *Line C D*. Fig. II.

VIII. An *Obtuse Angle* is that which is *Greater* than a *Right Angle*:

So the *Angle C B E* is an *Obtuse Angle*, it being greater than the *Right Angle A B C*, by the *Quantity* of the *Angle A B E*.

IX. An *Acute Angle* is that which is *Less* than a *Right Angle*.

So the *Angle E B D* is an *Acute Angle*, it being *Less* than the *Right Angle A B D*, by the *Quantity* of the *Angle A B E*.

X. A *Limit* or *Term* is the *End* of any thing.

Forasmuch as there is no *Quantity* (or *Magnitude*) of which *Geometry* treateth, but it hath *Bounds* or *Limits*: And as *Points* are the *Bounds* or *Limits* of *Lines*, so *Lines* are the *Bounds* or *Limits* of *Plains* or *Superficies*; and *Plains* (or *Superficies*) of *Solids* (or *Bodies*.)

XI. A *Figure* is that which is contained under *One Term*, or *Limit*; or *Many*.

So *A* is a *Figure* contained under one *Line* or *Limit*: *B* is a *Figure* under *Three Lines* or *Limits*: *C* under *Four*: *D* under *Five*, &c. which are their respective *Bounds* or *Limits*. Fig. III.

A 2

XII. A

XII. A Circle is a Plain Figure contained under One Line, which is called a Circumference or Periferie; unto which all Right Lines drawn from one certain Point within the Figure unto the Circumference, are equal one to the other.

Fig. IV. So the Figure B C D contained under One crooked Line, is a Circle, whose Circumference or Periferie is B C D. In the middle whereof there is a Point A, from which all the Right Lines, A B, A C, A D, being drawn to the Circumference B C D, are Equal. And that Point A is called the Centre of the Circle B C D.

XIII. The Diameter of a Circle is any Right Line drawn through the Centre, and ending at the Circumference on either Side, dividing the Circle into Two Equal Parts.

Fig. V. So the Line E K F is a Diameter, because it passeth from the Point E of the Circumference on the one Side, to the Point F on the other Side; and passeth also by the Point K, which is the Centre of the Circle: And moreover, it divideth the Circle into Two equal Parts, viz. into the Part E G F above, and E H F below, the Diameter; which Two Parts are termed Semicircles.

XIV. A Section, Segment or Portion, of a Circle, is a Figure contained under one Right Line, and a Part of the Circumference; Greater or Lesser than a Semicircle.

So the Right Line L M divideth the Circle E G F M H L into Two unequal Sections; namely, into the Section L G M above, Greater, and the Section L H M below, Lesser, than a Semicircle.

XV. The Semidiameter of a Circle, is half of the Diameter of that Circle.

So K E or K F are Semidiameters of the Circle E G F H: And so is any Right Line drawn from K the Center to the Circumference; which Lines are frequently called the Radius of the Circle.

XVI. Right

XVI. Right Lined Figures, are such Figures as are contained under Right Lines; of which none can consist of less than Three, and those are called Trilaterals, or Triangles.

And Triangles are denominated partly from the Differences of their Sides, and partly from the Differences of their Angles: As for the Differences of their Sides, they may be, (1.) All Equal, and such a Triangle is called an Equilateral Triangle, as the Fig. VI. Figure N. Or, (2.) Two Sides may be Equal, and the third unequal; and such a Triangle is called an Isosceles Triangle, as the Figure O. Or, (3.) All the Three Sides may be unequal, and such a Triangle is called a Schalenum Triangle, as the Figure P. And these are the Distinctions in relation to their Sides.

Now for the Distinction of Triangles in relation to their Angles, they are Three also: For, (1.) If a Triangle have One Right Angle, it will have Two Acute ones, as the Figure Q, where the Angle at A is a Right Angle, and the Angles at B and C are Acute, and such a Triangle is called Orthogonium. Or, (2.) If the Triangle have all the Angles Acute, as the Figure R; all whose Angles at D E and F are Acute; such a Triangle is called an Oxogonium Triangle. Or, (3.) If the Triangle have One Obtuse Angle, as the Figure S, whose Angle at I is Obtuse, and the other Two at G and H are Acute; such a Triangle is called Ambligonium. And these are the Denominations in relation to their Angles.

XVII. Of Four-sided Figures (or Quadrilaterals) a Quadrāt, or Square, is that whose Sides are Equal, and Angles Right.

And as Triangles have their various Denominations from the Species of their Sides and Angles, so have Quadrilateral Figures also. For, (1.) If all the Sides be Equal, and all the Angles Right Angles, as the Figure O, such a Figure is called a Quadrāt or Square. But, (2.) If of the Four Sides, Two be longer than the other, each to its Correspondent (or Opposite) and the Angles all Right Angles, as the Figure P; such a Figure is called a Parallelogram, or Long Square, (and sometimes) a Rectangle. But, (3.) If all the Sides be Equal, but the Angles Unequal, that is, Two Acute, and Two Obtuse, each to his Correspondent (as the Figure R) such Figure is called a Rhombus or Diamond Form. And farther, (4.) If such a Figure have Two Sides Longer, and Two Sides Shorter; and also Two Angles Acute, and Two Obtuse, each

each to it opposite corresponding, as the Figure Q, such a Figure is called a *Rhomboides*, or Diamond-like Figure. But, (5.) If *Quadrilateral Figures* have all their *Sides*, and all their *Angles*, *Unequal*, as the Figures S and T, such Figures are called *Trapezia*, or Table Forms.

P R A-

PRACTICAL PROBLEMS, GEOMETRICAL.

P R O B L E M I.

To divide a Right Line A B, into Two Equal Parts A E and B E, and at Right Angles.

Practice. **F**irst, Open your Compasses to any Distance greater than half the Length of the given Line A B. *Fig. VIII.*

2. With that Distance set one Foot in A, and with the other describe the obscure Arch *b c*,—and (with the same Distance) One Foot set in B, with the other describe the obscure Arch *d e*, crossing the former Arch in the Points C and D.

3. Through the Points C and D, draw a Right Line C D, which will divide the given Line A B into Two Equal Parts in the Point E, and at Right Angles. The Angle A E C on one Side thereof, being equal to the Angle C E B on the other Side.

P R O B. I.

Upon any Point (as O) taken in the Right Line Q R, to erect a Perpendicular O S.

Practice. **F**irst, Open the Compasses to any small Distance; and setting one Foot in the given Point O, with the other Foot make Marks on either Side of O, as at T and V.

2. Open the Compasses to any Distance, greater than the former; and setting one Foot in T, with the other describe the Arch *b b*. —Also, with the same Distance, set one Foot in V, and with the other describe the Arch *g g*, crossing the former Arch *b b* in the Point S. *Fig. VIII. IX*

3. Draw

3. Draw the Line O S, and it will be *Perpendicular* to the given Line Q R.

P R O B. III.

From the End of a Line X Z, to erect a Perpendicular Z A.

Fig. X. *Practice.* **F**irst, The Compasses being opened to any small Distance, set one Foot in Z, and with the other describe the Arch B C D, and upon it set the same Distance from B to C, and from C to D.

2. The Compasses still continuing at the same Distance, set one Foot in C, and with the other describe the Arch F D: Also set one Foot in D, and with the other describe the Arch C E, cutting the Arch D F in A.

3. From Z draw the Line Z A, which will be *Perpendicular* to the Line X Z.

P R O B. IV.

Another Way to erect a Perpendicular upon the End of a Line.

Fig. XI. *Practice.* **F**irst, With any small Distance of the Compasses set one Foot in H, and with the other describe the Arch I K, and set that same Distance from I to K.

2. Upon K (with the same Distance) describe the Arch I L M N.

3. Upon this Arch set the same Distance from I to L, from L to M, and from M to N.

4. A Line drawn from H, through N, shall be *Perpendicular* to the Line G H.

A Third Way.

Fig. XII. 1. **T**HE Compasses opened to any small Distance, set one Foot in O, and pitch the other Foot down at Pleasure, as at Q; and one Foot resting upon Q, turn the other Foot about, till it cross the Line P O in S, and also describe the small Arch *b a*.

2. Lay a Ruler from S to Q, and it will cut the Arch *b a* in R.

3. A Line drawn from O, through R, shall be a *Perpendicular* to P O.

P R O B.

P R O B. V.

From a given Point above, as P, to let a Perpendicular fall upon a given Right Line under it, N O.

IN the Performance there are Two Varieties: For, (1.) The given Point may be scituate over (or about) the Middle; or over (or near) the End of the given Line

In the First Case.

Practice. **L**ET N O be the Right Line given; and let P be the Point above, from whence the Perpendicular is to be let fall.

1. Open the Compasses to a Distance greater than is the nearest Distance between the Point above P, and the Line upon which the Perpendicular is to fall; and with that Distance, setting one Foot in P, with the other describe the Arch of a Circle, which will cut the given Line in Two Points, R and S.

2. Divide the Space between R and S (by the first Probl.) into Two equal Parts in the Point Q, then will a Right Line, drawn from P to Q, be Perpendicular to the given Line N O.

Fig. XIII.

Note, To avoid the dividing of the Space between R and S into Two equal Parts, to find the Point Q, (if you have room either above or below the given Line) you may set one Foot of the Compasses in S, and opening the other to any convenient Distance, describe an Arch yy ; and removing the Compasses to R, describe the Arch zz , crossing the former in A; and so, a Line drawn from A, through the given Point, shall be Perpendicular to the given Line N O.

In the Second Case.

Let V be the Point given,

Fig. XIV.

Practice. **F**irst, From any Part of the given Line N O, as from T, draw a Right Line to the given Point V, which divide into Two equal Parts in X.

2. Set one Foot of the Compasses in X, and with the Distance X T, describe the Semicircle V O T, cutting the given Line in O, so shall O be the Point in the Line N O; from which, if you draw a Line to V, it will be Perpendicular to N O.

C

P R O B. V.

P R O B. VI.

Unto a given Right Line P Q, to draw another Right Line S T, which shall be Parallel to it; and at the Distance of the given Line M.

Fig. XV. Practice. **F**irst, Take in your Compasses the Length of the given Line of Distance M.

2. Set one Foot in O, (near to one end of the give Line P Q,) and with the other describe the Arch *a a*: Also, upon the Point L, (near the other end of the given Line) describe the Arch *b b*; then,

3. By the Convexity (or Tops) of those Two Arches, if you draw a Right Line S T, it will be Parallel to the given Line P Q, and at the Distance of the Line M.

P R O B. VII.

To a given Line V W, to draw another Right Line Y A, which shall pass through a given Point R, and Parallel to V W.

Fig. XVI. Practice. **F**irst, Set one Foot of the Compasses in the given Point R, and with the other describe the Arch *c c*, so that it may only touch the given Line V W; and with the same Distance, set one Foot in any Point of the Line V W, as at X, and with the other describe the Arch *d d*.

2. A Line drawn through the given Point R, by the Convexity of the Arch *d d*, as the Line Y R A, that Line shall be a Parallel to the Line V W.

P R O B. VIII.

A second Way to draw a Line Parallel to a given Line A B, which shall pass through a given Point P.

Fig. XVII.

Practice. **F**irst, With the Distance P A, upon the Point B, describe the Arch *e e*.

2. With the Distance A B, upon P, describe the Arch *f f*, crossing the Arch *e e* in the Point C.

3. A Right Line drawn from the given Point P, through C, shall be Parallel to the given Line A B.

P R O B.

P R O B. IX.

To make an Angle D F E, equal to a given Angle A B C.

- Practice.* **F**irst, Upon the Angular Point B, at any Distance, describe an Arch $g g$. Then,
2. Having drawn another Line, as F E, upon the end F (with the same Distance) describe the Arch $h h$.
 3. Take the Distance $g g$ in your Compasses, and set it from h to h : Then,
 4. A Line drawn from F through h , as F h D, shall make the Angle D F E equal to the Angle A B C.

Fig.
XVIII.

P R O B. X.

To divide an Angle G H K into Two Equal Parts.

- Practice.* **F**irst, Upon the Angular Point H (with any Distance of the Compasses) describe an Arch cutting the Two Sides, containing the Angle, in the Points h and s . Fig. XIX.
2. The Compasses opened to the same (or any other) Distance, set one Foot in h , and with the other describe the Arch $k k$; and (with the same Distance) one Foot set in s , describe the Arch $i i$, crossing $k k$ in the Point L.
 3. Join H L, and so is the Angle G H K divided into Two Equal Parts, by the Line H L.

P R O B. XI.

How to divide a given Right Line M N, into any Number of Equal Parts: Suppose Five.

- Practice.* **F**irst, From one end of the given Line, as N, draw the Line N O at Pleasure, making the Angle O N M. Fig. XX.
2. Upon the Point M (by Prob. IX.) make the Angle N M P equal to the Angle O N M; or (by Prob. VIII.) through the Point M, draw the Line M P Parallel, to the Line O N.
 3. With any small Distance of the Compasses, one Foot being set in N, run over Four of those Distances upon the Line N O, at the Points 1, 2, 3, 4. — Do the like upon the Line M P: Then,

C 2

4. If

4. If you draw the obscure Lines 1 4, 2 3, 3 2, and 4 1, they will cross the given Line M N in the Points *d*, *c*, *b*, *a*, dividing it into *Five* Equal Parts, as was required.

P R O B. XII.

To make an Equilateral Triangle A B C, whose Sides shall be Equal to a given Line O.

Fig.
XXI.

Practice. **M**Ake the Side B C equal to the given Line O, and with the same Distance of the Compasses, setting one Foot in B, with the other describe the Arch *m m*; and set also on C, describe the Arch *l l*, crossing the former Arches in A: Then join A B and A C, so have you constituted a Triangle A B C, whose Three Sides are severally equal to the given Line O.

P R O B. XIII.

To make a Triangle G H K, whose Three Sides shall be Equal to the Three given Right Lines D, E, F.

Fig.
XXII.

Practice. **F**irst, Make the Line G H equal to the given Line D.

2. Take the given Line E in your Compasses, and setting one Foot in H, with the other describe the Arch *n n*.

D —————
E —————
F —————

2. Take the Line F in your Compasses; and setting one Foot in G, with the other describe an Arch *o o*, crossing the former Arch in the Point K.

4. Join G K and H K; so shall you have constituted a Triangle; whose Three Sides G H, H K, and G K, are equal to the Three given Lines D E and F.

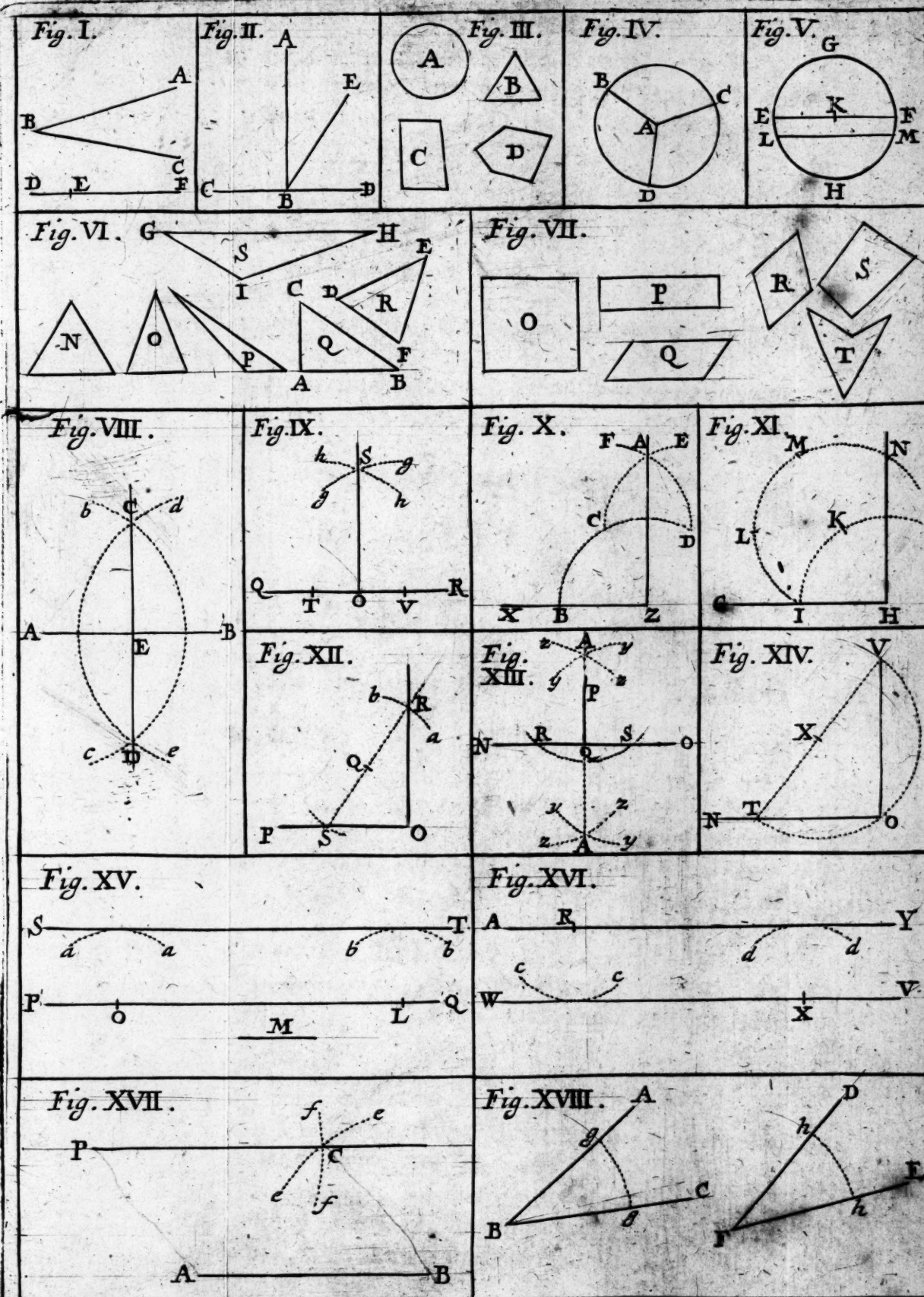
P R O B. XIV.

To make a Geometrical Square B C D E, whose Sides shall be equal to the given Line A.

Fig.
XXIII.

Practice. **F**irst, Make the Line B C for one Side of the Square, equal to the given Line A, and on one End thereof,

as



as on C (by Prob. IV.) erect the Perpendicular C D, equal also to the Line A.

2. With the same Distance of A, set one Foot in B, and with the other describe the Arch *a a*, and on D, and describe the Arch *b b*, crossing *a a* in the Point E.

3. Join B E and D E, which will constitute the Geometrical Square B C D E.

P R O B. XV.

To make a Parallelogram (or Long Square) G H I K, whose Length and Breadth shall be equal to Two given Lines E and F.

Practice. **F**irst, Make the Line G H equal to the given Line F, and on the end H, (by Prob. IV.) erect the Perpendicular H I, equal to the given Line E. Fig. XXIV.

2. With the Distance of the Line F, set one Foot in G, and with the other describe the Arch *c c*; and with the Distance of the Line E, with one Foot in I, describe the Arch *d d*, crossing *c c* in K.

3. Join I K and G K, and so have you formed the Parallelogram G H I K, whose Sides are equal to the given Lines E and F.

P R O B. XVI.

To make a Rhombus, M N O P, whose Four Sides shall be equal to a given Line L.

Practice. **F**irst, Make M N, (for one Side of the Rhombus) equal to the given Line L, and with that Length, set one Foot of the Compasses in N, and with the other describe the Arch M O P Q. Fig. XXV.

2. Set the same Distance upon this Arch, from N to O, and from O to P.

3. Join M O, O P, and P N, so shall you have constituted a Rhombus, whose Four Sides are all equal to the given Line L.

P R O B. XVII.

To make a Rhomboyades C D E F, whose Sides shall be equal to Two given Lines A and B; and the Acute Angles thereof C and E, equal to a given Angle Z.

Fig.
XXVI.

Practice. **F**irst, Make the Side of the Rhomboyades C D equal to the Line B.

2. On the end C, (by Prob. IX.) make an Angle F C D, equal to the given Angle Z; and the Side C F equal to the given Line A.

3. Upon the Point F, (with the Length of the given Line B) describe an Arch *h h*; and upon D, (with the Distance A) describe the Arch *g g*, crossing *h h* in the Point E.

4. Join F E and D E, and so have you constituted a Rhomboyades, whose Sides are equal to the Lines A and B, and its Acute Angles C and E, equal to the given Angle Z.

P R O B. XVIII.

To make a Trapezia H I K L, (or Figure of Four unequal Sides) which shall have one Angle at I, equal to a given Angle G; and the Four Sides equal to Four (possible) Right Lines given, viz. to the Lines O, D, E, F.

Fig.
XXVII.

Practice. **F**irst, Make the Line H I (for one of the Sides of the Trapezia) equal to one of the given Lines, as O.

2. Upon the Point I (by Prob. IX) make an Angle H I K equal to the given Angle G, making the Side K I equal to the given Line D.

3. Take another of the given Lines in your Compasses (as the Line E) and setting one Foot upon H, with the other describe the Arch *k k*; also take the fourth given Line F, and setting one Foot of the Compasses in K, with the other describe the Arch *m m*, crossing *k k* in the Point L.

4. Join H L and K L; so shall you have constituted a Trapezia, whose Four Sides are equal to the Four given Lines O, D, E, F; and one of the Angles, viz. I, equal to the given Angle G.

P R O B.

P R O B. XIX.

To divide a Circle A F C G, into any Number of Equal Parts;
not exceeding Ten.

- Practice.* **F**irst, Describe any Circle, and cross it with Two Diameters A C and F G, crossing each other at Right Angles (by Prob. I.) in the Centre E. Fig. XXVIII.
2. Make A B and A D, equal to E F, and join B D; so is B D the Third Part of the Circle.
 3. Join A and F; so will A F be the Fourth Part.
 4. Upon H, and Distance H F, describe the Arch F I, and join F I; which F I is the Fifth Part.
 5. E F, E G, E A, E C, either of them, are the Sixth Part.
 6. H D or H B are the Seventh Part.
 7. Draw a Line from the Centre E through the Point M, extending it to K; join K A, which be the Eighth Part.
 8. Divide the Arch D A B into Three Equal Parts at S, and join S D, which will be the Ninth Part.
 9. E I is the Tenth Part.

P R O B. XX.

To any Three Points given, A, B, C, (which are not in one Right Line,) to find a Centre O, upon which a Circle may be described, which shall pass through all the given Points A, B, C.

- Practice.* **F**irst, Set one Foot of Compasses in one of the given Points, as in A, and extend the other Foot to another of the given Points, as to B, and on A, with the Distance A B, describe an Arch of a Circle G F D. Fig. XXIX.
2. The Compasses open still to the same Distance, set one Foot in B, and with the other Foot cross the former Arch in the Points D and E, and through them draw the Right Line D E.
 3. Set one Foot of the Compasses in the third given Point C, (being still open to the former Distance) and with the other Foot cross the Arch first drawn in the Points F and G, through which Points draw the Right Line F G, which will cut the Line D E in the Point O, which is a Centre; on which,

4. If you set one Foot of the Compasses, and extend the other to any of the Three given Points, the Circle so described shall pass through all of them.

P R O B. XXI.

Two Points *X* and *Y*, within the Circle *ABD* being given, how to find the Centre of the Arch of a Great Circle *A X Y L*, which shall pass through those Two given Points *X* and *Y*.

Fig.
XXX.

Definition. A Great Circle of the Sphere, is such a Circle as divideth the Sphere or Globe into Two Equal Parts; and so the Arch of a Great Circle described upon a Plain, is such an Arch as divideth the Periferie or Circumference of the Fundamental or Primitive Circle, within which it is described, into Two Equal Parts:

Practice. LET *GBHD* be a Primitive Circle given, whose Centre is *C*, and let the Two Points within the same (through which the Arch of the Great Circle is to pass) be *X* and *Y*.

First, Through one of the given Points, as *X*, and the Centre *C*, draw a Right Line *XCE*, extending it infinitely towards *F*.

Secondly, Upon this Line, from the Centre *C*, erect the Perpendicular *CB*, and from *B*, through *X*, draw *BXG*; and from *G*, through *C*, draw the Diameter *GCH*.

Thirdly, Through the Points *B* and *H*, draw a Right Line, extending it till it cut the Line *XC* in *R*, so have you found a Third Point, viz. *R*, through which the Arch of the Great Circle must pass: And now, having Three Points, *X Y* and *R*, you may through them (by the last Problem) draw the Arch *AXYL R*, whose Centre will be at *K*; and this Arch doth divide the Primitive Circle into Two equal Parts in the Points *A* and *L*: And that it doth so is evident; for that the Right Line drawn from *A* to *L*, doth pass through the Centre *C*.

A Compendium.

It will often fall out that the third Point *R* will fall very remote from the Centre *C* of the Primitive Circle, notwithstanding (in all Cases) the Work may be performed without finding it at all.

For,

Having found the Points E and G, as before, take the Distance E B, and set it upon the Circle from G to F, and from F let fall a Perpendicular to C E, as F O, extending it (if Need be) infinitely towards M; for in some Point of that Line will the Centre be. And to find that Point,

Divide the Line supposed between the Two given Points, X and Y, into Two Equal Parts at Right Angles, and that Line extended will cut the Line O M in the Centre, as here in K, as before.

P R O B. XXII.

About a Triangle D E F, to describe a Circle.

Practice. **F**irst, (by Prob. I.) Divide any Two of the Sides of the Triangle, as D E and D F, each into Two equal Parts, at Right Angles in the Points G and H; through which Points the Lines *k k* and *m m* being drawn at Right Angles, will cross each other in the Point $\odot$, which will be the Centre of the Circle. Fig. XXXI.

P R O B. XXIII.

Within a Triangle G H K, to Inscribe a Circle.

Practice. **D**ivide any Two of the Angles of the given Triangle, as the Angles at G and H into Two equal Parts, (by Prob. X.) by the Lines H O and G P, crossing each other in the Point $\odot$, for that Point shall be the Centre upon which the Greatest Circle that the Triangle is capable to receive must be described. Fig. XXXII.

GEOMETRICAL THEOREMS.

I. *If a Right Line do fall upon Two Parallel Right Lines, it maketh the Alternate Angles Equal.*

Fig. **XXXIII.** *S*O the Right Line PQ , falling upon the Two Parallel Right Lines LM and NO , doth make the Alternate Angles Equal. As the Angle PRM , equal to the Angle NSQ ; and PRL , equal to QSO , *Elem. L. 1. P. 29.*

II. *If divers Right Lines, be cut by divers other Right Lines, which are Parallel one to the other, the Segments are Proportional.*

Fig. **XXXIV.** Let the Two Lines RS and RT , be cut by the Four Parallel Lines V, X, Y, Z . I say then, the inter Segment Ra and Rb ; as also aT and bS , are proportional one to the other: For if Ra be one third Part of RT , Rb shall be one Third of RS , &c. because the Right Line aX , cutteth off one third Part of the Space $ÆVTZ$, and therefore it cutteth off a third Part from every Line drawn within that Space.

III. *If Two Right Lines be multiplied into one another, there is made of them a Right Angled Parallelogram.*

Fig. **XXXV.** Let the Two Sides to be multiplied be $A 7$ and $B 9$. If of the Lines A and B , a rectangled Parallelogram be made, it will be such a Figure as $CDEF$: Or, if $A 7$ and $B 9$, be multiplied each by other, the Product will be 63 , and so many little Parallelograms are there contained in the larger Parallelogram $CDEF$.

IV. *If*

Fig. XIX.

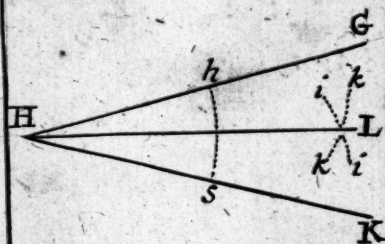


Fig. XX.

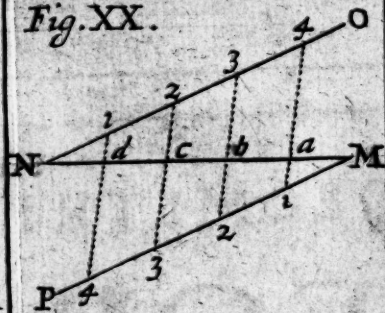


Fig. XXVII.

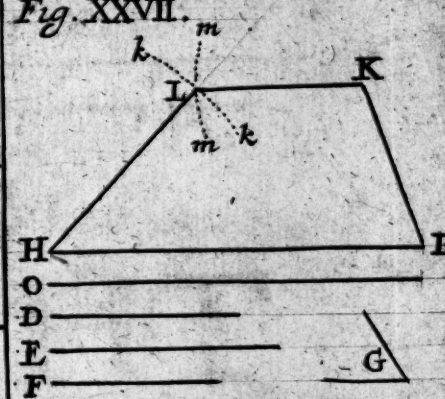


Fig. XXI.

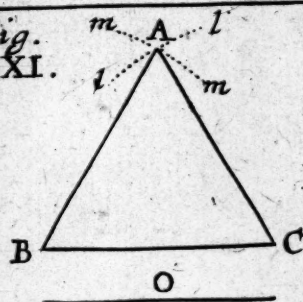


Fig. XXII.

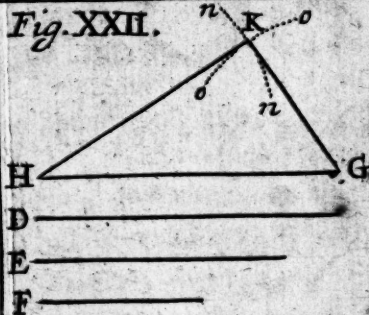


Fig. XXVIII.

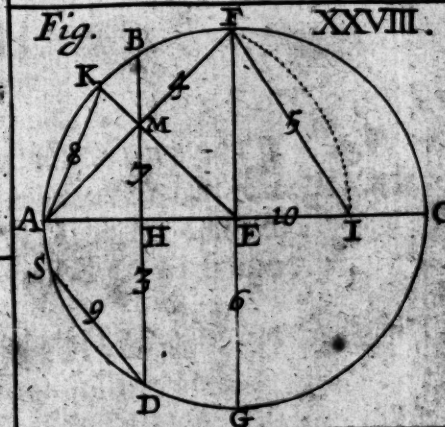


Fig. XXIII.

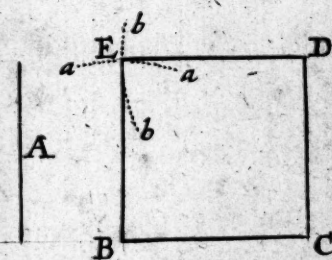


Fig. XXIV.

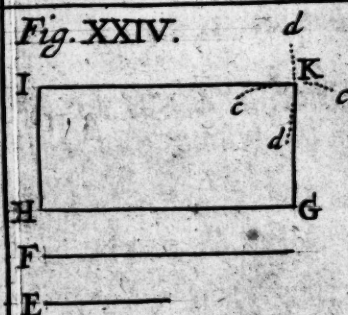


Fig. XXIX.

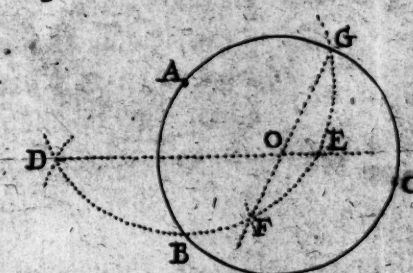


Fig. XXV.

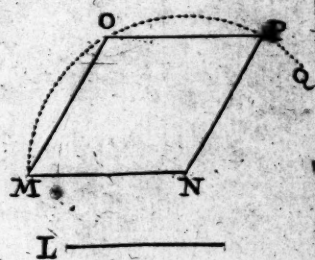


Fig. XXVI.

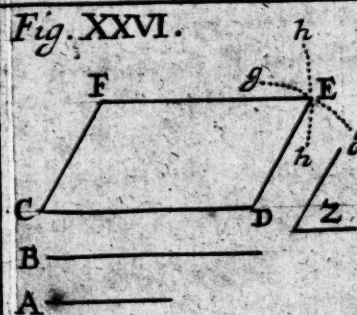


Fig. XXXI.

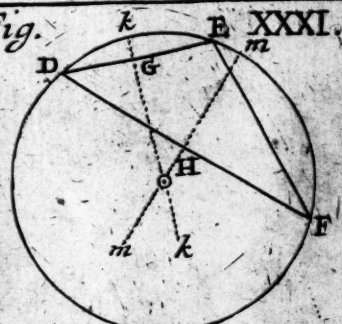


Fig. XXXII.

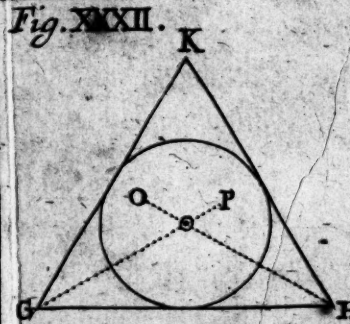
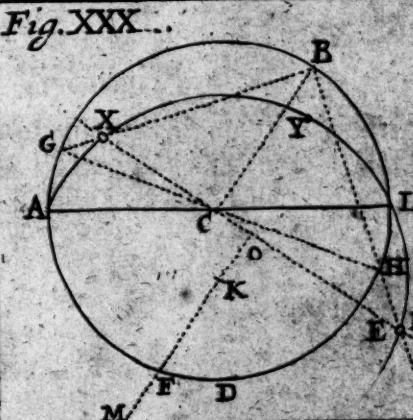


Fig. XXX.



IV. If Two Rectangled Figures, be made of One of the Sides of that Figure, and of any Two Segments of the other Side of the same Figure; those Two Parallelograms added together, shall be equal to that Figure.

So if the Parallelogram G L I M be made of the Side B 9, and a Segment of the other Side; namely, 4, that Parallelogram shall produce 36, for 9 times 4 is 36. And if another Parallelogram shall be made of the whole Side B 9, and the other Segment of the other Side A 7, namely, 3, they will produce the other Parallelogram L H M K 27, for 3 times 9 is 27: And these Two Parallelograms 36 and 27 added together; do make 63 equal to the first.

V. If Four Right Lines be proportional, (that is, as the first is to the second, so is the third to the fourth) the Right-angled Figure made of the Two Means, (or Middle Terms) shall be equal to the Rectangled Figure, made of the Two Extream Terms.

Let there be Four Proportionals; as N 4, O 6, P 8, Q 12. I Fig. say, that the Right angled Figure R S T V, made of the Two Means, O 6, and P 8, shall be equal to the Rectangled Figure X Y Z Æ, made of the Two Extreams, N 4, and Q 12: For, as 6 times 8 is 48, so 4 times 12 is 48 also. And from hence will follow

C O N S E C T A R Y I.

If Four Right Lines (or Numbers) be proportional; if Three of them be given, the Fourth is also given. For,

The Rectangle Figure made of the Two Means, being divided by one of the Extreams, the Quotient shall be the other Extream.

—As the Rectangle Figure, made of the Two Means 6 and 8, is 48, this divided by 4, the Lesser Extream, shall give you in the Quotient 12, for the Greater Extream.—Or, if the proportional Terms given had been (by Transposition) 4, 8, 6, 12, the Rectangle made of the Means 8 and 6 would be 48; which divided by 12, the Greater Extream, would give in the Quotient 4, the Lesser Extream.

CONSECTARY II.

Hence also it followeth, that equal Rectangled Figures, have their Sides reciprocally proportional: That is,

Fig. XXXVI. As the Lesser Side of the First Figure, is to the Lesser Side of the Second Figure; so is the Greater Side of the Second, to the Greater Side of the First, & Contra. As in the Equi-rectangular Figures R S T V, and X Y Z Æ, appeareth: For,

$$\begin{array}{l} \text{So is} \\ \text{As R T : is to X Y : : Y Æ : to R S} \\ \text{Or, As X Y : is to R T : : R S : to X Z} \end{array}$$

$$\begin{array}{ccccccc} 6 & : & 4 & : : & 12 & : & 8 \\ 4 & & 6 & & 8 & & 12 \end{array}$$

VI. If Three Lines or Numbers be proportional, (that is, as the First is to the Second, so shall the Second be to a Fourth) the Square made of the Means, is Equal to the Oblong made of the Extrems.

As let the proportional Numbers be 4 : 8 : 8, the Proportion will be

$$\text{As } 4 : \text{ is to } 8 : : \text{ So is } 8 : \text{ to } 16.$$

So the Square of the Two Means 8 is 64 : So also the Oblong made of the Two Extrems 4 in 16 is 64 also.

VII. In a Plain Triangle, a Line drawn Parallel to the Base, cutteth the Sides thereof proportionally.

Fig. XXXIV. As in the Plain Triangle R T S (in the Scheme of the Second hereof) if c d be Parallel to the Base T S, it cutteth off from the Side R S one third Part, and it cutteth from the Side R T one third Part also: And so they shall be proportional by the second hereof. For,

$$\begin{array}{l} \text{As R T : to R S : : So is R c : to R d. And} \\ \text{As R c : to c T : : So is R d : to d S. Also,} \\ \text{As R c : to R d : : So is c T : to d S.} \end{array}$$

VIII. If divers Plain Triangles be compared together, All Equi-angled Triangles have the Sides about (or containing) the Equal Angles proportional: And the contrary, Eucl. Lib. 6. P. 4.

For Illustration,

Fig. XXXVII. (1.) Let A B C and A D E be Two Plain and Equi-angled Triangles: So that the Angles at B and D, at A and A, and also

also at C and E, be *Equal* one to another, each to its Correspondent : I say, the *Sides* about the *Equal Angles* are *Proportional*: For,

1. As AB : is to BC :: So is AD : to DE .
2. As AB : is to AC :: So is AD : to DE .
3. As AC : is to CB :: So is AE : to ED .

DEMONSTRATION.

(2.) Because the *Angles* BAC and DAE are equal, therefore if AB be applied to AD , then AC shall of Necessity fall in AE ; and by such Application shall such a *Figure* be made.

In which *Figure*, because that AB and AC do meet together, and also the *Angles* at B and D , and at C and E , are *Equal*; therefore the other *Sides* BC and DE shall be *Parallel*, (*by the first hereof*). But in a plain *Triangle* a Right Line, *Parallel* to the *Base*, cutteth the *Sides* proportionally (*by the seventh hereof*); therefore in the *Triangle* ADE , the Right Line BC being *Parallel* to the *Base* DE , cutteth the *Sides* AD and AE proportionally; and therefore it follows, that

As AB : is to AD :: So is AC : to AE .

(3.) Again, By the Point B let the Right Line BF be drawn *Parallel* to the *Base* AE , and it shall cut the other Two *Sides* DA and DE proportionally in the Points B and F , (*by the last hereof*) and the Proportion will be

As AB : AD :: FE : DE ; or,
 AB : AD :: BC : DE .

For FE and BC are *Equal*, (*by the second hereof*). And since it is that,

As AB : AD :: AC : AE ,

And so BC to DE , they shall also be,

As AC : AE :: BC : DE .

For those Two Things, which are agreeable to a third Thing, are agreeable one to another: Therefore it generally follows,

(1.) As

Clavis Mathematica.

$$(1.) \text{ As } A B : A D :: B C : D E$$

$$(2.) \text{ As } A B : A D :: A C : A E$$

$$(3.) \text{ As } A C : A E :: B C : D E$$

Or, by transposing the *Second* and *Third Terms* of the *Proportion*, thus;

$$(1.) \text{ As } A B : B C :: A D : D E$$

$$(2.) \text{ As } A B : A C :: A D : A E$$

$$(3.) \text{ As } A B : B C :: A E : D E$$

And thus it is *demonstrated*, that all *Plain Equiangled Triangles* (as these, $A B C$ and $A D E$ are) have their *Sides* comprehending their *Equal Angles*, proportional.

IX. If *several Plain Triangles*, (*how many soever*) be *compounded*, and be *cut* by *Parallel Right Lines*, the *Inter-Segments* are *Proportional*. *As thus*:

Fig. XXXVIII. If the *Two Triangles* $H F I$ and $I F K$ be *compounded*, and be *cut* with the *Parallel Lines* $G L M$, and $H I K$, their *Inter-Segments* are *proportional*. For,

$$\begin{aligned} &\text{As } G L : H I :: L M : I K. \text{ Or,} \\ &\text{As } G L : L M :: H I : I K \end{aligned}$$

(*by the second and seventh hereof*) For the *Triangles* $F G L$ and $F H I$ are *Equiangled*, (*by the first hereof*) because $G L$ and $H I$ are *Parallel*: Therefore,

$$\text{As } F L : F I :: G L : H I$$

(*by the seventh hereof*) but (*by the same seventh hereof*)

$$\text{As } F L : F I :: L M : I K.$$

And, those that are agreeable to a third, are also agreeable between themselves. Therefore,

$$\text{As } G L : H I :: L M : I K.$$

X. If *any one Side* of a *Plain Triangle* be *continued*, the *outward Angle* (*made by that Continuation*) is *equal* to the *Two inward and opposite Angles* of the *same Triangle*.

Fig. XXXIX. If, in the *Plain Triangle* $N O P$, the *Side* $N P$ be *continued* to Q , the *outward Angle* $O P Q$ shall be *equal* to the *Two inward Angles*, $O N P$ and $N O P$: For, If

If from the Point P, a Right Line P R, be drawn *Parallel* to N O, the outward Angle O P Q shall be compounded of the Angles O P R, and R P Q; but the Angles R P Q and R P O, are equal to the Two inward Angles O N P and P O N; that is to say, the Angle R P Q, to the Angle O N P, and the Angle O P R, to the Angle N O P, (*by the first hereof*) because of the the *Parallels* N O and P R; and therefore the Angle O P Q is equal to the Two inward and opposite Angles, O N P and N O P: Which was to be demonstrated.

XI. The Three Angles of every Plain (or Right Lined) Triangle, are equal to Two Right Angles.

As in the Plain Triangle N O P (in the Diagram before of the tenth) I say, the Three Angles N O P, O P N and O N P, are together equal to Two Right Angles. Fig. XXXIX

For the Angles (how many soever) meeting in One Point, in one and the same Right Line, are equal to Two Right Angles.

But the Three Angles N O P, O P N and O N P, are equal to the Three Angles, meeting in the Point P, upon the same Right Line N Q. For the Angle O P N is common to both, and the Angles R P Q and N O P (*by the last.*) Therefore, the Three Angles N O P, O P N and O N P, are Equal to Two Right Angles. And from hence will follow these

C O R O L L A R I E S.

1. That there can be but One Right, or One Obtuse, Angle, in any Plain Triangle.

2. And if One Angle be Right, or Obtuse, the other Two shall be Acute.

3. That the Third Angle of any Plain Triangle is the Complement of the other Two to Two Right Angles.

4. That if Two Triangles be Equiangled in any Two of their Angles, they are wholly Equiangled.

XII. In every Right-angled Plain Triangle, The Square made of the Side which subtendeth (or is opposite to) the Right Angle, shall be Equal to both the Squares, which are made of the Two Sides which subtend the Right Angle. Eucl. Lib. 1. Pr. 47.

As in the Plain Right-angled Triangle S T V, Right-angled at T, I say, the Sides S T and T V, including the Right Angle S T V, are

are equal in Power to the Hypotenuse VS ; that is, the Squares of the Sides ST and TV ; namely, the Squares $SWT\text{Æ}$, and $VTBC$, added together, are equal to the Square of the Hypotenuse VS ; to wit, the Square $XYSV$.

For, if from the Right Angle at T , be let fall the Perpendicular TAZ , then out of the Square $XYSV$ is made Two Rect-angled Figures, $ASYZ$ and $AVXZ$, the Rectangle (or Oblong) $ASYZ$ equal to the Square $SWT\text{Æ}$, and the Rectangle $XZVA$, equal to the Square $VTBC$. — And therefore the Square $VXYZ$, compounded of those Two Oblongs, is equal to the Two Squares $VTBC$ and $ST\text{Æ}W$.

But that the Two Oblongs $AZY S$ and $AVXZ$ are equal to the Squares $SWT\text{Æ}$ and $VTBC$, shall thus be proved.

If Three Right Lines be Proportional, (as by the sixth hereof) the Square of the Means is equal to the Oblong made of the Two Extreams.

But the Three Right Lines SY , (equal to SV) ST and SA , are Proportional; that is,

$$\text{As } SY (=SV) : ST :: ST : SA :$$

Therefore the Square of ST is equal to the Oblong made of $SY (=SV)$ and SA ; for the Triangles STV and TAS are Equiangled, because of the common Angle at S , and the Two Right Angles at T and A : Therefore (by the eighth hereof)

$$\text{As } SV : ST :: ST : SA.$$

In the same manner it is also proved, that the Oblong $VXZA$ is equal to the Square $VTBC$: For the Triangles TSA and ATV are Equiangled, because of their common Angle at V , and the Two Right Angles at T and A . Therefore,

$$\text{As } SV : TV :: TV : AV.$$

And so (by the sixth hereof) the Square of TV is equal to the Oblong $ZXA V$. Which was to be demonstrated.

Hence

Hence it followeth: That,

If in a Right-angled Plain Triangle, any Two Sides be given, the Third may be said to be given also.

As if the *Two Sides* including the *Right Angle* T V 8, and T S 6 were given, their *Squares* 64 and 36 added together, make 100; the *Square Root* whereof is 10, for the *Third Side* (or *Hypotenuse*) S V — On the contrary, if the *Hypotenuse* S V 10, and one of the containing *Sides* T S 6, be given; substract the *Square* of 6, viz. 36, from the *Square* of 10, viz. 100, the *Remainder* will be 64; the *Square Root* whereof is 8, for the other containing *Side* T V. Fig. XL.

XIII. *In every Plain Triangle (as well Right as Oblique angled) the Sides are in Proportion one to the other, as are the Sines of the Angles opposite to those Sides, & contra.*

DEMONSTRATION.

Let the *Triangle* A B C, be inscribed in a *Circle*, and from the *Centre* D, draw the *Radii* D F, D E, D G; bisecting as well the *Peripheries* as the *Subtenses*; and let there be also drawn the *Radius* D C. Now, because the *Angle* at the *Centre* E D C is equal to the *Angle* in the *Peripherie* A B C, and C D F, equal to C A B (by the twentieth of the third of Eucl.) Therefore shall the *Halves* of the *Sides* be as *Sines*; and what *Proportion* the *Side* C A hath to the *Side* C B, the same shall the *Sine* H C, have to the *Sine* C I: For what *Proportion* the *Whole* hath to the *Whole*, the same shall the *Half* have to the *Half*. Which was to be demonstrated. Fig. XLI.

And from hence follow these

CONSECTARIES.

If the Angles of a Triangle be given, the Reason of the Sides is also given.

I. *If One Side be given, besides the Angles, both the other Sides are also given.*

E

III. If

III. If Two Sides of a Triangle be given, with an Angle opposite to one of them, the Angle opposite to the other of them is also Given.

Theorems Extraordinary.

I. If in a Circle Two Right Lines be inscribed, cutting each other, the Rectangles of the Segments of each Line, are equal: And the Angle at the Point of Intersection, is measured by the half Sum of its intercepted Arches.

II. If to a Circle Two Right Lines be adscribed from a Point without, the Rectangles of each Line from the Point assigned, to the Convex and Concave are equal: And the Angle at the assigned Point is measured by the half Difference of its intercepted Arches.

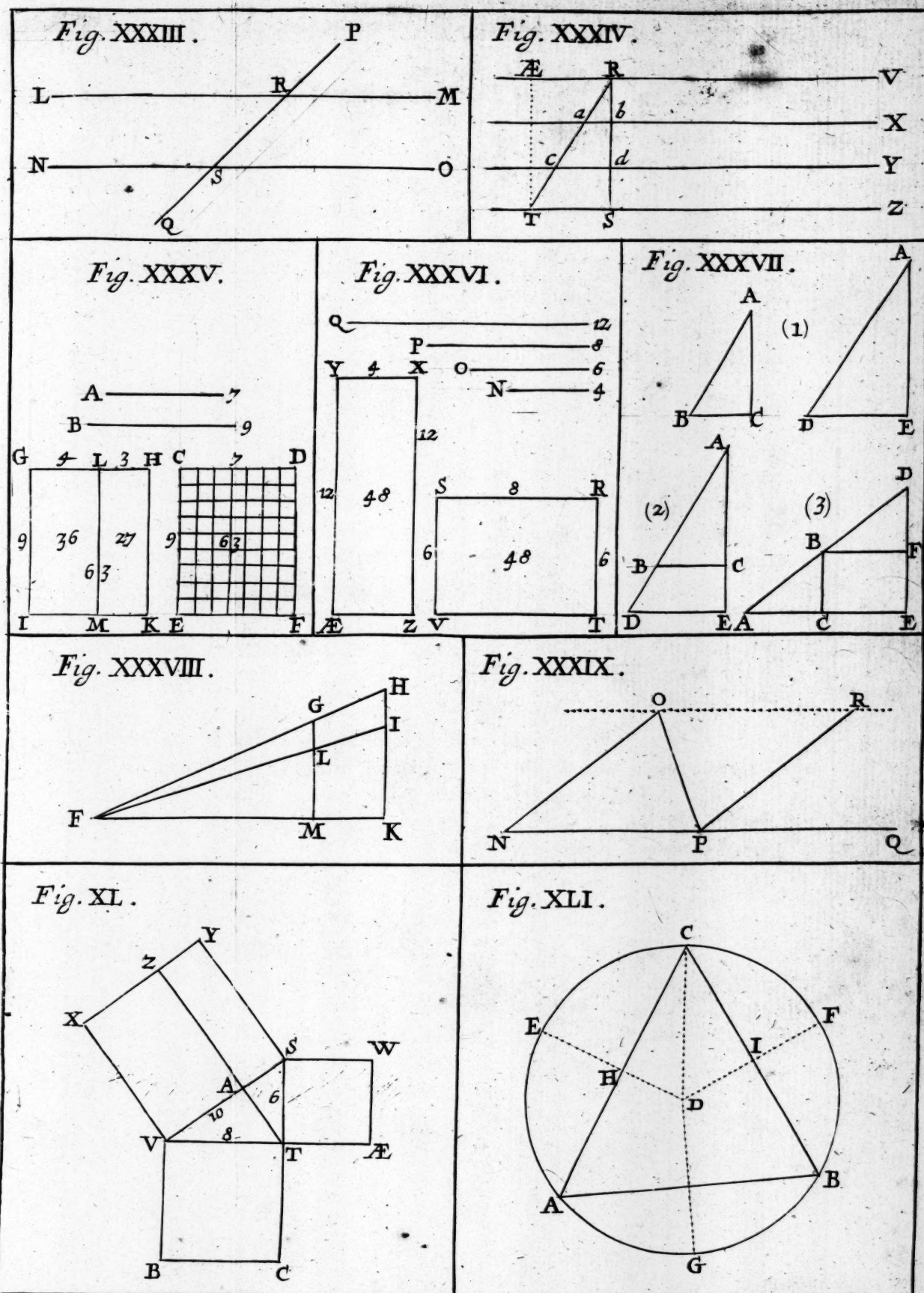
III. If in a Circle Three Right Lines shall be inscribed, one of them cutting the other Two: Then the Rectangles of the Segments of each Line, so cut, are directed proportional to the Rectangles of the respective Segments of the Cutter.

IV. If a Plain Triangle be inscribed in a Circle, the Angles are one half of what their opposite Sides do subtend: And if it hath one Right Angle, then the longest Side of that Triangle shall be the Diameter of the Circle.

V. If in a Circle, any Plain Triangle be inscribed, and a Perpendicular be let fall upon one of the Sides, from the opposite Angular Point. Then, as the Perpendicular, to one of the adjacent Sides; so is the other adjacent Side, to the Diameter of the circumscribing Circle.

The End of the First Part.

JANU A



JANUA MATHEMATICA.
THE
G A T E
TO THE
Mathematical Sciences
O P E N E D.

P A R T II.

O F
TRIGONOMETRY.
W H E R E I N

The Doctrine of the Dimension of
PLAIN and SPHERICAL TRIANGLES
is Succinctly Handled, Geometrically
Demonstrated,

And { Arithmetically,
Geometrically, } Performed.
Instrumentally, }

By *William Leybourn*, Philomathemat.

L O N D O N :
Printed *Anno Domini*, MDCCIV.

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JANUA MATHEMATICA.

SECTION I.

Of Plain (or Right Lined) Triangles.

CHAP. I.

DEFINITIONS.

OF *Triangles* there are Two Kinds; viz. *Plain*, (or *Right-lined*) and *Spherical*, (or *Circular*.) Either of which do consist of Six Parts; namely, of Three *Sides*, and as many *Angles*; but in this Place we shall only treat of the *Plain*.

I. A *Plain* (or *Right Lined*) *Triangle* consisteth of Three *Sides*, and as many *Angles*: And such are the Two Figures *CBA* and *CDB*; in which, in the first Figure *AB*, *BC* and *CA*, are the Three *Sides* of the *Triangle CBA*; and *CAB*, *ACB* and *BCA*, are the Three *Angles* of the same *Triangle CBA*. Fig. I.

¶ And Note here] That an *Angle* (in any Case) is always noted with Three Letters; the middlemost whereof represents the *Angular Point*. So in the *Triangle ABC*, if I would express the *Angle* at *C*, I would say, The *Angle ACB*; or, The *Angle BCA*.

Also in the second Figure *DBC*, the Lines *DC*, *DB* and *CB*, are the Three *Sides* of the *Triangle DBC*; and the *Angles BDC*, *CBD* and *BCD*, are the *Angles* of the same *Triangle DBC*.

II. Any

Fig. I.

II. Any Two Sides of a Triangle, are called the Sides of that Angle contained by (or comprehended between) them: So the Sides B C and C A are the Sides containing the Angle B C A.

III. Every Side of a Triangle, is the subtending Side of that Angle which is opposite unto it. As in the Triangle A B C, B C is the subtending Side of the Angle C A B, the Side B A subtends the Angle B C A, and the Side C A subtends the Angle C B A.

¶ And here Note again,] That the greatest Side, always, subtends the greatest Angle, the lesser Side the lesser Angle, and equal Sides subtend equal Angles.

IV. The Measure of an Angle is the Arch of a Circle described upon the Angular Point, and is intercepted between the Two Sides containing the Angle, (increasing the Sides, if Need be). So in the Triangle A B C, the Measure of the Angle C B A, is Arch C F.

Fig. II.

V. Every Circle is divided into 360 Degrees, and every Degree into 60 Minutes, (or rather into 100 or 1000 Parts, &c.) Which Degrees are so much the Greater, by how much the Circle is Greater; and those Arches which contain the same Number of Degrees in equal Circles, are Equal: But in unequal Circles they are termed Like-Arches. So the Arches C F and D E are Equal Arches, they being equal Parts of the same Circle D H F K. But the Arches C F and O P are Like-Arches: For, as C F is 40 Parts (or Degrees) of the Greater Circle D E F C, so is O P 40 Degrees of the Lesser Circle P O G.

VI. A Quadrant (or Quarter) of a Circle, is an Arch of 90 Degrees. As is the Quadrant (or Arch) H F.

VII. The Complement of an Arch less than a Quadrant, is so much as that Arch wanteth of 90 Degrees. So the Complement of the Arch C F 40 Degrees, is the Arch H C 50 Degrees.

VIII. The Excess of an Arch Greater than a Quadrant, is so many Degrees as that Arch exceedeth 90 Degrees. So the Arch D H C being 140 Degrees, is the Arch H C, 50 Degrees more than the Quadrant D H.

IX. A Semicircle is an Arch of 180 Degrees. As is the Semicircle D H F.

X. The Complement of an Arch less than a Semicircle, to a Semicircle, is so much as that Arch wants of 180 Degrees. So the Complement of the Arch D H C 140 Degrees, to the Semicircle D H F 180 Degrees, is the Arch C F 40 Degrees.

XI. The

Of Right Lined Triangles.

31

XI. The opposite Angles made by the crossing of Two Diameters in a Circle, (or any Two other Right Lines crossing each other) are equal. So the Angles C B F and C B E, (made by the Intersection of the Two Diameters D F and C E in the Center B) are equal. Fig. II.

XII. An Angle is either Right or Oblique.

XIII. A Right Angle is that whose Measure is a Quadrant or 90 Deg. So the Angles H B D, and H B F, are Right Angles, their Measures being the Quadrants D H and H F.

XIV. All Oblique Angles are either Acute or Obtuse.

XV. An Acute Angle, is that whose Measure is less than 90 Deg. So the Angles H B C 50 Deg. and C B F 40, are Oblique Acute Angles.

XVI. An Obtuse Angle, is that whose Measure is more than a Quadrant or 90 Deg. So the Angle D B C (consisting of 90 and 50 Deg. viz. of 140 Deg) is an Oblique Obtuse Angle.

XVII. The Complements of Angles are the same, as are the Complements of Arches.

XVIII. All Angles concurring (or meeting) together upon One Right Line, all of them being taken together, are equal to a Semicircle, or 180 Deg. So the Angle D B H 90 Deg. H B C 50 Deg. and C B F 40 Deg. (made by the concurring, or meeting, of the Three Lines D B, H B and C B, upon the Diameter D F, in the Center B) are all of them Equal to the Semicircle D H C F, or 180 Deg.

XIX. A Triangle hath some of its Sides Equal, or else they be all Unequal.

XX. A Triangle of some Equal Sides is either Equicrural or Equilateral.

XXI. An Equicrural Triangle is that which hath only Two Equal Sides. And such is the Triangle D B E, whose Sides B D and B E are Equal.

XXII. An Equicrural Triangle is Equi-angled at the Base. So in the Equicrural Triangle B D E, the Angles B D E and B E D, at the Base D E, are Equal, viz. each of them 70 Deg. for the Angle D B E being 40 Deg. that taken from 180 Deg. leaves 140 Deg. the half, whereof 70 Deg. is equal to the Angle B D E or D E B, and all the Three Angles equal to 180 Deg. or Two Right Angles.

This is Demonstrated in the XIth Theorem hereof.

XXIII. An

Fig. II.

XXIII. *An Equilateral Triangle, is that whose Sides are all Equal, and whose Angles contain (each of them) 60 Deg. So the Triangle E B K hath its Sides B E, B K and E K, all of them equal; and the Angles E B K, E K B and K E B equal also, and each of them equal to 60 Deg. and consequently all of them equal to 180 Deg.*

XXIV. *A Triangle is either Right Angled or Oblique Angled.*

XXV. *A Right-angled Triangle, is that which hath one Right Angle. And such is the Triangle C A B, Right-angled at A.*

XXVI. *An Oblique-angled Triangle is that which hath all its Angles Oblique. And such is the Triangle B C D.*

XXVII. *An Oblique-angled Triangle, is either Acute-angled, or Obtuse-angled.*

XXVIII. *An Oblique Acute-angled Triangle is that which hath all its Three Angles Acute. And such are the Triangles D B E, and E B K.*

XXIX. *An Oblique Obtuse-angled Triangle is that which hath One Obtuse, and Two Acute Angles. And such is the Triangle D B C, whose Angle D B C is Obtuse, and the Angles B D C and B C D Acute.*

C H A P. II.

Of Right Lines, applied to a Circle.

FOrasmuch as the Ratio or Proportion of an Arch Line to a Right Line, is as yet unknown, yet it is absolutely necessary that Right Lines be applied to a Circle, for the Calculation of Triangles, wherein Arch Lines come in Competition: For the Angles of Plain (or Right-lined) Triangles are measured by Arches of Circles.

Now, the Right Lines applied (or relating) to a Circle, are Chords, Sines, Tangents, Secants and Versed Sines.

Fig. III.

1. A Chord, or Subtense, is a Right Line, joining the Extremities of an Ark, as A C is the Chord of the Arks A B C and A D C.

2. A Right Sine, which is singly called a Sine, is a Right Line, drawn from one end of an Ark, perpendicular to the Diameter drawn through to the other End: Or, it is half the Chord of twice the Ark; so A E is the Right Sine of the Arks A B and A D. The Radius (or Sine of 90 Deg.) is called the Whole Sine,

Sine, and is the greatest of all *Sines*: For the *Sine* of an *Ark* greater than a Quadrant, is less than the *Radius*; so FG is the whole *Sine* or *Radius*.

3. A *Versed Sine* is the Segment of the *Radius* between the *Ark* and its *Right Sine*; so EB is the *Versed Sine* of the *Ark* AB , and of the *Ark* AGD .

4. The *Secant* of an *Ark*, is a *Right Line*, drawn from the Center through one end of an *Ark*, till it meet with the *Tangent*: That is, a *Right Line* touching the Circle at the nearest end of that Diameter which cuts the other end of the *Ark*. FM is the *Secant*, and BM the *Tangent*, of the *Ark* AB , or of AD .

5. The *Difference* of an *Ark* from a Quadrant, (or 90 Deg.) whether it be Greater or Less, is called the *Complement* of that *Ark*; so GA is the *Complement* of the *Arks* AB and AGD , and HA is the *Sine* of that *Complement*: GI the *Tangent* of that *Complement*: and FI the *Secant* of that *Complement*. All which (for Brevity) we write *Co-Sine*, *Co-Tangent*, *Co-Secant* of the *Ark*.

6. The *Difference* of an *Ark* from a Semicircle (or 180 Deg.) is called its *Supplement*; so the *Ark* AB is the *Supplement* of the *Ark* DGA , to a Semicircle.

7. That Part of the *Radius* which is between the Centre and its *Right Sine*, is equal to the *Co-Sine*. As FE is equal to HA , and FO is equal to the *Co-Sine* of the *Ark* DS .

8. If an *Ark* be Greater or Less than a Quadrant, the Sum or Difference, accordingly, of the *Radius* and *Co-Sine*, is equal to the *Versed Sine*: For FD and HA together, are equal to DE , the *Versed Sine* of the *Ark* DGA ; and FB less by HA (or EF) is equal to EB , which is the *Versed Sine* of the *Ark* AB .

C H A P. III.

Some Short Problems, to make Canons of Sines, Tangents and Secants.

P R O B. I.

The Sine of an Ark being given, how to find out the Sine of the Complement.

B C being given, to find A C.

Fig. IV. **B**Ecause the Triangle A C B is a Rectangle (by the Definition of a Sine) and the Sides A C, B C, are of the same Power as the Hypotenuse; that is, the Radius A B: Therefore, if the Square of the Sine B C be subtracted from the Square of the Radius A B, there remains the Square of A C, whose Side is the Right Line A C, the Sine sought for.

P R O B. II.

The Sine of an Ark being given, and likewise the Sine of the Complement, to find the Sine of half the Ark.

R Q and A Q being given, to find B O or R O.

Fig. V. As A B : is to B O, so is :: B O : to B G; therefore B O will be the Square Side of the Plain from the Radius A B and B G, the half Versed Sine. For Q B, the Versed Sine of the Ark B R, is given; because A Q, the Sine of the Complement, and A B, the Radius, are given by the Supposition.

P R O B. III.

The Sines of Two Arks, and the Sines of the Complements being given, to find the Sine of the Sum.

R Q, Q A, and S T, T A, being given, to find S P.

Fig. VI. As A R is to R Q, so is A T to T G, or C P.
As A R is to A Q, so is S T to S C.

B T and C P together make S P, the Sine of the Sum of Two Arks.

P R O B.

Of Right Lined Triangles.

35
Fig. VI.

P R O B. IV.

The same being given, to find the Sine of the Difference.

R Q, Q A, and S P, P A, are given, to find S T.

As A Q is to Q R, so is A P to P O, from whence you may find O S. As A R is to A Q, so is O S to S T.

To these join the *Theorems*.

Theorem I. *The least Sines are, almost, in the same Ratio as their Arks.*

This *Theorem* will be prov'd to be true hereafter in the continued Bisections; but the least *Arks* are about one of the first Scruples, or less, and are almost in the same *Ratio* as their *Sines*, because they are, almost, contiguous amongst themselves, and almost of the same Quantity, as it appears, tho' not to every Search, yet to a very profound one.

Theorem II. *If the same Line be cut into unequal Parts in Number, the Number of the Parts of the first Section, is to the Number of the second, (reciprocally) as one Part of the second Section is to one Part of the first Section.*

Divide the same *Line*, first into Four, then into Three Parts; then it will be as 4 to 3, so $\frac{1}{4}$ Part to $\frac{1}{3}$ reciprocally; the Reason is, because 3 in $\frac{1}{4}$ makes 1, and 4 in $\frac{1}{3}$ makes 1; and because the Products are equal, the Multiplicands will be reciprocally proportionable.

The Construction of the Canon of Sines.

The *Sine* of the whole Quadrant is call'd the *Radius*, for it is the Semidiameter of a Circle. Set in the *Canon* a *Radius* of 100000 Parts, or 100000.00 for Necessity of Calculation; but for the better making of the *Canon* take a *Radius* 100000.00000 Parts; for by that Means, the Errors which oftentimes happen amongst the Right Hand Figures may safely be corrected without any Prejudice to the *Canon*.

F 2

Then

Fig. VI.

Then biseſt the Quadrant, and look for the *Sine* of the *Bisegment* by *Prob. 2.* and its *Co-sine* by *Prob. 1.* then biseſt this *Bisegment* again, and look for the *Sine* of the second *Bisegment* by *Prob. 2.* and the *Co-sine* by *Prob. 1.* then biseſt this second *Bisegment*, and look for its *Sine* or *Co-sine* by *Prob. 2.* and *1.* and then biseſt the third *Bisegment*, &c. and continue biseſting 13 times, till you find a *Sine* of $\frac{1}{8192}$ Part of the whole Quadrant, as 'tis here ſet in the *Table*. Now we come to the least *Arks*, where the Truth of the first *Theorem* is illuſtrated: For as the *Ark* of the Quadrant $\frac{1}{4096}$ is double to the *Ark* $\frac{1}{8192}$, ſo is its *Sine* almoſt to that *Sine*.

The <i>Sine</i> of the Quadrant.	100000.00000
The <i>Sine</i> of $\frac{1}{2}$ of the Quad.	70710 67811 ✕
The <i>Sine</i> of $\frac{1}{4}$ of the Quad.	38268.34323 ✕
The <i>Sine</i> of $\frac{1}{8}$ Part of the Q.	19509.03220 ✕
The <i>Sine</i> of $\frac{1}{16}$ Part of the Q.	9801.71403 ✕
$\frac{1}{32}$	
$\frac{1}{64}$	
$\frac{1}{128}$	
$\frac{1}{256}$	
$\frac{1}{512}$	
$\frac{1}{1024}$	
$\frac{1}{2048}$	
$\frac{1}{4096}$	
$\frac{1}{8192}$	

This

Of Right Lined Triangles.

37

Fig. VI.

This least *Sine* being thus found out, the *Sine* of one of the first Scruples, that is, of $\frac{1}{60}$ part of the whole Quadrant, or of one hundredth part of a Degree; that is, of $\frac{1}{100}$ part of the whole Quadrant is to be found. Therefore by the *second Theorem*, as $\frac{5400}{100}$ is to 8192, so the quantity of the first Part of this Division is to the quantity of the first Part of that Division, and by the first *Theorem*; so is the *Sine* of $\frac{1}{8192}$ Part, which you have in the Table at the *Sine* of $\left\{ \frac{1}{100} \right\}$ part of Degree.

Therefore the *Sine* of the first Minute, or of the first hundredth part thus form'd by *Prob. 1.* extract the *Sine* of the Complement, to wit, of the *Ark* 89 Deg. $\left\{ \frac{5400}{100} \right\}$ then by *Prob. 3.* find out the *Sine* of the second Minute, and its *Co-sine* by *Prob. 1.* and from thence you will find the *Sine* of the Sum of 2 Min. and 1 Min. that is 3 Min. by *Prob. 3.* and its *Co-sine* by *Prob. 1.* and from the *Sine* and *Co-sine* 2 Min. or from the *Sines* and *Co-sines* 3 Min. and 1 Min. you may look for the 4th *Sine* by *Prob. 3.* and the *Sine* of the Complement by *Prob. 1.* Likewise from the *Sines* and *Co-sines* of 2 Min. and 3 Min. or 4 Min. and 1 Min. you may find the 5th *Sine* and *Co-sine* by *Prob. 3.* and 1. &c. even to $\left\{ \frac{5400}{100} \right\}$ or 1 Deg. and from the *Sine* of 1 Deg. you may by the same means find all the *Sines* of the 90 whole Deg. and from the *Sines* and *Co-sines*, before found, of 60 single Minutes, it will be easie by 3d *Prob.* taking the 4th *Prob.* too, when it will be useful, to extract the single *Sines* of all the single Minutes interspers'd.

A Deduction of the Tangents and Secants from the Tables of Sines.

The *Tangents* are made thus:

As A C, the *Co-sine*, is to C B the *Sine*; so is A E, the *Radius*, Fig. VII. to E D the *Tangent*.

But the *Secants* thus:

As A C, the *Co-sine*, is to A B the *Radius*; so is A E, the *Radius*, to A D the *Secant*.

By this way the whole *Canons* of *Tangents* and *Secants* are extracted from the *Canon* of *Sines*.

I pass

Fig. VII. I pass by all Rules of Calculation, for I don't undertake to make new Canons; forasmuch as some of the most excellent Artists, by their Study and Industry have sav'd me that Trouble, it sufficiently answers my Design, if the Reason of *Syntax*, whatever it be, be only understood, and the Truth of the Numbers put in the *Canon* which the Propositions above abundantly prove.

C H A P. IV.

Of the Affections of Right-lined Triangles, in order to the Calculating (or Resolving) of them.

1. **A** Plain Triangle is contained under Three Right Lines, and is either *Right-angled*, or *Oblique*.

2. In all Plain Triangles, Two Angles being given, the third is also given; and One Angle being given, the Sum of the other Two is given; because, *The Three Angles together are equal to Two Right Angles*. By the 32 Pro. Euclide. Lib. I. and by the 11th of Sect. I. hereof. Therefore,

In a Plain Right-angled Triangle, One of the Acute Angles is the Complement of the other to 90 Deg.

3. In the Resolution of Plain Triangles, the Angles only being given, the Sides cannot be found; but only the (*Reason*, or) Proportion of them: It is therefore necessary that one of the Sides be known.

4. In a Right-angled Triangle, Two Terms (besides the Right Angle) will serve to find the third; so that One of the Terms be a Side.

5. In Oblique-angled Plain Triangles there must be Three Things given to find a fourth.

6. In Right-angled Plain Triangles there are Seven Cases, and Five in Oblique: For the Solution of which, the Four following AXIOMS are sufficient.

AXIOME

Of Right Lined Triangles.

39

AXIOME I.

In a Right-angled Plain Triangle: *The Rectangle made of Radius, and one of the Sides containing the Right Angle, is equal to the Rectangle, made of the other containing Side, and the Tangent of the Angle thereunto adjacent.*

DEMONSTRATION.

In the Right-angled Plain Triangle B E D, draw the Arch F E; Fig. VIII. then is B E Radius, and D the Tangent of the Angle at B: Make C A parallel to D E, then are the Triangles A B C and B D E like, because of their Right Angles at A and E, and their common Angle at B. Therefore,

$$\text{As } B A : B E :: A C : E D.$$

And, B A in E D is equal to B E in A C.

That is, B A in $\angle$ B is equal to Radius in A C.

Which was to be demonstrated.

AXIOME II.

In all Plain Triangles: *The Sides are proportional to the Sines of their opposite Angles.*

DEMONSTRATION.

In the Plain Oblique Triangle C B D, extend B C to F, making Fig. IX. B F equal to D C, and describe the Arches F G and C H; then are the Perpendiculars F E and C A, the Sines of the Angles at D and B; and the Triangles B E F and B A C are like, because of their Right Angles at E and A, and their common Angle at B. Therefore,

$$\text{As } B C : C A :: B F : F E.$$

That is,

$$\text{As } B C : \angle D :: D C \text{ (equal to } B F) : \angle B.$$

Which was to be demonstrated.

AXIOME

A X I O M E III.

In all Plain Triangles : *As the half Sum of the Sides, is to their half Difference; so is the Tangent of the half Sum of their opposite Angles, to the Tangent of their half Difference.*

Otherwise,

In every Plain Triangle: *As the Sum of the Two Sides, is to their Difference; so is the Tangent of the half Sum of the Opposite Angles, to the Tangent of half their Difference.*

D E M O N S T R A T I O N.

Fig. X. In the Oblique angular Triangle A B C, let the known Sides be B A and B C; and the Angle A B C, comprehended by them: Where it is *Obtuse* in the *superior*, but *Acute* in the *inferior*, Diagram.

Continue the Side A B to H; so that H B may be equal to B C, and join C and H, —and make B I equal to A B: Also, from the Points B and I, draw the Right Lines B D and I G, parallel to the Side A C.

Then shall the exterior Angle C B H be equal to the Two interior and opposite Angles (*by the 32th of the 1st Euclid.*) For the Angle C B D is equal to the Angle A C B; and the Angle D B H, to the Angle C A B.

(Moreover, from the Point B, let fall a Perpendicular B E, which shall bise^t C H at the Point E; then making B E the Radius, upon B, describe the Arch M E L. Therefore shall C E be the *Tangent* of half the *Sum* of the opposite *Angles*: And D E (to which F E is equal) the *Tangent* of half their *Difference*.

Now, because A C, B D, and I G, are *Parallel*, therefore shall C D, D G, F H, be *Equal*: As also, D F and G H: Therefore I say,

By Equality of Proportion:

As A H, the Sum of the Two Sides,
Is to I H, their Difference;
So is C E, the Tangent of half the opposite Angles,
To D E, the Tangent of half their Difference.

Otherwise,

Of Right Lined Triangles.

41

Fig. VIII.

Otherwise,

As the Sum of the Two Sides,
Is to the greater Side doubled;
So is the Tangent of half the Sum of the opposite Angles,
To the Sum of the Tangents of the half Sum, and half Difference of the Angles.

Otherwise,

As the Sum of the Two Sides,
Is to the lesser Side doubled;
So is the Tangent of half the Sum of the opposite Angles,
To the Difference of the Tangents of the half Sum, and the half Difference of the Angles.

A X I O M E IV.

In all Plain Triangles: *As the Base, is to the Sum of the other Sides; so is the Difference of those Sides, to the Difference of the Segments of the Base.*

D E M O N S T R A T I O N.

In the Triangle BCD, let fall the Perpendicular CA; extend BC to F, and draw FG and DH. Fig. XI.

Then is BF equal to the Sum of the Sides CD and CB; and HB equal to the Difference of those Sides; and GB is equal to the Difference between AB and AD, the Segments of DB, the Base: And the Triangles HDB and BGF are like; because of their equal Angles at D and F; the Arch HG being the double Measure of them both: And their common Angle at B.

Therefore,

As BD : BF :: HB : GB.

That is,

As DB : FB (the Sum of the Sides) :: HB (the Difference of the Sides CB and CD) : GB (the Difference of the Segments of the Base.)

G

AD.

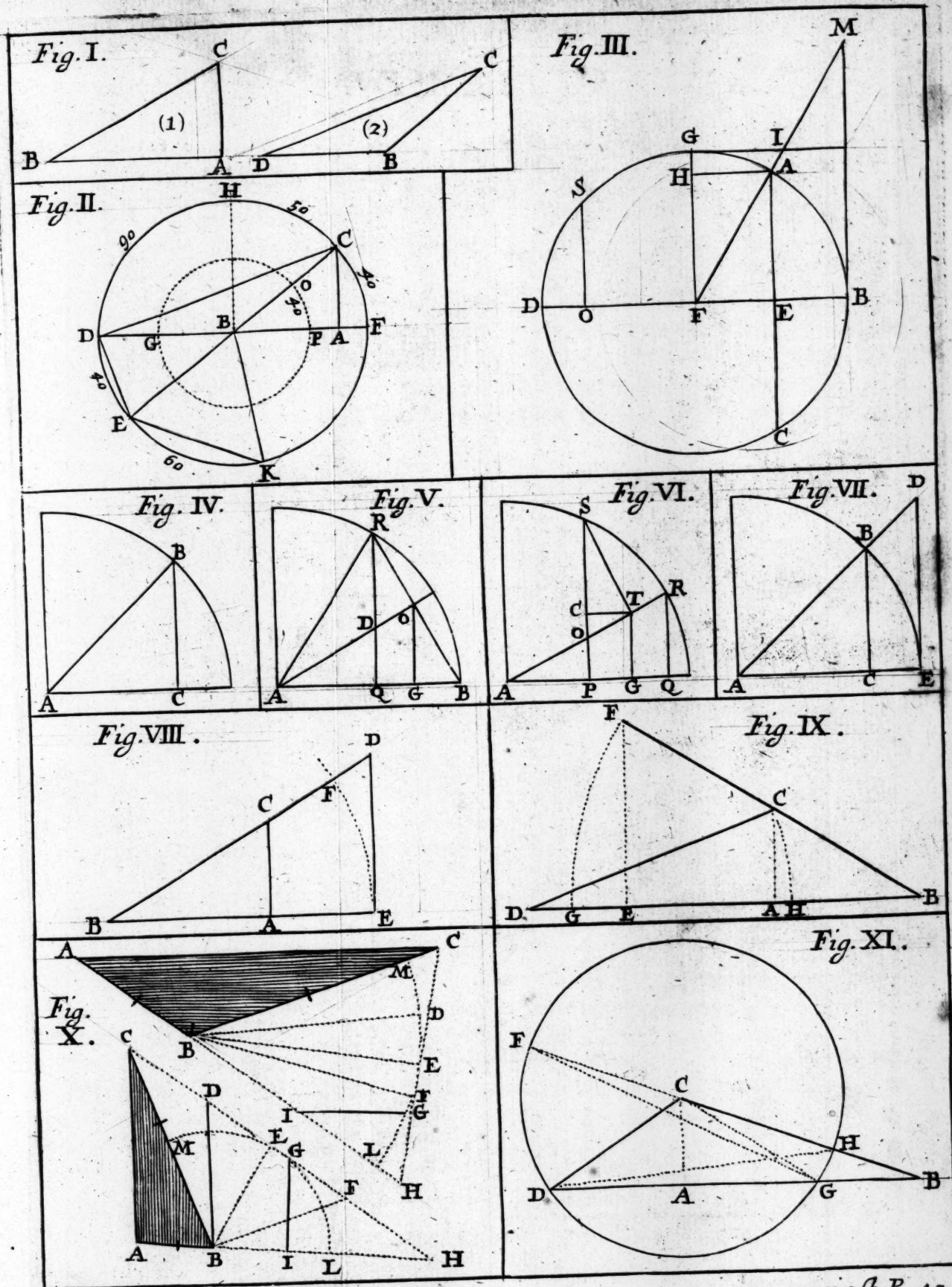
ADVERTISEMENT.

A Table to Reduce Sexagenary Minutes, to Centesimal Parts; and the contrary.

<i>Sexag. Minut.</i>	<i>Cent. Parts.</i>	<i>Sexag. Minut.</i>	<i>Cent. Parts.</i>
1	.01.67	31	.51.67
2	.03.33	32	.53.33
3	.05.	33	.55.
4	.06.67	34	.56.67
5	.08.33	35	.58.33
6	.10.	36	.60.
7	.11.67	37	.61.67
8	.13.33	38	.63.33
9	.15.	39	.65.
10	.16.67	40	.66.67
11	.18.33	41	.68.33
12	.20.	42	.70.
13	.21.67	43	.71.67
14	.23.33	44	.73.33
15	.25.	45	.75.
16	.26.67	46	.76.67
17	.28.33	47	.78.33
18	.30.	48	.80.
19	.31.67	49	.81.67
20	.33.33	50	.83.33
21	.35.	51	.85.
22	.36.67	52	.86.67
23	.38.33	53	.88.33
24	.40.	54	.90.
25	.41.67	55	.91.67
26	.43.33	56	.93.33
27	.45.	57	.95.
28	.46.67	58	.96.67
29	.48.33	59	.98.33
30	.50.	60	1.00.00

Whereas in this *Part* of this *Book*, which treateth of *Trigonometry*, where the *Angles* of *Right-lined*, and the *Sides* and *Angles* of *Spherical Triangles* are measured by *Arches* of *Circles*; and those *Arches* are usually numbred (or accounted) by *Degrees*, *Minutes*, *Seconds*, &c. Of which one *Degree* contains 60 *Minutes*, One *Minute* 60 *Seconds*, &c. which is called the *Sexary Division* of the *Degree*. Or otherwise, the *Degree* is supposed to be divided into 100 or 1000 *Parts*, which is called the *Centesimal*, *Millesimal*, &c. *Division*, (and is the better of the Two in many *Respects*.)

Now, whereas in the following *Trigonometrical Calculations* of this *Book*, and also in the other *Parts* (which concern the *Doctrine of Triangles* applied to *Practice* in several *Parts* of the *Mathematicks*) I have, sometimes, used the *Sexagenary*, and sometimes the *Centesimal*, Way of *Dividing*.



CHAP. V.

IN *Right-angled Plain Triangles*, I call those *Sides*, which comprehend the *Right Angle*, the *Legs*; and the *Side* subtending the *Right Angle*, I call the *Hypotenuse*; and the *Triangle* which I shall make use of in the *Resolving* the Seven CASES following shall be that *Diagram* noted with M, in Fig. XII. Whose *Sides* and *Angles* are,

Diagram
M.

Angles $\left\{ \begin{array}{l} A \quad 90.00 \\ B \quad 31.27 \\ C \quad 58.73 \end{array} \right\}$ Deg. and Cent.

P R O P O R T I O N.

As Log. A C : Radius :: Log. A B : t C. [By *Axi. I.*
G 2
O P E.

O P E R A T I O N.

As Log. C A, 142.72 Parts	2.1544848
Is to Rad. Tang. 45 Deg.	<u>10.</u>
So is Log. A B, 235.00 Parts	12.3710678
To Tangent C, 58.73 Deg.	<u>10.2165830</u>

Whose Compl. 31.27 Deg. is the Angle at B.

Fig. XII. C A S E II. The Angles C and B, and the Leg. A B given; To find the Leg. A C.

P R O P O R T I O N.

As Rad. : Leg. A B : : t B : Log. A C. [By Axi. I.

O P E R A T I O N.

As Radius, Tang. 45 Deg.	10.
To Log. A B, 235.00 Parts	<u>2.3710678</u>
So Tang. B, 31.27 Deg.	9.7883979
To Log. A C, 142.72 Parts	<u>12.1544657</u>

Fig. XII. C A S E III. The Hypotenuse B C, and the Leg. A B given; To find the Angle C.

P R O P O R T I O N.

As Leg. B C : Rad. : : Leg. A B. s C. [By Axi. II.

O P E R A T I O N.

As Hypo. B C, 274.93	2.4392537
Is to Radius, Sine 90 Deg.	<u>10.</u>
So is Leg. A B, 235.00	12.3710678
To Sine C, 58.73 Deg.	<u>3.9318141</u>

Fig. XII. C A S E IV. The Hypotenuse B C, and the Angles B and C, given; To find the Leg. A B.

P R O P O R T I O N.

As Radius : Hyp. B C : : s C, : Leg. A B. [By Axi. II.

O P E

Of Right Lined Triangles.

45

OPERATION.

As Radius, Sine 90 Deg.

To Hypotenuse B C, 274.93 Parts

So is Sine C, 58.73 Deg.

To the Leg. B A, 235.00 Parts

10.

2.4392537

9.9218293

12.3710830

CASE V. The Angles B and C, and the Leg. A C, given; Fig. XII.
To find the Hypotenuse B C. Case V.

PROPORTION.

As s B : Leg. A C :: Radius : Hypotenuse B C. [By Axi. II.]

OPERATION.

As Sine B, 31.27 Deg.

Is to Leg. A C, 142.72 Parts

So Radius, Sine 90 Deg.

To the Hypotenuse B C, 274.93 Parts

9.7152273

12.1544848

10.

2.4392575

CASE VI. The Hypotenuse B C, and the Leg. A C, given; Fig. XII.
To find the other Leg. A B. Case VI.

PROPORTIONS.

1.) As Leg. B C : Rad. :: Leg. A C : s B. [By Case I.]

2.) As t B : Leg. A C :: Rad. : Leg. A B. [By Case I.]

OPERATIONS.

1.) As Hypotenuse B C, 274.93 Parts

To the Radius, Sine 90 Deg.

So is the Leg. A C, 142.72 Parts

To the Sine B, 31.27 Deg.

2.4392537

10.

12.1544848

9.7152311

2.) As the Tangent of B, 31.27 Deg.

Is to the Leg. C A, 142.72 Parts

So is Radius; Tang 45 Deg.

To the Leg. A B, 235.00 Parts

9.7833980

12.1544848

10.

2.3700868

CASE.

Fig. XII. CASE VII. The two Legs, B A and C A, given; To find the
Case VII. Hypotenuse B C.

P R O P O R T I O N S.

- 1.) As Leg. A B : Rad. :: Leg. A C : r B. [By Case I.
- 2.) As r B : Leg. A C :: Rad. : Leg. B C. [By Case V.

O P E R A T I O N S.

- 1.) As the Leg. A B, 235.00 Parts, 2.3710678
 Is to Radius, Tang. 45 Deg. 10.
 So is the Leg. A C, 142.72 Parts 12.1544848
 To Tangent B, 31.27 Deg. 9.7834170
- 2.) As Sine B, 31.27 Deg. 9.7151857
 Is to Leg. A C, 142.72 Parts 12.1544848
 So is Rad. Sine 90 Deg. 10.
 To the Hypotenuse B C, 274.93 Parts 2 4392991

The End of the Seven Cases of Right-angled Plain Triangles.

C H A P. VI.

Of the Solution (or Mensuration) of Oblique-angled Plain Triangles.

Fig. XII. **T**HE Triangle which I shall make use of in the Solution of
Diagram the Five CASES of Oblique-angled Triangles, shall be
S. that noted with S; whose Sides and Angles are

Side $\left\{ \begin{array}{l} B D, 1270 \\ C D, 865 \\ B C, 632 \end{array} \right\}$ Parts.

Angle $\left\{ \begin{array}{l} C, 115.18 \\ B, 38.05 \\ D, 26.77 \end{array} \right\}$ Degrees and Cent.

CASE

Of Right Lined Triangles.

47

CASE I. Two Sides, D C, and C B, with the Angle at D, Fig. XII. opposite to C B, given; To find the Angle at B, opposite to C D. Case I.

PROPORTION.

As Log. C B : s D :: Log. C D, : s B. [By Axiom II.

But here it must be known whether the Angle sought be Acute or Obtuse.

OPERATION.

As Log. C B, 632 Parts

Is to the Sine of $\angle D$, 26.77 Deg.

So is the Logar of C D, 865 Parts

2.800717

9.653608

2.927016

12.590624

9.789907

To the Sine of $\angle B$, 38.05 Deg.

CASE II. Two Sides, D C, and B C, with the Angle C, comprehended by them, given; To find the other Angles B or D. Case II.

The given Angle C being above 90 Deg. viz. 115.18 Deg. Subtract it from 180 Deg. the Remainder is 64.82 Deg. Which must be made use of in the Calculation: And then, This is the

PROPORTION.

As half the Sum of the given Sides D C and B C,

Is to half the Difference of those Sides;

So is the Tangent of half the Sum of the Angles at B and D,

To the Tangent of half the Difference of those Angles.

By Axiom III. Then,

To the half Sum of these Angles D and B, if you Add the half Difference thus found, you will have the Greater Angle; but being subtracted, it will give you the Lesser Angle.

O P E-

O P E R A T I O N.

Parts.

The Side { C D, is	—	865.0
C B, is	—	632.0
Their Sum		1497.0
The half Sum		748.5
Their Difference		233.0
The half Difference		116.5
The Sum of the Angles at B and D	—	64.82 Deg.
Their Difference		11.82 Deg.
The half Sum		32.41 Deg.
The half Difference		5.64 Deg.

Being thus prepared, say,

As the Log. of the half Sum of C B and C D, 748.5	2.8741918
Is to the Log. of half their Difference, 116.5	2.0663259
So is the Tang. of half the Sum of B and D, 32.41	9.8026808

11.8690067

To the Tangent of 5.64 Deg.

8.9948149

This 5.64 Deg. is half the Difference between the Angle B and D,— Which added to the half Sum of those Angles 32.41 Deg. gives 38.05 Deg. for the *Greater Angle* at B. But being subtracted therefrom, it leaves 26.77 Deg. for the *Lesser Angle* at D.

Fig. XII. C A S E III. Two Angles C and D, with the Side C B, opposite to D, given; To find either of the other Angles.

P R O P O R T I O N.

As $s D : \text{Log. } C B :: s C : \text{Log. } D B$. [By *Axiom II.*

O P E R A T I O N.

As the Sine of the Angle at D, 26.77 Deg.	9.6536081
To the Logar of the Side C B, 632 Parts;	2.3007171
So is the Sine of the Angle C, 115.18 Deg. (or 64.82)	9.9566369
	12.7573540
To the Log. of the Side D B, 1270 Parts	3.1037549

C A S E

Of Spherical Triangles.

49

CASE IV. Two Sides, DB and CB, with the Angle B, comprehended by them, being given; To find the Side CB. Fig. XII.
Case IV.

- 1.) Find the Angle C, by the Third Axiome.
- 2.) Find the Side CB, by the Second Axiome.

CASE V. The Three Sides DC, CB, and DB, being given; To find the Angle at D. Fig. XII.
Case V.

To the resolving of this Case, Two Operations are required. The First, To find the Segments of the Base DG and GB, by these

PROPORTIONS.

- (1.) As the Log. of the Base DB,
Is to the Log. of the Sum of the Sides CD and CB:
So is the Log. of the Difference of those Sides DE,
To the Log. of DG. By Axiome 4.
And GD subtracted from DB, shall be equal to GB,
and half GB, is equal to AG or AB. Then,

- (1.) As Log. DC,
Is to Radius:
So is Log. DA,
To the Sine of the Angle DCA. By Axiome 2.
And AG and AB together, are equal to GB; and therefore,
the Angle at B, may be found in the same manner.

OPERATION.

The Sum of the Sides CD and CB, is	1497
Their Difference is	233
The Base BD is	1270

Being thus prepared, say,

As the Log. of the Base DB, 1270	3.103804
Is to the Log. of the Sum of CD and CB, 1497	3.175222
So is the Log. of the Difference of CD and CB, 233	12.367356
	5.542578
To the Log. of the Segment DG 274.65	2.438774

H

Sub.

Fig. VI. Subtract DG 274.65, from the whole Base DB 1270, the Remainder will be GB 995.35; the half whereof, 497.675, is equal to AG or AB .

Then,

As the Logar. of DC , 865,	2.9370161
Is to the Radius, Sine 90 Deg.	10.
So is the Log. of AD , 772.32	12.8877973
To the Sine of DCA , 63.23 Deg.	9.9507812
Whose Complement to 90 Deg. is 26.77 Deg. for the Angle CDB .	

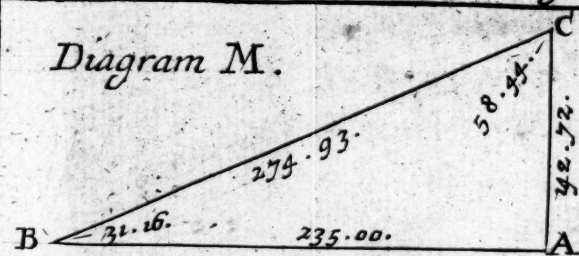
In the same manner; may the Angle at B be found, by, first, finding the Angle ACB , which you will find to be 51.95 Deg. Whose Complement to 90 Deg. 38.05 Deg. is the Quantity of the Angle at B .

The End of the Five Cases of Oblique-angled Plain Triangles.

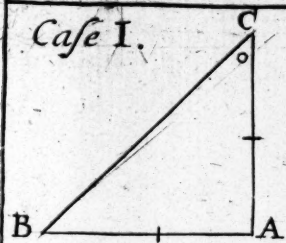
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Figure XII.

Diagram M.

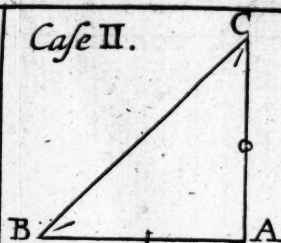


Case I.



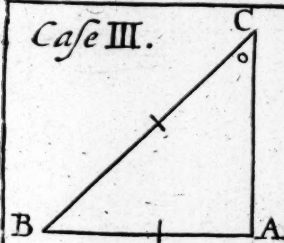
$$AC:R::AB:tC$$

Case II.



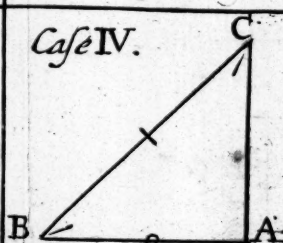
$$R:AB::tB:AC$$

Case III.



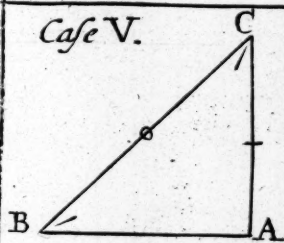
$$BC:R::AB:sC$$

Case IV.



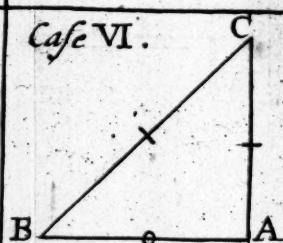
$$R:BC::sC:AB$$

Case V.



$$sB:AC::R:BC$$

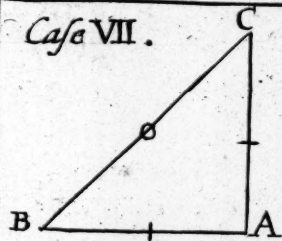
Case VI.



$$BC:R::AC:sB$$

$$tB:AC::R:AB$$

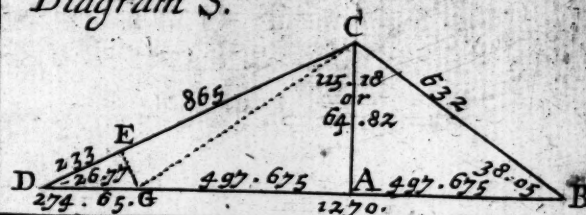
Case VII.



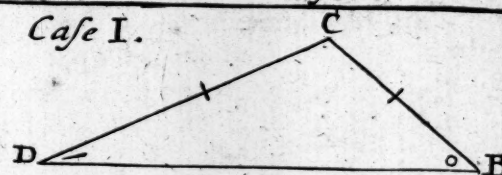
$$AB:R::AC:tB$$

$$sB:AC::R:BC$$

Diagram S.

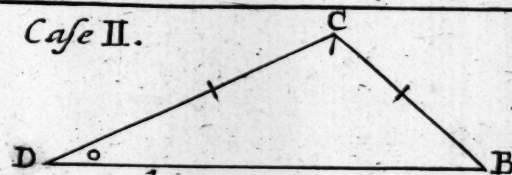


Case I.



$$CB:sD::CD:sB$$

Case II.

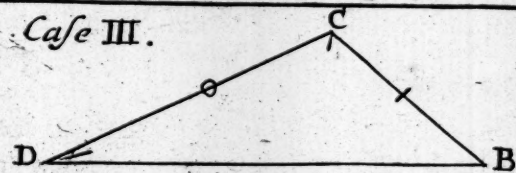


$$\frac{1}{2}DC + BC:$$

$$\frac{1}{2}DC - BC:$$

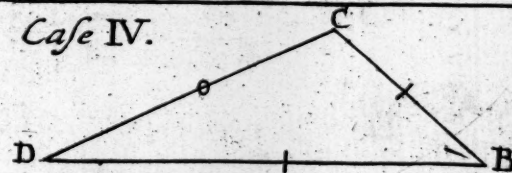
$$::t\frac{1}{2}B + D:t\frac{1}{2}\text{Differ.}$$

Case III.



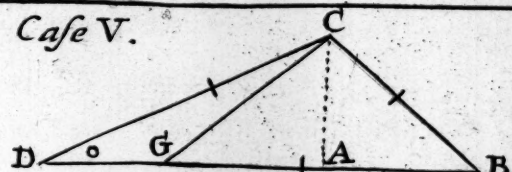
$$sD:CB::sC:DB$$

Case IV.



$$\text{Find } \begin{Bmatrix} C \\ CB \end{Bmatrix} \text{ by } \begin{Bmatrix} \text{Axion} \\ I. \end{Bmatrix}$$

Case V.



$$DB:CD + CB::$$

$$::CD - CB:DG.$$

$$DB - DG = GB \frac{1}{2} GB = AB.$$

ANNUA MATHEMATICA.

SECTION II.

Of Spherical Triangles.

CHAP. I.

Definitions and Theorems, necessary to the right understanding of the Doctrine of the Dimension of Spherical Triangles.

A Spherical Triangle (as well as a Plain, or Right-lined Triangle) consisteth of Six Parts; namely, of Three Sides, and as many Angles: The Affections whereof shall be demonstrated from these Twenty Theorems following.

- I. The Three Sides of every Spherical Triangle are the Arches of Three Great Circles of the Sphere, every one of them being less than a Semicircle, or consisting of fewer Degrees than 180, which is the Measure of a Semicircle. By the 9th of Sect. II.
- II. A Great Circle of the Sphere, is such a Circle as divideth the whole Sphere, or Globe, into Two Equal Parts or Hemispheres; and so is in all Parts distant from the Poles thereof, by a Quadrant or 90 Deg. of the Great Circle.
- III. If one Great Circle of the Sphere, do pass by the Poles of another Great Circle, those Two Great Circles do intersect, or cut, each other at Right Angles: And the contrary.

Thus, Let A E C be a Great Circle of the Sphere; (and let it represent the Horizon of any Place) whose Poles let be B and D, (the Zenith and Nadir of the same Place, equi-distant from the Horizon A E C 90 Deg.) by which Poles B, and D, let another Great Circle pass; namely, B E D, one of the Colures (or any Azimuth, or Vertical Circle). Now, I say, that the Great Circle

H 2

BED, Fig. XIII.

Fig. XIII. B E D, cutteth the Great Circle A E C, at Right Angles in the Points E and F. For, if upon the Pole E or F another Great Circle A B C D be described, it is manifest, that A B, B C, C D, and D A, shall be the Measures of the Angles at E and F: (by the 4th of Sect. II.) But the Arches A B, B C, C D, and D A, are Quadrants (by the 3d hereof) and therefore, the Angles at E and F, are Right Angles (by the 13th of Sect. II.) Which was to be demonstrated.

IV. The Measure of a Spherical Angle, Is the Arch of a Great Circle, described upon the Angular Point; and intercepted between the Two Sides, they being continued out till they be Quadrants: (by the First hereof.)

Fig.
XIV.

So, In the Spherical Triangle A B C, the Measure of the Spherical Angle at A, is not the Arch B C, but the Arch E C, intercepted between the Two Sides A B and A C, continued till they be Quadrants; that is, to the Points E and D; because the Arch B C is not described upon the angular Point A, but the Arch E D is; (by the 1st hereof) and therefore, the Arch B C, cannot be the Measure of the Angle B A C, (by the 4th of Sect. II.)

V. The Sides of a Spherical Triangle, being continued till they meet together, do make Two Semicircles, and at their Intersection, do comprehend an Angle, Equal and Opposite to the First Angle.

Fig. XV. Thus, In the Triangle A B C, the Sides A B and A C, of the Angle B A C, being continued to D, do make the Semicircles A B D and A C D, which do comprehend the Angle B D C, equal to the Angle B A C, because the same Arch G H (being distant from A and D 90 Deg.) measureth both those Angles, (by the 4th hereof.)

VI. Every Spherical Triangle, hath from every Angle thereof, another Triangle opposite thereunto, whose Base and Angle opposite to the Base, are the same; and the other Parts of that Triangle, are the Complements of the Parts of the other Triangle.

Thus, the Triangle A B C, hath another Triangle B D C, opposite thereunto; whose Base B C, and Angle B D C, opposite thereunto, are the same, (by the 5th hereof.) And the Sides B D and C D, are the Complements of the Sides A B and A C, to a Semicircle.

Of Spherical Triangles.

53

circle: And the *Angles* D B C, and D C B, are the *Complements* Fig. XV. of the *Angles* A B C, and A C B, to Two Right Angles, or 180 Deg. (by the 18th of Sect. II.)

VII. The Sides of a Spherical Triangle may be changed into Angles, and the contrary; the Complement of the Greatest Side, or Greatest Angle, to a Semicircle, being taken for the Greatest Side, or Greatest Angle.

Thus (in the Figure of Theorem IV. hereof) the Spherical Tri- Fig. XIV. angle A B C, obtuse-angled at B: Let D E be the Measure of the Angle at A, and let F G be the Measure of the Acute Angle at B, (being the Complement of the Obtuse Angle at B, the Greatest Angle in the Triangle:) And let H I be the Measure of the Angle at C: Now,

$\left. \begin{matrix} K L \\ L M \\ K M \end{matrix} \right\}$ is equal to $\left\{ \begin{matrix} D E \\ F G \\ H I \end{matrix} \right\}$ because $\left\{ \begin{matrix} K D \\ L G \\ K I \end{matrix} \right\}$ and $\left\{ \begin{matrix} L E \\ F M \\ M H \end{matrix} \right\}$ are Quadrants, and $\left\{ \begin{matrix} L D \\ L F \\ K H \end{matrix} \right\}$ their common Complement is

Therefore, the Sides of the Triangle K L M are equal to the Angles of the Triangle A B C; taking for the Greatest Angle A B C, the Complement thereof F B G.

It may also be demonstrated, That the Sides of the Triangle A B C are equal to the Angles of the Triangle K L M: (by the Converse of the former.) For, the Side

$\left\{ \begin{matrix} A B \\ B C \\ A C \end{matrix} \right\}$ is equal to $\left\{ \begin{matrix} O P \\ F H \\ D I \end{matrix} \right\}$ the Measure of the Angle $\left\{ \begin{matrix} M I K \\ L M K \\ D K I \end{matrix} \right\}$ which is the Complement of the Obtuse Angle M K L.

—For that $\left\{ \begin{matrix} A D \\ A P \\ B F \end{matrix} \right\}$ and $\left\{ \begin{matrix} C I \\ O B \\ C H \end{matrix} \right\}$ are Quadrants, and their common Complement is $\left\{ \begin{matrix} C D \\ A O \\ C F \end{matrix} \right\}$.

Therefore, The Sides may be turned into Angles, and the contrary. Which was to be demonstrated.

VIII. A Right-angled Spherical Triangle hath One Right Angle, or more than One.

IX. Suppose the Angles at A and D, viz. B A C and B D C, to be Right Angles. Then,

The Triangle $\left\{ \begin{matrix} B A C \\ B D C \\ C D E \end{matrix} \right\}$ hath $\left\{ \begin{matrix} \text{One Right Angle, and Two Acute, Two Ob-} \\ \text{tuse Angles, and One Right, One Ob-} \\ \text{tuse, and Two Acute.} \end{matrix} \right\}$

X. A

Fig. XV. X. A Right-angled Spherical Triangle, bath, from the Right Angle, a Right-angled Triangle opposite thereunto, with Two Obtuse Angles, & contra.

As in the Triangles B A C, and B D C.

XI. The Sides of a Right-angled Spherical Triangle, with Two Acute Angles, are either of them less than Quadrants.

As the Sides of the Triangle B A C, are all of them, less than Quadrants.

XII. The Two Sides of a Right-angled Spherical Triangle, with Two Obtuse Angles, are greater than Quadrants; and the third Side, is less than a Quadrant.

As in the Triangle B C D, Obtuse-angled at B and C; the Sides D B and D C are greater than Quadrants; and the Side B C lesser.

XIII. A Right-angled Spherical Triangle, with Two Acute Angles, bath (from the Acute Angle) opposite to it, a Right-angled Spherical Triangle, with One Acute, and One Obtuse, Angle.

As in the Right-angled Spherical Triangle E D F, having the Angles at E and F Acute, is opposite to the Right-angled Triangle C D E, with the Acute Angle E C D, and the Obtuse C E D.

XIV. The Sides subtending the Right Angles of a Spherical Triangle, having divers Right Angles, are Quadrants.

As in the Triangle A G H, If the Two Great Circles A G and A H, do cut the Great Circle G H at Right Angles in the Points G and H; then A is the Pole of the Great Circle G H (by the third hereof) and A G and A H are Quadrants (by the second hereof.) But if the Angle at A be also a Right Angle, then G H is also a Quadrant, (by the 13th of Sect. II. and by the 4th hereof.)

XV. A Spherical Triangle, having divers Right Angles, bath either Two or Three Right Angles; and so of the Sides, it bath Two or Three of them Quadrants.

As if the Angle at A be put for a Right Angle, the Spherical Triangle A G H shall have the Three Right Angles at A, G and H; and

Of Spherical Triangles.

55

and therefore the *Three Sides*, A G, A H and H G, shall be *Fig. XV.*
Quadrants. --- But, If you put the *Angle* at A for an *Acute Angle*,
 then the *Spherical Triangle* A G H shall have *Two Right Angles*
 at G and H; and thereupon the *Two opposite Sides*, A G and
 A H, *Quadrants*.

XVI. If the *third Angle* of a *Spherical Triangle*, having *Two Right*
Angles, be *Acute*; the *third Side* is less than a *Quadrant*; but if
Obtuse, then it is greater than a *Quadrant*.

As in the *Spherical Triangle* G H I, *Acute-angled* at G, the
third Side H I is less than a *Quadrant*. ---- But in the *Spherical*
Triangle A G I, *Obtuse-angled* at G, the *third Side* A I is more
 than a *Quadrant*.

XVII. An *Oblique Spherical Triangle*, consisteth simply of *Acute* or
Obtuse Angles, or of both of them.

XVIII. A *Spherical Triangle*, with *Two Obtuse Angles*, and *One*
Acute Angle; is opposite to a *Spherical Triangle*, simply *Acute-*
angled, & *contra*.

As if the *Angles* at A and D, be supposed *Acute*; then the *Tri-*
angle B C D, with *Two Obtuse Angles* at B and C, and *One Acute*
Angle at D, is opposite to the simply *Acute-angled Triangle* A B C.

XIX. A *Spherical Triangle*, with *Two Acute Angles*, and *One*
Obtuse; is opposite to a *Spherical Triangle*, simply *Obtuse-an-*
gled, & *contra*.

As if the *Angles* at A and D, be supposed *Obtuse*; then the *Tri-*
angle A B C, with *Two Acute Angles* at B and C, and *One Obtuse*
Angle at A, is opposite to the simply *Obtuse-angled Triangle*
 B D C.

XX. The *Three Angles* of every *Spherical Triangle*, are greater
 than *Two Right Angles*.

This is evident in *Spherical Triangles*, having more *Right*, or
Obtuse Angles, than *One*: But in *Acute-angled Triangles*, it may
 be thus *Demonstrated*.

D E.

DEMONSTRATION.

Fig. XVI. In the Right-angled Spherical Triangle A B C, Right-angled at C, and Acute-angled at A and B.

The Measure of $\left\{ \begin{array}{l} B A C \\ A B C \text{ or } B D E \end{array} \right\}$ is the Arch $\left\{ \begin{array}{l} E F \\ H I \end{array} \right\}$ (by the fourth hereof.) But the Arches E F and E D together, are equal to a Quadrant; therefore the Arches F E and H I, added together, are more than a Quadrant; and consequently, the Angles answering to those Arches, namely, the Angles B A C and A B C together, are more than a Quadrant. But the Angle A C B is a Right Angle, by the Proposition: Therefore, in the Spherical Triangle A B C, consisting of Two Acute Angles, the Three Angles together are greater than Two Right Angles.

Fig.
XVII.

Again, In the Acute-angled Triangle K L M

The Measure of the Acute Angle $\left\{ \begin{array}{l} K L M \\ M K L \\ L M K \end{array} \right\}$ is the Arch $\left\{ \begin{array}{l} N O \\ V X \\ R Q \end{array} \right\}$

But these Three Arches N O, V X, and R Q together, are more than Two Quadrants: For P Q, and P V, (being the Complements of the Two Arches, Q R and V X) added together, are less than the Arch N O, by the Proposition: Therefore the Arch N O, being the Measure of the third Angle, is more than the Complements of the other Two Angles added together, and consequently, the third Angle is greater than the Complements of the other Two Angles. And therefore, In Acute-angled Spherical Triangles, the Three Angles are greater than Two Right Angles. Which was to be demonstrated.

C H A P. II.

Such Affections of Great Circles of the Sphere, as relate to the Solution of Spherical Triangles.

1. **A** Great Circle of the Sphere, Is such a Circle, as divideth the whole Body of the Globe or Sphere, into Two Equal Parts.

Of Spherical Triangles.

57

2. A *Spherical Triangle* is that part of the *Superficies* of the *Globe*, as lyes between the *Arches* of Three *Great Circles* of the *Sphere* intersecting one another.

Fig.
XVII.

3. A *Spheric Angle* is the same with the mutual *Aperture* or *Inclination* of the *Plains* of such *Two Great Circles* which constitute the *Angle*.

4. When one *Circle* falls upon another *Circle*, or when the *Arches* of *Two Great Circles* intersect each other, the *Sum* of the *Angles* made thereby is equal to *Two Right Angles*: And the *Vertical Angles* made thereby are mutually *Equal*.

5. In all *Spherical Triangles*, the *Greater Angle* is always oppos'd to the *Greater Side*.

6. An *Isofceles Triangle*, hath its *Two Angles* at the *Base* mutually *Equal*; and, on the contrary, if a *Triangle* hath *Two Angles Equal*, it hath *Two Sides* also *Equal*.

7. *Two Triangles* mutually *Equilateral*, are also *Equiangular* one to the other.

8. If there be *Two Triangles*, and in each one *Angle*, and the *Two Sides* including it, respectively equal: Or, if *One Side*, and the *Two Angles* adjacent, be severally equal, then are those *Two Triangles* equal.

9. An *Arch* of a *Great Circle*, is the shortest *Distance* between *Two Points* on the *Surface* of a *Globe*: And so, any *Two Sides* of a *Spherical Triangle* taken together, are *Greater* than the *Third*.

10. All *Great Circles* cut each other into *Two equal Parts*; for their *common Section* is a *Diameter* of the *Sphere*, and consequently the *Two Sections* of the *Peripheries* of *Two Great Circles* are at a *Semicircle's Distance*. Hence it follows, That

11. Every *Side* of a *Spherical Triangle*, is less than a *Semicircle*. So *DB* is less than the *Semicircle* *DBC* or *DAC*.

Fig.
XVIII.

12. The opposite *Angles* at the *Sections* of *Two Circles*, are *Equal*; as the *Angle* at *D*, is equal to that at *C*; for the same *Plains* constitute both *Angles*.

13. In any *Spherical Triangle*, if the *Sum* of the *Legs* containing an *Angle* be *Greater*, *Equal* to, or *Lesser* than a *Semicircle*, the internal *Angle* at the *Base*, is (accordingly) *Greater*, *Equal* to, or *Lesser* than the outward opposite; and consequently, the *Sum* of the *Two internal Angles* at the *Base*, are *Greater*, *Equal* to, or *Lesser* than *Two Right Angles*.

I

D E-

D E M O N S T R A T I O N.

If DB and BA together be *Greater, Equal* or *Lesser* than DC , then BA is *Greater, Equal* to, or *Lesser* than BC ; and therefore the Angles at C and D are *Greater, Equal* or *Lesser* than the Angle BAC ; and the Angles BDA and DAB , *Greater, Equal* to, or *Lesser* than the Angles BAC and DAB , equal to Two Right Angles.

C O R O L L A R Y.

In an *Isocheles Triangle*, if one of the *Equal Legs* be *Greater, Equal* to, or *Lesser* than a *Quadrant*, the Angle at the *Base* is *Greater, Equal* to, or *Lesser* than a *Right Angle*.

14. The *Sum* of the *Three Sides* of a *Triangle* is less than a *Whole Circle*, or 360 Deg. For BA is *Less* than BC and AC . Therefore DB , DA and BA together, are *Lesser* than DBC and DAC .

15. If from the Point of an Angle, as a *Pole*, you describe a *Great Circle*; or, if you describe a Circle at 90 Deg. on the angular Point, the *Ark* of that Circle so described, which is intercepted between the *Legs* of the Angle, is the *Measure* of that Angle.

Fig. XIX. 16. The *Poles* of the *Sides* of any *Triangle* $GH D$, constitute another *Triangle* $n x m$, which we may call *Supplemental* to the *Triangle* $GH D$, for the *Supplements* of the Angles and *Sides* of the *Triangle* $n x m$ are equal to the *Sides* and *Angles* of the *Triangle* $GH D$.

D E M O N S T R A T I O N.

From the Points G, H, D , as *Poles*, describe Three *Great Circles*, $x A Y$, $R T m n$, $x B n Z$; then is $Y m$ equal to a *Quadrant*, and equal to $A x$, because m is the *Pole* of $H G Y$, and x or E , the *Pole* of $G A$; therefore $m x$, equal to $A Y$, equal to the *Supplement* of CA , equal to the Angle $H G D$; and the *Quadrant* $Z n$, equal to $B X$; therefore $n x$, equal to $B Z$, equal to the *Supplement* of the Angle $H D G$, and $n T$, equal to a *Quadrant*, equal to $m R$; therefore $n m$, equal to $T R$, equal to the *Supplement* of the Angle $D H G$.

Now that the *Triangle* $n E m$, constituted between the Three next *Poles*, hath its Three *Sides* and *Angles*, equal to the *Angles* and *Sides* of the *Triangle* $GH D$, save that the *Greatest Side* $n m$ is the *Supplement* of the *Greatest Angle* H , and the Angle E , of the Side $G D$.

17. Any

17. Any Angle of a Triangle, with the Difference of the other Two, is Lesser than Two Right Angles: For $x n$ is Lesser than $x m$ and $n m$, that is, Two Right Angles, wanting D, is Lesser than Two Right Angles, wanting G, and Two Right Angles, wanting H. Therefore, G and H wanting D, is less than Two Right Angles.

18. If Two Triangles are mutually Equiangular, they are also mutually Equilateral; for, because they are Equiangular, their Supplemental Triangles are Equilateral (by the 16th) and therefore Equiangular (by the 7th). And therefore the proposed Triangles are Equilateral (by the 16th.)

19. The Three Angles of every Spherical Triangle, are Greater than Two Right Angles, and Lesser than Six Right Angles. For, $n x$ and $x m$ and $n m$ together, are Lesser than Four Right Angles, (by the 14th.) that is, Six Right Angles, wanting D, and G, and H, lesser than Four Right Angles: That is, Two Right Angles are lesser than D, and G, and H. Also, the Sum of the Internal-angles is less than the Sum of the Internal and External Angles taken together, for both of them make but Six Right Angles.

20. Of several Arches of Great Circles falling from the same Point of the Spheres Surface, on another Circle, the Greatest is that which passeth through the Pole of the Circle; and the next to this, is Greater than that which is farther off. For suppose P the Pole of the Circle C $\propto$ D, and $\propto$ the Pole of D P C; then is A D Greater than A B, A B Greater than A E, A E Greater than A C: And the Ark B $\propto$ C Greater than the Ark B P, and B P Greater than B D. Fig. XX.

21. A Great Circle passing through the Poles of another Great Circle, cuts it at Right Angles; And on the contrary, If it cut it at Right Angles, it passeth through its Poles: The Angle P B O is equal to a Right Angle, equal to P G D, equal to P D B, equal to $\propto$ A C.

22. In an Oblique-angled Triangle, if the Angles at the Base are like, or of the same Kinds; that is, both Acute, or both Obtuse, the Perpendicular falls Within the Triangle, and the Quadrantal Arch without. But if they be unlike, the Perpendicular falls Without; and the Quadrantal Arch Within the Triangle. For the Triangle A E F hath the Angles at E and F Acute, and the Perpendicular A C falls Within, and the Quadrantal Arch A $\propto$ Without. Also, the Triangle B A G hath the Angles at B and G, Obtuse, and the Perpendicular A D Within, and the Quadrantal

Fig. XX. Arch A ϖ Without. But the Triangle B A E hath the Angles at B and E of different Kinds; and the Perpendicular A C falls Without, and the Quadrantal Arch A ϖ Within, the Triangle.

C H A P. III.

Of the Mensuration, or Solution, of Right-angled Spherical Triangles.

Fig. XXI. I. **I**N a Right-angled Spherical Triangle, there are (besides the Right Angle) Five other Parts; whereof those Three which are more remote from the Right Angle, the Lord Nepeir changeth into their Complements: — As in this Triangle A B C, Right-angled at A: For the Three Remote Parts, to wit, the Angles B and C, and the Side C B, he takes their Complements: These Three Complements, with the Sides C A and B A, do make Five Parts: Which, by an artificial Term, he calls

C I R C U L A R.

Viz. The $\left\{ \begin{array}{l} \text{Side A B,} \\ \text{Side A C,} \\ \text{Complement of the Angle at B,} \\ \text{Complement of the Angle at C,} \\ \text{Complement of the Side B C.} \end{array} \right.$

But the Right Angle at A is set aside from being any of the Circular Points.

II. In the Solution of a Right-angled Spherical Triangles, there are always Two other Parts, or Terms, given (besides the Right Angle) to find out a Fourth.

III. These Three Terms (namely, the Two that are Given, and the Third which is Required) must be first looked upon according to their Circular Parts.

IV. Of which, One is named the Middle (or Mean) Part; the other Two are called the Extream Parts, borrowing their Appellation from the Scituation of the Terms themselves: For, of Three Terms, One must (of necessity) be in the Middle, and the other Two in the Extreams: Therefore the Circular Part

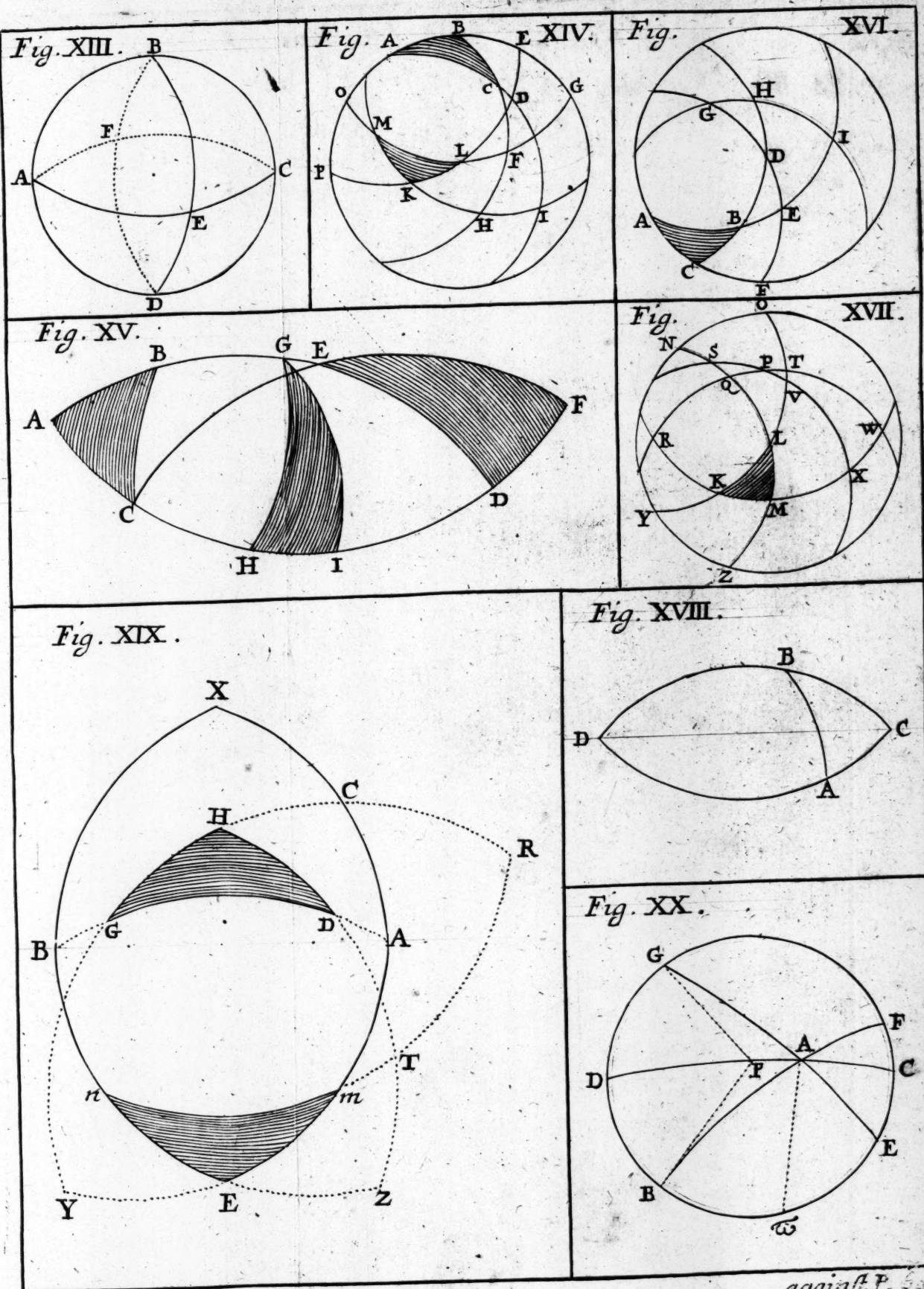


Plate VI. From Page 51, to Page 60.

against P. 50

Part of the *Middle Term*, is called the *Middle Part*, and the *Circular Parts* of the *Extream Terms* are called the *Extream Parts*.

V. But the *Extream Parts* may be twofold, either *Conjunct*, or *Disjunct*: For those *Three Terms* (besides the *Right Angle*) do come in question, according as the *Two Extreams* are from either *Part* immediately *joined* to a third *Mean*, or are *dis-joined* from the same, by a *Side* or *Angle* interposed on both *Sides*: So are their *Circular Parts* named, *Extreams Conjunct*, or *Extreams Disjunct*.

VI. But of those *Three Terms* which may fall in question, we will subject all their Varieties in their *Circular Parts*, according as every one of them ought (in respect of each other) to be called, The *Middle Part*; and which, The *Extreams Conjunct* or *Disjunct*, as in this following *Synopsis* is fully demonstrated.

The Analysis or Synopsis.

<i>Middle Part.</i>		<i>Extreams Conjunct.</i>	<i>Extreams Disjunct.</i>
If the	Side A B	Side A C and Comp. B	Com. B C and Com. C
	Side A C	Side A B and Comp. C	Com. B C and Com. B
	Compl. B	Side A B and Comp. B C	Side A C and Com. C
	Compl. C	Side A C and Comp. B C	Side A B and Com. B
	Compl. B C	Comp. B and Comp. C	Side A B & Side A C.

Which *Synopsis* is thus to be Read and Understood: Example of the *First Line*. If the *Side A B* be the *Middle Part*, then is the *Side A C*, and *Comp. of the Angle B*, the *Extreams Conjunct*. And the *Comp. of the Side B C*, and *Comp. of the Angle C*, the *Extreams Disjunct*. And so of all the rest.

And here it is to be noted, That the *Sides A B* and *A C*, are supposed to be joined together, (as one entire Part,) because the *Right Angle* at *A*, is not reckoned amongst the *Circular Parts*.

VII. Therefore, In the *Resolution* of a *Right-angled Spherical Triangle*, to know the *Mean*, and *Extream Parts*, you must observe, That

1. If One of the *Three Terms* (which, besides the *Right Angle*, come in question) doth stand alone by it self, severed from the

Fig. XXI.

the other *Two* on both *Sides*; (as the *Side* B C, from the *Sides* C A and B A, by the *Angles* B and C interposed) that shall be the *Middle Term*; and so its *Circular Part* shall be called the *Middle Part*; and the other *Two Circular Parts* are the *Extreams Disjunct*. But,

2. If the *Three Terms* do immediately adhere together, the *Middle Term* doth easily shew the *Middle Part*, and the *Extream Terms*, the *Extream Parts Conjunct*.

These Things being all rightly understood, the whole *Trigonometry of Sphericals* will be absolved by this *One Proposition*; which therefore we will call *Catholick* or *Universal*.

Proposition Universal.

The Sine of the Middle Part and the Radius, are Reciprocally Proportional, with the Tangents of the Extream Parts Conjunct; and with the Co-sines (or Sines Complements) of the Extreams Disjunct. That is,

As the *Radius*,

To the *Tangent* of one of the *Extreams Conjunct*;

So is the *Tangent* of the other *Extream Conjunct*,

To the *Sine* of the *Middle Part*: & contra.

Then also,

As the *Radius*,

To the *Co-sine* of one of the *Extreams Disjunct*;

So is the *Co-sine* of the other *Extream Disjunct*,

To the *Sine* of the *Middle Part*: & contra.

C O R O L L A R Y

- I. If the *Middle Part* be Sought, the *Radius* shall be in the *First Place* of the *Proportion*: But if one of the *Extream Parts* be Sought, then the other *Extream* shall be in the *First Place*. Of the *Second* and *Third Places*, it mattereth nothing how they be disposed.

II. If

Of Spherical Triangles.

63

II. If the *Extreams* (in any *Proportion*) be *Distinct* from the *Middle Part*, the *Proportion* will be performed by *Sines* only: But if the *Extreams* be *Conjunct* to the *Middle Part*, it must be performed by *Sines* and *Tangents* jointly. Fig. XXI.

The *Demonstration* of the *Universal Proposition* is obvious enough: For, where the *Extream Parts* are *Disjunct*, the *Proportions* differ nothing from the common ones: And in the *Extreams Conjunct*, where it is commonly said,

As *Radius*, to the *Tangent*,

We here say,

As the *Co-Tangent*, to the *Radius*.

And likewise *Inversly* and *Contrarily*; which is plainly the same thing: Because,

The *Radius* is a *Mean Proportional*, between the *Tangent* of an *Arch*, and the *Tangent Complement* of the same *Arch*.

Note, That when a *Complement* in any *Proportion* doth chance to concur with a *Complement* in the *Circular Parts*, you must then (always) take the *Sine* it self, or the *Tangent* it self; instead of the *Co-sine*, or *Co-Tangent*, in the *Circular Parts*: Because the *Co-sine* of the *Co-sine*, is the *Sine* it self, and the *Co-tangent* of the *Co-tangent*, is the *Tangent* it self,

As is the *Sixth Case* following; where *C B* and *C A* are given, and the *Angle* at *B* is required: Here *C A* is the *Middle Part*, and *C B*, and *B*, are the *Extreams Disjunct*: Wherefore (by the second Part of the foregoing *Corollary*) the *Proportion* will be performed by *Sines* only.—And (by this last) because the *Two Extreams*, *CB*, and *B*, fall upon *Complements* in the *Circular Parts*; therefore, instead of *Co-sine B C*, and *Co-sine B*, you must say *Sine B C*, and *Sine B*.

These Things premised, we will exemplifie in the Solution of: *Right-angled Spherical Triangles* in all the Cases thereof.

C H A P.

C H A P. IV.

The Analogies, or Proportions, for the Solution of the several Cases of Right-angled Spherical Triangles, by the Universal Proposition.

Fig.XXI. **F** O R the Performance hereof, I shall make use of this Right-angled Spherical Triangle A B C, Right-angled at A, the Diagram B. Quantities of whose Sides and Angles are adfix'd to their respective Circular Parts in the Diagram not'd with B, in Fig. XXI. And also in this Table, both in Sexagenary Degrees and Minutes; and in Decimal Parts also.

	D.	M.	Cent.
B C —	66	30	66.50
C A —	51	30	51.50
B A —	50	10	50.16
∠ B —	58	35	58.58
∠ C —	56	52	56.85

And in every Case I shall distinguish the Two Given Terms, (besides the Right Angle, which is, always the *third*) by marking the Sides or Angles Given, by a short Stroak, (_), and the Term Required, I shall mark with (o.) All which are to be seen in Figure XXI.

The XVI Cases of Right-angled Spherical Triangle, Resolved,
The Hypotenuse B C, and the Angle at C, given ; To find

Fig.XXI. **C A S E I.** The Opposite Side A B, the Middle Part.
Case I.

As Radius 90 Deg.

To Sine C, 56 Deg. 52 Min.

So Sine B C, 66 Deg. 30 Min.

To Sine B A, 50 Deg. 10 Min.

10 0000000
9.9229334
9.9623978
19.8853312

C A S E

Of Spherical Triangles.

65

CASE II. The Side Adjacent A C, Extream Conjunct.

Fig. XXI.

Case II.

As Co-tangent B C, 23 Deg. 30 Min.

9.6383019

To Radius, 90 Deg.

10.

So Co-fine C, 33 Deg. 8 Min.

19.7376611

To Tangent C A, 51 Deg. 30 Min.

10.0993592

CASE III. The other Angle B, Extream Conjunct.

Case III.

As Co-Tangent C, 33 Deg. 8 Min.

9.8147277

To Radius, 90 Deg.

10.

So Co-fine B C, 23 Deg. 30 Min.

19.6006997

To Co-Tangent B, 31 Deg. 25 Min.

9.7859720

The Hypotenuse B C, and Side A C, given, to find

CASE IV. The Opposite Angle at B, Ex. Disj.

Case IV.

As Sine B C, 66 Deg. 30 Min.

9.9623978

To Radius, 90 Deg.

10.

So Sine C A, 51 Deg. 30 Min.

19.8935444

To Sine B, 58 Deg. 35 Min.

9.9311466

CASE V. The Adjacent Angle C, Middle Part.

Case V.

As Radius, 90 Deg.

10.

To Tangent C A, 51 Deg. 30 Min.

10.0993948

So Co-tangent B C, 23 Deg. 30 Min.

9.6383019

To Co-fine C, 33 Deg. 8 Min.

19.7376967

CASE VI. The other Side A B, Extream Disjunct.

Case VI.

As Co-Sine C A, 38 Deg. 30 Min.

9.7941496

To Radius, 90 Deg.

10.

So Co-fine B C, 23 Deg. 30 Min.

19.6006997

To Co-fine B A 39 Deg. 50 Min.

9.8065501

Fig. XXI. The Side A C, and the Angle opposite there to B, being Given;
To find

Case VII. CASE VII. The other Side B A, Middle Part.

As Radius, 90 Deg.	10.
To Tangent C A, 51 Deg. 30 Min.	10.0993948
So Co-Tangent B, 31 Deg. 25 Min.	9.7859004
To Sine B A, 50 Deg. 10 Min.	9.8852952

Case VIII. CASE VIII. The other Angle at C, Extream Disjunct.

As Co-sine C A, 38 Deg. 30 Min.	9.7941496
To Radius, 90 Deg.	10.
So Co-sine B, 31 Deg. 25 Min.	19.7170526
To Sine C, 56 Deg. 52 Min.	9.9229030

Case IX. CASE IX. The Hypotenuse B C, Extream Disjunct.

As Sine B, 58 Deg. 35 Min.	9.9311522
To Radius, 90 Deg.	10.
So Sine C A, 51 Deg. 30 Min.	19.8935444
To Sine B C, 66 Deg. 30 Min.	9.9623922

The Side C A, and the Angle C, adjacent thereto, given; To find

Case X. CASE X. The other Side A B, Extream Conjunct.

As Co-tangent C, 33 Deg. 8 Min.	9.8147277
To Radius, 90 Deg. 8 Min.	10.
So Sine C A, 51 Deg. 30 Min.	19.8935444
To Tangent B A, 50 Deg. 10 Min.	10.0788167

Case XI. CASE XI. The other Angle B: Middle Part.

As Radius, 90 Deg.	10.
To Sine C, 56 Deg. 52 Min.	9.9229334
So Co-sine C A, 38 Deg. 30 Min.	9.7941496
To Co-sine B, 31 Deg. 25 Min.	9.7170830

CASE

Of Spherical Triangles.

67

CASE XII. The Hypotenuse BC, Extream Conjunct. Fig. XXI.

As Tangent CA, 51 Deg. 30 Min.	10.0993948	Case XII.
To Radius, 90 Deg.	10.	
So Co-fine C, 33 Deg. 8 Min.	19.7376611	
To Co-tangent BC, 23 Deg. 30 Min.	9.6382663	

The Two Sides, AC and AB, given; To find

CASE XIII. Either Angle, as C; Extream Conjunct. Case XIII.

As Tangent BA, 50 Deg. 10 Min.	10.0787534	
To Radius, 90 Deg.	10.	
So Sine CA, 51 Deg. 30 Min.	19.8935444	
To Co-tangent C, 33 Deg. 8 Min.	9.8147910	

CASE XIV. The Hypotenuse CB: Middle Part. Case XIV.

As Radius, 90 Deg.	10.	
To Co-fine AC, 38 Deg. 30 Min.	9.7941496	
So Co-fine BA, 39 Deg. 50 Min.	9.8065575	
To Co-fine CB, 23 Deg. 30 Min.	19.6007071	

The Two Angles B and C, given; To find

CASE XV. Either of the Sides, as AC: Extream Disjunct. Case XV.

As Sine C, 56 Deg. 52 Min.	9.9229334	
To Radius, 90 Deg.	10.	
So is Co-fine B, 31 Deg. 25 Min.	19.7170526	
To Co-fine CA, 38 Deg. 30 Min.	9.7941192	

CASE XVI. The Hypotenuse BC: Middle Part. Case XVI.

As the Radius, 90 Deg.	10.	
To Co-tangent C, 33 Deg. 8 Min.	9.8147277	
So Co-tangent B, 31 Deg. 25 Min.	9.7859004	
To Co-fine BC, 23 Deg. 30 Min.	19.6006281	

K 2

Note,

Fig. XXI. *Note*, These are the *Proportions* answerable to the *Universal Proposition*, yet may (many of them) be varied; so that the *Radius* may be brought into the *first Place*, and that by the latter part of the foregoing *Corollary*: Which says—*Radius is a Mean Proportional between the Tangent of an Arch, and the Tangent Complement of the same Arch.*—So that (in the XIIIth C A S E) where it is said,

As the *Tangent* B A, Is to *Radius*:
So is *Sine* C A, To *Co-tangent* C.

It is all one, as if you should say,

As *Radius*, 90 Deg.

	10.
To the <i>Co-tangent</i> B A, 39 Deg. 50 Min.	9.9222466
So the <i>Sine</i> of C A, 51 Deg. 30 Min.	9.8935444
To the <i>Co-tangent</i> of C, 33 Deg. 3 Min.	x9.8157910

The like Course may be taken in the *Second, Third, Tenth and Twelfth CASES.*

And thus you have the whole *Doctrine* of the *Dimension* of *Right-angled Spherical Triangles*, performed by Help of this one *Catholick Proposition.*

C H A P. IV.

Some Propositions concerning Oblique-angled Spherical Triangles, in order to the Solution of them.

IN *Oblique-angled Spherical Triangles* there are XII *Cases*, Ten of which may be resolved by the *Universal Proposition*; but then the *Oblique Triangle* must be reduced into Two *Right-angled Triangles* by help of a *Perpendicular* let fall, sometimes *within*, sometimes *without*, the *Triangle*: And to know whether it fall *within* or *without*, the subsequent *Rules* are to be observed.

RULE I. *If the Angles at the Base of the Triangle be both of the same Affection; that is, both Acute or Obtuse; the Perpendicular let fall from the Vertical Angle shall fall within: But if of different Affections, without.*

As

Figure XXI.

Diagram A.

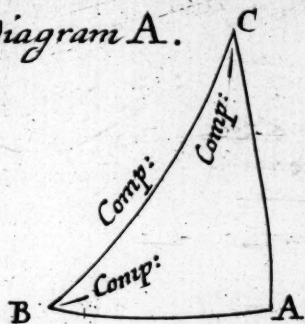
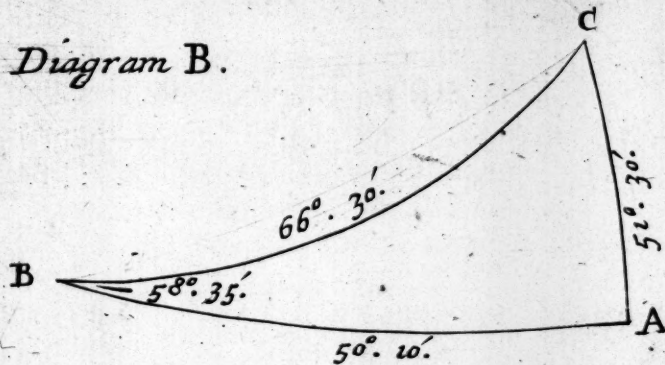
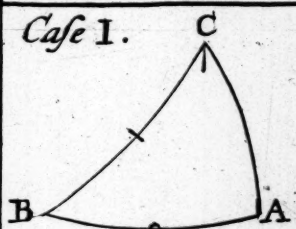


Diagram B.

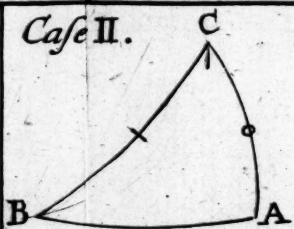


Case I.



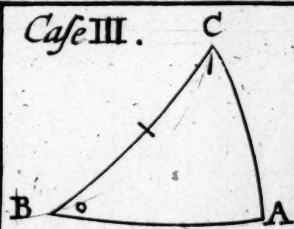
$$R : sC :: sBC : BA$$

Case II.



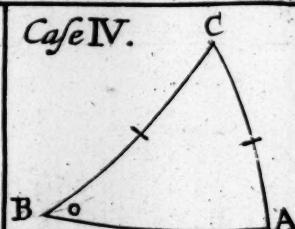
$$ctBC : R :: csC : tCA$$

Case III.



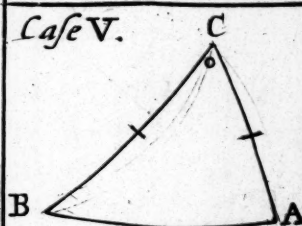
$$ctC : R :: csBC : ctB$$

Case IV.



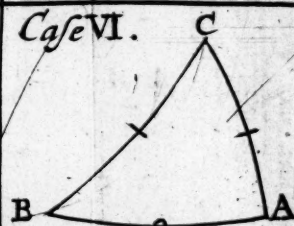
$$sBC : R :: sCA : sB$$

Case V.



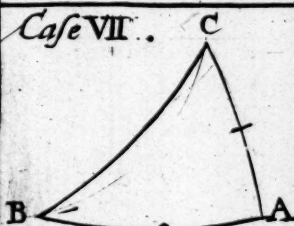
$$R : tCA :: ctBC : csC$$

Case VI.



$$csCA : R :: csBC : csBA$$

Case VII.



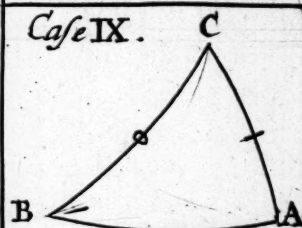
$$R : tCA :: ctB : sBA$$

Case VIII.



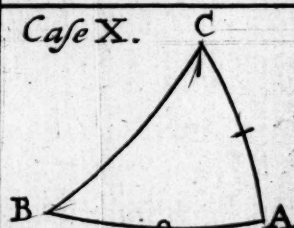
$$csCA : R :: csB : sC$$

Case IX.



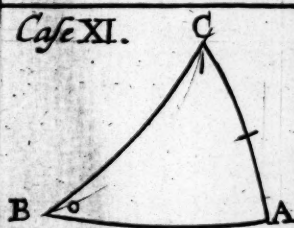
$$sB : R :: sCA : sBC$$

Case X.



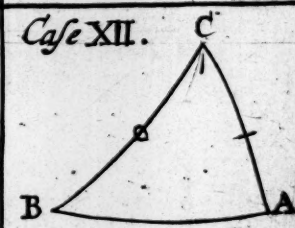
$$ctC : R :: sCA : tBA$$

Case XI.



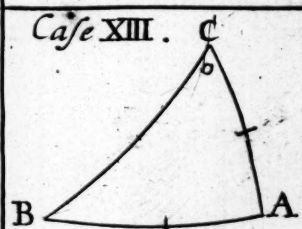
$$R : sC :: csCA : csB$$

Case XII.



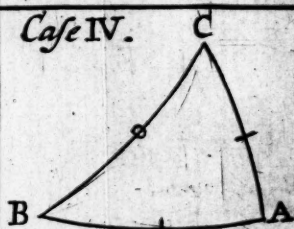
$$tCA : R :: csC : ctBC$$

Case XIII.



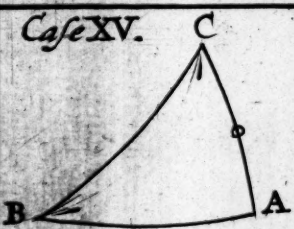
$$tBA : R :: sCA : ctC$$

Case XIV.



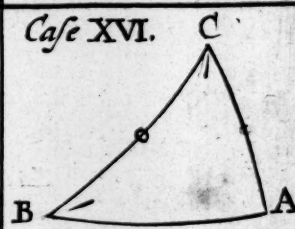
$$R : csCA :: csBA : csCB$$

Case XV.



$$sC : R :: csB : csCA$$

Case XVI.



$$R : ctC :: ctB : csBC$$

As in the *Oblique-angled Triangle* ABC , whose *Angles* at B and C are both *Acute*, the *Perpendicular* AD shall fall within the *Triangle*: For, if it fall not within, it must be the same with one of the *Sides*, or else it must fall without the *Triangle*: If it be the same with either of the *Sides*, then the *Angle* at B or C must be a *Right Angle*; which is contrary to the Proposition: If it fall without the *Triangle*, as suppose at E , then the *Angle* AEB shall be a *Right Angle*: But the *Angle* ABE is *Obtuse*, for it is the Complement of the *Acute Angle* ABC , and therefore the Side AE is greater than a Quadrant: And the *Angle* ACE being *Acute*, AE shall be also less than a Quadrant: But, that the same Side should be both *More* and *Less* than a Quadrant, is absurd: And therefore, in this Case, the *Perpendicular* shall fall within the *Triangle*.

Fig.
XXII.

But, In the *Triangle* AEB , *Obtuse-angled* at B , and *Acute* at E , the *Perpendicular* AD shall fall without the *Triangle* upon the Side EB , contrived: Or, if otherwise, it must be the same with one of the *Sides*, or fall within the *Triangle*: It cannot be the same with either of the *Sides*, for then the *Angle* at B or E should be a *Right Angle*: And it cannot fall within the *Triangle*, because then the *Angles* at B and E must either be both *Obtuse*, or both *Acute*, as hath been already proved. If therefore the *Angles* at the Base be of different *Affections*, the *Perpendicular* shall fall without; as was to be proved.

Fig.
XXII.

However this *Perpendicular* falleth, it must be always *opposite* to a *known Angle*; and for better Direction herein, take this *General Rule*.

RULE II. Let your *Perpendicular* fall from the End of a Side given, and adjacent to an Angle given.

As in this *Triangle* ABC , if there were given the Side AB , and the *Angle* at A ; by the former, and this, *Rule*, the *Perpendicular* must fall from B upon the Side AC .

But if there were given the Side AC , and the *Angle* at A , the *Perpendicular* must fall from C , upon the Side AB , continued to E , — And to know whether the Side upon which the *Perpendicular* shall fall, must be continued or not, is no more than to ask whether the *Perpendicular* must fall *within* or *without* the *Triangle*. But, if the former Directions be not sufficient, the Calculation will determine it. For,

Fig.
XXIII.

RULE

Fig.
XXIII.

RULE III. If the Ark found at the first Operation (whether it be of a Side or an Angle) be more than the Arch given; the Perpendicular shall fall without; if less, within the Triangle.

And this will plainly appear in the Solution of the following Cases.

General Rules to be observed in the Second Operation of the Solution of Oblique-angled Spherical Triangles, when they are reduced into Two Right-angled Triangles.

After the first Operation, whereby either the Segments of the Base, or the Angle at the Cathetus, or Perpendicular, is found; a diligent Care being had to the Addition or Subtraction of them: The second Operation will be performed by one of the Four following Rules.

Fig.
XXIV.

RULE I. The Sines of the Complement of the Hypotenuses, to the Sines of the Complement of the Bases, are in direct Proportion. So,

$$csBA : csDA :: csBC : csDC.$$

RULE II. The Sines of the Bases, to the Tangents of the Angles at the Base, are in Reciprocal Proportion: So,

$$sBA : sDA :: ctB : ctD :: tB : tD.$$

RULE III. The Sines of the Complement of the Angles at the Base; to the Sines of the Complement of the Angles at the Cathetus (or Perpendicular) are in direct Proportion: So,

$$sBCA : sDCA :: csB : csB : csD.$$

RULE IV. The Tangents of the Hypotenuses, to the Sines of the Complement of the Angles at the Cathetus (or Perpendicular) are in Reciprocal Proportion: So,

$$csBCA : csDCA :: ctBC : ctDC :: tDC : tBC.$$

C H A P.

C H A P. V.

The Solution of Oblique-angled Spherical Triangles: By letting fall a Perpendicular, whereby the Oblique Triangle is Reduced into Two Right-angled.

CASE I. In the Oblique-angled Spherical Triangle ABC , there is given, the Two Sides AB and BC , with the Angle C , opposite to BA ; to find the Angle at B : If it be also known whether the enquired Angle be Acute or Obtuse.

Analogy.

$$\text{As } sBA : sBC :: sC : sA.$$

For by the Universal Proposition,

$$(1.) \text{As Rad.} : sAB :: sA : sBD.$$

$$(2.) \text{As Rad.} : sBC :: sC : sBD.$$

Therefore,

$$\text{As } sAB : sBC :: sC : sA.$$

CASE II. Two Angles, A and C , and the Side BC , opposite to the Angle A , given; To find the Side BA : If it be known whether the enquired Side be more or less than a Quadrant.

Analogy.

$$\text{As } sA : sC :: sBC : sBA.$$

CASE III. In the Oblique-angled Spherical Triangle ACD , there is given, the Two Sides AD and AC , with the Angle DAC , contained by them: To find the third Side DC .

In this Case the Perpendicular may fall from the Extremity of either Side, but opposite to the Angle given.

Analogies.

$$(1.) \text{As } cs. AC : \text{Rad.} :: cs. DAC : r. AB.$$

And $AD - AB = BD$ in Triangle I.

But $AD + AB = BD$ in Triangle II.

$$(2.) \text{As } cs. AB : cs. AC :: Ra. : cs. BC.$$

$$(3.) \text{As } Ra. : cs. BC :: cs. BD : cs. DC.$$

There-

Fig.
XXV.

Fig.
XXVI.

Fig.
XXVI.

Therefore,

$$\text{As } c s. AB : c s. AC :: c s. BD : c s. DC.$$

CASE IV. Two Sides AC and DC, with the Angle ADC, opposite to AC given; To find the third Side AD.

In this Case, Let the Perpendicular fall from the Concourse of the given Sides, upon the Side enquired, continued, if need be.

Analogies.

$$(1.) \text{As } c t. CD : \text{Rad.} :: c s. ADC : t. BD.$$

$$(2.) \text{As } c s. BD : c s. CD :: \text{Rad.} : c s. CB.$$

$$(3.) \text{As } c s. CB : \text{Rad.} :: c s. AC : c s. AB.$$

Therefore,

$$c s. CD : c s. BD :: c s. AC : c s. AB.$$

And, $BD + AB = AD$ in the first } Triangle.
And, $BD - AB = AD$ in the second

CASE V. Two Sides CA and DA, and their contained Angle CDA given; To find one of the other Angles.

In this Case the Perpendicular may fall from the Extremity of either of the Sides, opposite to the Angle given.

Analogies.

$$(1.) \text{As } c t. AC : \text{Rad.} :: c s. DAC : t. AB.$$

$$\text{And } AD - AB = BD \text{ in the first}$$

$$\text{And } AD + AB = BD \text{ in the second } \} \text{ Triangle.}$$

$$(2.) \text{As } c t. CAD : s. AB : \text{Rad.} : t. BC.$$

$$\text{As } t. BC. \text{Rad.} :: s. BD : ct. ADC.$$

Therefore,

$$\text{As } s. AB : ct. CAD :: s. BD : ct. ADC.$$

CASE VI. Two Angles, ADC and CAD, and the Side AC, opposite to one of them being given; To find the Side between them. If it be known whether the Side sought, or the Side opposite to the other given Angle, be Acute or Obtuse.

In this Case, Let the Perpendicular fall from the Extremity of the given Side, on the Side enquired, continued, if need be.

Analogies.

Of Spherical Triangles

73

Analogies.

- (1.) As *ct.* AC : Rad. :: *cs.* DAC : *t.* AB.
- (2.) As *ct.* CAD : *s.* AB :: Rad. : *t.* BC.
- (3.) As Rad. : *t.* BC :: *ct.* ADC : *s.* BD.

Fig.
XXVI.

Therefore,

As *ct.* CAD : *s.* AB :: *ct.* ADC : *s.* BD.

And, AB + BD = AD in the 1st } Triangle.
But, DB - AB = AD in the 2d }

CASE VII. Two Angles DAC and ADC, and the Side AC, opposite to one of them, (viz. D) being given; To find the other Angle at C. If it be known, whether the Angle enquired, or the Side opposite to the other given Angle, be *Acute* or *Obtuse*. Fig. XXVI. Case VII.

In this Case, Let the Perpendicular fall from the Angle enquired.

Analogies.

- (1.) As *ct.* CAD : Rad. :: *cs.* AC : *ct.* ACB.
- (2.) As *s.* ABC : *cs.* CAB :: Rad. : *cs.* BC.
- (3.) As *cs.* BC : Rad. :: *cs.* BDC : *s.* BCD.

Therefore,

As *cs.* CAB : *s.* ACB :: *cs.* BDC : *s.* BCD.

And, ACB + BCD = ACD in the 1st } Triangle.
But, BCD - ACB = ACD in the 2d }

CASE VIII. Two Angles, ACD and CAD, with the Side between them, AC, being given; To find the third Angle D. Fig. XXVI. Case VIII.

In this Case, Let the Perpendicular fall from the Extremity of the given Side, and opposite to the Angle enquired.

Analogies.

- (1.) As *ct.* CAB : Rad. :: *cs.* AC : *ct.* ACB.

And, ACD - ACB = BCD in the 1st } Triangle.
But, ACD + ACB = BCD in the 2d }

- (2.) As *s.* ACB : *cs.* CAB :: Rad. : *cs.* BC.
- (3.) As Rad. : *cs.* BC :: *s.* BCD : *cs.* CDB.

L

There-

Therefore,

$$\text{As } s. ACB : c s. CAB :: s. BCD : c s. CDB.$$

Fig. XXVI. *CASE IX. Two Sides, AC and CD, with the Angle ADC, opposite to one of them, (viz. AC) being given; To find their contained Angle ACD. If it be known, whether the enquired Angle, or the Angle opposite to the other given Side, be Acute or Obtuse.*

In this Case, Let the Perpendicular fall from the Angle enquired.

Analogies.

- (1.) As *ct. CDB* : Rad. : : *c s. CD* : *ct. BCD*.
- (2.) As *ct. CD* : *c s. BCD* : : Rad. : *t. BC*.
- (3.) As Rad. : *t. BC* : : *ct. AC* : *c s. ACB*.

Therefore,

$$\text{As } ct. CD : c s. BCD :: ct. AC : c s. ACB.$$

And $BCD + ACB = ACD$ in the 1st } Triangle.
But $BCD - ACB = ACD$ in the 2^d

Fig. XXVI. *CASE X. Two Angles, DAC and ACD, with the Side between them, AC, being given; To find either of the other Sides, DC or AD.*

In this Case, Let the Perpendicular fall from the Concourse of the Side given and sought, on the third Side, continued, if need be.

Analogies.

- (1.) As *ct. CAB* : Rad. : : *c s. AC* : *ct. ACB*.
And, $ACD - ACB = BCD$ in the 1st } Triangle.
But, $ACD + ACB = BCD$ in the 2^d
- (2.) As *ct. AC* : *c s. ACB* : : Rad. : *t. BC*.
- (3.) As *t. BC* : Rad. : : *c s. BCD* : *ct. CD*.

Therefore,

$$\text{As } c s. BCA : ct. AC :: c s. BCD : ct. CD.$$

CASE

Of Spherical Triangles.

75

CASE XI. *The Three Sides being given ; To find an Angle.*

Fig.
XXVI.
Case XI.

For the resolving of this *Problem*, there must be some Preparation made; for that the *Universal Proposition* of the Lord *Nepier's* is not, of it self, sufficient for the Solution of this or the following *Case*. And therefore, the said Lord *Nepier's*, to bring these *Cases* within some Compass of this his *Universal Proposition*, he first finds the Difference of the Segments of that *Side*; which being made the Base of the *Triangle*, is divided into Two Parts, by letting fall of a *Perpendicular*; and by help of this

ANALOGY.

As the Tangent of half the Base,
Is to the Tangent of half the Sum of the other Two *Sides*;
So is the Tangent of half the Difference of those *Sides*,
To the Tangent of half the Difference of the Segments of the Base.

Thus then,

In the *Oblique-angled Spherical Triangle* A C D, there is given, the Two *Sides* (or *Legs*) A C and C D, together with the Base A D; to find the *Angle* C A D.

Analogies.

As the Tangent of half A D,
Is to the Tangent of half A C and A D;
So is the Tangent of half the Difference of A C and A D,
To the Tangent of half A E.

And half A D + half A E = A B in the 1st
But half A E — half A D = B E or B D in the 2d } *Triangle.*

Fig.
XXVII.

Hence, to find the *Angle* at A.

As Rad. : *c t.* A C : : *t.* A B : *c s.* C A B.

CASE XII. *The Three Angles being given ; To find a Side.*

Fig.
XXVII.
Case XII.

This *Case* is but the *Converse* of the last beforegoing, and is to be Solved after the same manner: If so be that we convert the *Angles* into *Sides*. Which, how to perform, is shewed in the 12th *Case* of the following *Treat* of the *Solution of Oblique-angled Spherical Triangles*, without letting fall a *Perpendicular*.

Fig. And for the *Proportions* or *Analogies* for the Resolving of this
XXVII. Case there are also Two Ways proposed, to which I refer you.

C H A P. VI.

The foregoing XII Cases, of Oblique-angled Spherical Triangles, Geometrically Demonstrated, and Resolved, without letting fall a Perpendicular.

FOR the Performance of what is here promised, these Six *Theorems* following, being demonstrated, will clear.

T H E O R E M I.

In any Spherical Triangle, whether Right or Oblique-angled.

The Sines of the Angles are Proportional, to the Sines of their opposite Sides: & contra.

This is demonstrated in *Seet. 1. Chap. 4. of Right-lined Triangles*, and is the same in *Spherical Triangles*, whether Right or Oblique Angular.

T H E O R E M II.

In all Oblique-angled Spherical Triangles, whose Three Sides together are less than a Semicircle, or 180 Degrees.

*As the Sine of half the Sum of the Angles at the Base,
 Is to the Sine of the half Difference of those Angles;
 So is the Tangent of half the Base,
 To the Tangent of half the Difference of the Sides.*

And also,

*As the Co-sine of half the Sum of the Angles at the Base,
 Is to the Co-sine of half the Difference of those Angles;
 So is the Tangent of half the Base,
 To the Tangent of half the Sum of the Sides.*

Of Spherical Triangles.

77

DEMONSTRATION.

It is already proved; That,
The Sine of the Sum of the Angles B and E,
Is to the Sine of the Difference of those Angles B and E;
As the Tangent of half the Base B E,
Is to the Tangent of half B C.

Fig.
XXVIII.

And multiplying the latter part of this Proportion, by the Tangent of half the Base B E: It is,

As the Sine of the Sum of the Angles B and E,
Is to the Sine of the Difference of those Angles B and E;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangent of half B E, and the Tangent of half B C.

But, the Rectangle made of the Tangent of half B E, and the Tangent of half B C, is equal to the Rectangle made of the Tangent of the half Sum, and half Difference, of the Sides B A and A E: Therefore,

As the Sine of the Sum of the Angles B and E,
Is to the Sine of the Difference of the Angles B and E;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangent of the half Sum, and half Difference of the Sides B A and A E.

And the former Part of this Proportion, being multiplied by the Sine of the Sum of the Angles B and E: It is,

As the Square of the Sine of the Sum of the Angles B and E,
Is to the Rectangles made of the Sines of the Sum, and Difference of those Angles;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangents of the half Sum, and half Difference of the Sides B A and A E.

But, the Rectangle made of the Sines of the Sum and Difference of the Angles, is equal to the Rectangle made of the Sum and Difference of the Sines.

And therefore,

As the Square of the Sine of the Sum of the Angles B and E,

Is

Fig. XXVIII. Is to the Rectangle made of the Sum and Difference of the Sines;
 So is the Square of the Tangent of half B E,
 To the Rectangle made of the Tangent of the half Sum, and half
 Difference of the Sides B A and A E.

*H. As the Difference of the Sines of B and E,
 Is to the Sum of the Sines of the Angles B and E;
 So is the Tangent of the half Difference of the Sides,
 To the Tangent of the half Sum of the Sides B A and A E.*

And multiplying the former Part of this Proportion, by the Difference of the Sines of the Angles; and the latter Part thereof, by the Tangent of half the Difference of the Sides A B and A E: It will be,

*As the Square of the Difference of the Sines of B and E,
 Is to the Rectangle made of the Sum and Difference of the Sines
 of B and E;*

*So is the Square of the Tangent of the half Difference of B A
 and A E,
 To the Rectangle made of the Tangents of the half Sum, and half
 Difference of the Sides.*

But, by the First Section of this Demonstration it is proved, That,

*As the Square of the Sine of the Sum of B and E,
 Is to the Rectangle made of the Sum and Difference of the Sines;
 So is the Square of the Tangent of half B E,
 To the Rectangle made of the Tangents of the half Sum, and half
 Difference of the Sides.*

Therefore,

*As the Square of the Sine of the Sum of B and E,
 Is to the Square of the Difference of the Sine of B and E;
 So is the Square of the Tangent of half B E,
 To the Square of the Tangent of half the Difference of the Sides.*

And also,

*As the Sine of the Sum of the Angles B and E,
 Is to the Difference of their Sines;
 So is the Tangent of half the Base B E,
 To the Tangent of half the Difference of the Sides B A and A E.*

But,

Of Spherical Triangles.

79

But,

As the Sine of the half Sum of B and E,
Is to the Sine of their half Difference;
So is the Sine of the Sum,
To the Difference of the Sines.

Fig.
XXVIII.

And therefore,

As the Sine of the half Sum of the Angles B and E,
Is to the Sine of the half Difference of B and E;
So is the Tangent of half B E,
To the Tangent of half the Difference of the Sides B A and A E,
which is the first Part of the Proposition.

III. Having already proved,—That the Sum of the Sines of the Angles B and E, is to the Difference of the Sines of those Angles; as the Tangent of the half Sum of the Sides, is to the Tangent of their half Difference. Therefore,

If you multiply the former Part of this Proportion, by the Sum of the Sines of the Angles of B and E; and the latter Part thereof by the Tangent of the half Sum of the Sides of A B and A E: Then it will be,

As the Square of the Sum of the Sines of B and E,
Is to the Rectangle made of the Sum and Difference of the Sines;
So is the Square of the Tangent of half the Sum of the Sides;
To the Rectangle made of the Tangent of the half Sum, and half Difference of the Sides.

But,

As the Square of the Sine of the Sum of B and E,
Is to the Rectangle made of the Sum and Difference of the Sines;
So is the Square of the Tangent of half B E,
To the Rectangle made of the Tangent of the half Sum, and half Difference of the Sides B A and A E.

Therefore,

As the Square of the Sine of the Sum of B and E,
Is to the Square of the Sum of the Sines of those Angles;
So is the Square of the Tangent of half B E,
To the Square of the Tangent of the half Sum of the Sides B A and A E.

And,

Fig.
XXVIII.

And,

*As the Sine of the Sum of the Angles B and E,
Is to the Sum of the Sines of the Angles B and E;
So is the Tangent of half the Base B E,
To the Tangent of the half Sum of the Sides A B and A E.*

But,

*As the Co-sine of the Sum of B and E,
Is to the Co-sine of the Difference of the Angles B and E;
So is the Sine of the Sum of the Angles B and E,
To the Sum of the Sines of the said Angles.*

And therefore,

*As the Co-sine of the Sum of the Angles B and E,
Is to the Co-sine of the Difference of the Angles B and E;
So is the Tangent of half the Base B E,
To the Tangent of the half Sum of the Sides A B and A E.
Which was to be Demonstrated.*

THEOREM III.

In all Spherical Triangles:

*As the Difference of the Versed Sines, of the Sum and Difference
of any Two Sides (including an Angle),
Is to the Diameter;
So is the Difference between the Versed Side of the Third Side,
and the Versed Sine of the Difference of the other Two Sides,
To the Versed Sine of the Angle comprehended by the said Two Sides.*

DEMONSTRATION.

Fig.
XXIX.

Let the Sides of the Triangle Z S P be known, and let the Vertical Angle be S Z P: Then shall Z S, the one Side, be equal to Z R, and P R equal to their Sum; and P B the Versed Sine of P R, and P C, is the Difference of the Sides Z S and Z P, and the Versed Sine of P C, is P M.

Now then, M B is the Difference between B P, the Versed Sine of P R, the Sum of the Sides, and P M the Versed Sine of P C, the Difference of the Sides.

M H is the Difference between P H the Versed Sine of P S, and P M the Versed Sine of P C the Difference of the Sides: Q V is

Of Spherical Triangles.

81

is the Diameter, and O V the Versed Sine of P Z S the Angle Sought: And the Right Lines N C, K L, and R G, being Parallel, by the Work, their Inter-segments M B and R C, and also M H and S C, are proportional.

Fig.
XXIX.

And therefore,

$$\begin{aligned} M B : M H :: R C : S C, \\ R C : S C :: Q V : O V, \end{aligned}$$

Therefore, $M B : M H :: Q V : O V,$
Or, $M B : M H :: \text{half } Q V : \text{half } O V.$

Which was to be Demonstrated.

T H E O R E M IV.

In all Spherical Triangles;

As the Rectangle of the Sines of the Sides containing the Angle enquired,

Is to the Square of the Radius;

So is the Difference between the Versed Sine of the Base, and the Versed Sine of the Difference of the other Two Sides, To the Versed Sine of the Angle sought.

D E M O N S T R A T I O N.

Let the Sides of Triangle A E K be given, and let the Angle at A be enquired; and from O, let fall the Perpendicular O B.

Fig.
XXX.

Now then, O E being the Difference of the Sides A E and A K, equal to A O; the Right Line O Q is the Right Sine thereof, and E Q the Versed Sine. In like manner, S M is the Right Sine, and E M the Versed Sine of E S; that is, of the Base E K; and M Q or O B, is the Difference of those Versed Sines.

O K, is the Versed Sine of the Angle O A K, in the Measure of the Parallel O F; and D X is the Versed Sine of the same Angle in the measure of a Great Circle, whose Diameter is H D. Now then, because of their like Arches, it shall be

$$\text{As } D C : D X :: O L : O K:$$

And because N E and O K are Parallel, as also E C and O B, the Angles B O K and C E N are equal: And the Triangles E C N and O K B like; and therefore the Sides N E, E C, O B, M Q and O K, are Proportional: And it will be

M

As

Fig.
XXX.

$$\text{As } LO : DC :: OK : DX, \\ NE : EC :: OB : OK.$$

It will also be

$$\text{As } LO * NE : DC * EC :: OK * OB : DX * OK.$$

Or rejecting the common Altitude OK, it will be

$$\text{As } LO * NE : DC * EC :: OB : DX,$$

The Versed Sine of the Angle sought. Which was to be Demonstrated.

THEOREM V.

In all Spherical Triangles,
As the Rectangle of the Sines of the Sides containing the enquired Angle,
Is to the Square of the Radius;
So is the Rectangle of the Sines of the half Sum, and half Difference of the Base, and Difference of the Legs,
To the Rectangle made of the Radius, and half the Versed Sine of the Angle enquired.

DEMONSTRATION.

Fig.
XXXI. It is already proved by the last Theorem,
That, As $LO * NE$: to the Square of the Radius:
So is $OB : DX$.

And therefore also,

$$LO * NE : \text{Sq. of Rad.} :: \text{half } OB : \text{half } DX.$$

And now in the following Diagram, O E G H, the Sum of ES and OE, is the double Measure of the Angle BSO; and the Arch OS is the Difference between the Base ES and EO, the Difference of the Sides AK and AE; And,

$$\text{As } R : \text{half } OS :: OH : \text{half } OB. \text{ And,} \\ \text{Half } OS, \text{ and half } OH, \text{ is equal to } OB * \text{Rad.}$$

Therefore,

$$LO * NE : \text{Sq. Rad.} :: \text{half } OB * \text{Rad.} : \text{half } DX * \text{Rad.}$$

And

Of Spherical Triangles.

83

And also,

$$LO * NE : Sq. R. :: half OS * half OH * : half DX * R.$$

Which was to be Demonstrated.

Fig.
XXXI.

THEOREM VI.

In all Spherical Triangles,

As the Rectangle of the Sines of the Sides containing the enquired Angle,

Is to the Square of Radius ;

So is the Rectangle of the Sines of the half Sum, and half Difference of the Base, and Difference of the Legs,

To the Square of the Sine of half the Angle enquired.

DEMONSTRATION.

It is already proved by the last Theorem, That

$$LO * NE : Sp. R. :: half OS * half OH : half DX * R.$$

Fig.
XXXI.

But the Rectangle made of Radius, and half the Versed Sine of an Arch, is equal to the Square of the Sine of half the Arch :

As in the foregoing Diagram ; let the Arch given be DT, then is DX the Versed Sine of that Arch ; and DF the Right Sine of half the Arch ; and the Triangles DFR and DTX are like.

Therefore,

$$DR : DF :: half DT (= DF) : half DX :$$

And, $DR * half DX$, is equal to the Square of DF.

Therefore,

$$LO * NA : Rq. :: half OS * half OH : the Square of DF.$$

Which was to be Demonstrated.

C H A P. VII.

The Solution of the Twelve Cases of Oblique-angled Spherical Triangles, without any Regard had to a Perpendicular let fall, whereby to reduce it into Two Right-angled Triangles.

Fig. XXXII. **A**ND the Spherical Triangle which I shall make use of is that noted with Z S P, whose Sides and Angles, both in Sexagenary Degrees and Minutes, and Decimal Parts also, are as in this Table is expressed.

	De.	Mi.	Centef.
S P —	42	04	42.07
Z S —	30	00	30.00
P Z —	24	04	24.06
∠ Z —	104	00	104.00
∠ S —	36	08	36.13
∠ P —	46	18	46.30

Fig. XXXII. **CASE I.** Two Sides, Z S and Z P, with the Angle P, opposite to one of them, being given; To find the Angle S, opposite to the other.

Analogy.

As the Sine of Z S, 30 Deg. Co-Ar.	0.3010299
Is to the Angle P, 46 Deg. 18 Min.	9.8591186
So is the Sine of Z P, 24 Deg. 4 Min.	9.6104465
To the Sine of S, 36 Deg. 8 Min.	9.7705850

Fig. XXXII. **CASE II.** Two Angles, S and P, with the Side Z P, opposite to one of them, being given; To find the Side Z S, opposite to the other

Analogy.

As the Sine of the Angle S, 36 Deg. 8 Min. Co-Ar.	0.2293936
Is to the Sine of Z P, 24 Deg. 4 Min.	9.6104465
So is the Sine of the Angle P, 46 Deg. 18 Min.	9.8591180
To the Sine of Z S, 30 Deg.	9.6989581

CASE

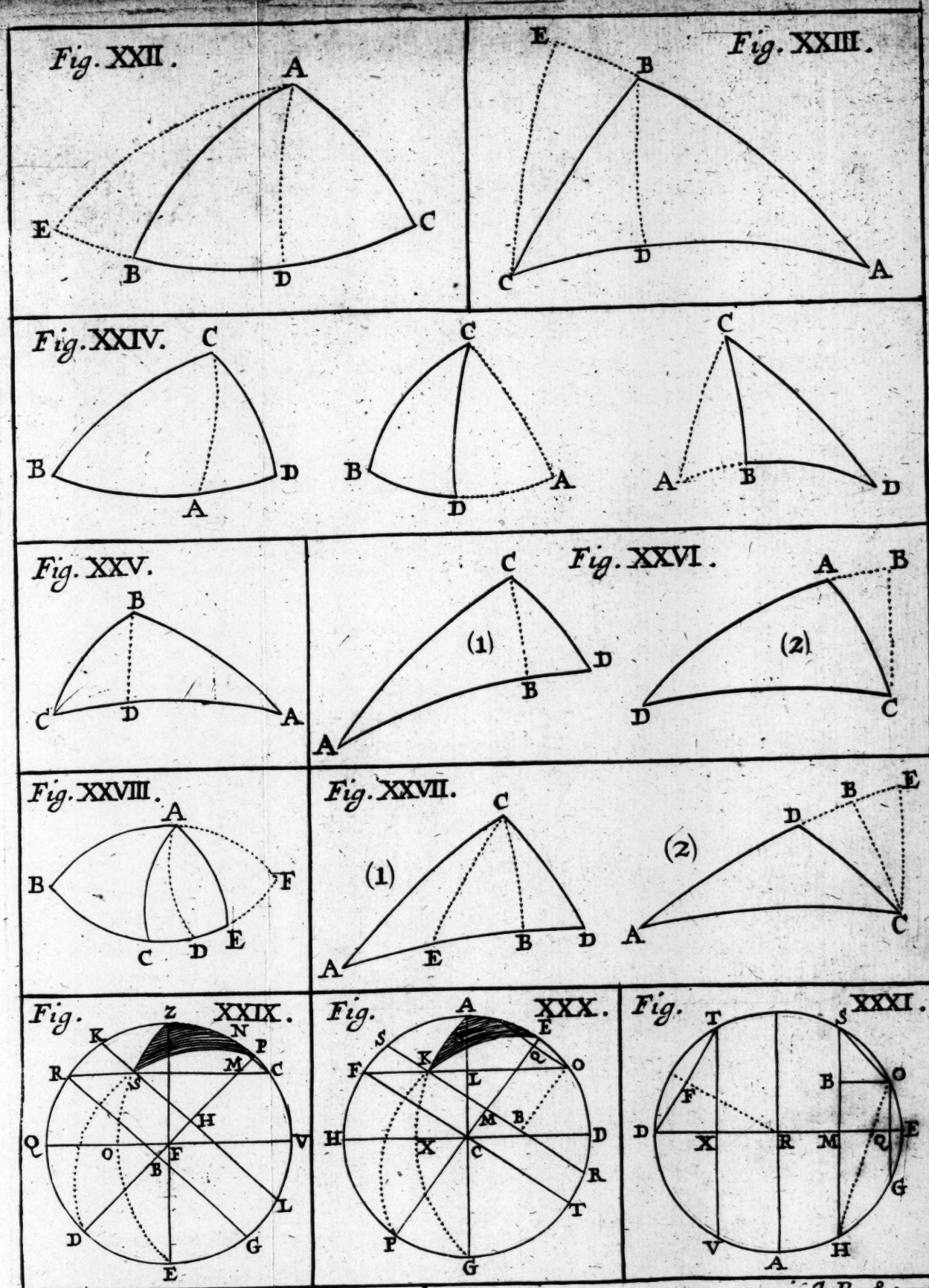


Plate VIII. From Pag. 68. to Pag. 84. against P. 84.

Of Spherical Triangles.

85

CASE III. Two Sides, ZP and ZS, with the Angle SZP, Fig. XXXII. S and P. To find the other Angles, Case III.

Sides	$\left\{ \begin{array}{l} ZS \\ ZP \end{array} \right.$	d.	'
		30	00
		24	04
Sum		54	04
Difference		5	56
Half Sum		27	02
Half Difference		2	58
Half the Angle Z.		52	00

Analogy.

First Operation.

As the Sine of half the Sum of the Sides 27 d. 2' Co-Ar. 2.3424577
 Is to the Sine of half the Difference of the Sides 2.58 8.7139520
 So is the Co-tangent of half Z, 52 d. 9.8928098
 To the Tangent of half the Difference of the $\angle$'s S and P, 5 d. 5 m. $\times 8.9492195$

Second Operation.

As Co-sine of half the Sum of the Sides 27 d. 2 m. $\left\{ \begin{array}{l} \text{Co-Ar.} \\ \text{To Co-sine of half their Difference 2 d. 58 m.} \end{array} \right. 0.0502479$
 So Co-tangent of half $\angle$ Z, 52 d. 9.8928098
 To Tangent of half the Sum of the $\angle$'s S and P $\left\{ \begin{array}{l} 41.31' \\ \text{And subtracted, gives the lesser } \angle \text{ S.} \end{array} \right. \times 9.9424752$

	d.	m.
Which half Sum of the Angles S and P	41	13
Added to half Difference of the Angles	5	05
Gives the Quantity of the greater $\angle$ P	46	18
And subtracted, gives the lesser $\angle$ S.	36	08

CASE

Fig. XXXII. CASE IV. Two Angles, S and P, and their contained Side SP, being given; To find the other Two Sides, ZS and ZP.
Case IV.

Angles	{	S	36	08
		P	46	10
Sum			82	18
Difference			10	02
Half Sum			41	09
Half Difference			5	1

Analogy.

First Operation.

As Sine of half the Sum of the Angles S and P	}	0.1817525
41 d. 9 m. Co-Ar.		
Is to the Sine of half their Difference 5 d. 1 m.		8.9402960
So is the Tangent of half the Side S P 21 d. 5 m.		9.5856859
To the Tangent of half the Diff. of ZS and ZP	}	18.7077344
2 d. 55 m.		

Second Operation.

As Co-sine of half the Sum of the Angles S and P	}	0.1232111
41 d. 9 m. Co-Ar.		
To Co-sine of half their Difference 5 d. 1 m.		9.9983331
So is the Tang. of half the Side S P 21 d. 5 m.		9.5863624
To Tan. of half the Sum of ZS and ZP		19.7076060
27 d. 2 m.		

	d.	m.
To this half Sum of ZS and ZP	27	02
Add half their Difference	2	55
Their Sum is the Greater Side ZS	29	57
And subtracted gives the Lesser ZP	24	07

CASE

Of Spherical Triangles.

87

CASE V. Two Sides ZS, and ZP; with the Angle P, opposite to one of them, being Given; To find the third Side SP.

Fig. XXXII. Case V.

First Operation.

As the Sine of the Side ZS, 30 d. Co-Ar. 0.3010299
 Is to the Sine of the Angle P, 46 d. 18 m. 9.8591185
 So is the Sine of the Side ZP, 24 d. 4 m. 9.6104465
 To the Sine of the Angle at S, 36 d. 8 m. 9.7705950

Then,

The Sides are			The Angles are		
ZS	30	00	P	46	18
ZP	24	04	S	36	08
Their Sum	54	04		82	26
Their Difference	5	56		10	10
Half Sum	27	02		41	13
Half Difference	2	58		5	05

Second Operation.

As the Sine of the half Difference of the Angles } 1.0525439
 S and P, 5 deg. 5 min. Co-Ar. }
 Is to the Sine of half the Sum of those Angles 41 d. 13 m. 9.8188250
 So is the Tangent of half the Difference of the Sides } 8.7139520
 ZS and ZP, 2 deg. 58 min. }
 To the Tangent of half SP, 21 deg. 4 min. 9.5853209
 Whose double is 42 deg. 9 min. for SP.

CASE VI. Two Angles, S and P, with the Side SZ, opposite to one of them, being given; To find the third Angle Z.

Fig. XXXI. Case VI.

First Operation.

As the Sine of the Angle at P, 46 d. 18 m. Co-Ar. 0.1408813
 Is to the Sine of the Side ZS, 30 deg. 9.6989700
 So is the Sine of the Angle at S, 36 deg. 8 min. 9.7700063
 To the Sine of the Side ZP, 24 deg. 4 min. 9.6098576

Then,

Fig.
XXXII.

Then,

The Sides			The Angles	
Z S	30 00		S	36 08
Z P	24 04		P	46 18
Their Sum	54 04		82 26	
Their Difference	5 56		10 10	
Half Sum	27 02		41 13	
Half Difference	2 58		5 05	

Second Operation.

As the Sine of half the Difference of the Sides Z S } 1.2860479
and Z P, 2 d. 58 m. Co-Ar.

Is to the Sine of half the Sum of the Sides, 41 d. 13 m. 9.8188250

So is the Tangent of half the Difference of the An- } 8.9491675
gles S and P, 5 d. 5 m.

To the Co-tangent of half the Angle Z, 52 deg. 10.0540404

Whose Double, 104 deg. is the Angle Z required.

Fig. XXXII. CASE VII. Two Angles, S and P, and a Side Z S, opposite to
One of them, being given; To find the Side between them, SP.
Case VII.

First Operation.

As the Sine of the Angle at P, 46 deg. 18 min. Co-Ar. 0.1409913

Is to the Sine of the Side Z S, 30 deg. 9.6989700

So is the Sine of the Angle at S, 36 deg. 8 min. 9.7706061

To the Sine of the Side Z P, 24 deg. 4 min. 9.6105676

Then,

The Sides				The Angles		
Z S	30	00		S	36	08
Z P	24	04		P	46	18
Their Sum	54	04			82	26
Their Difference	5	56			10	10
Half Sum	27	02			41	13
Half Difference	2	58			5	5

Second

Of Spherical Triangles.

89

Second Operation.

As the Sine of half the Difference of the Angles } 1.0525439
 S and P, 5 deg. 5 min. *Co-Ar.*
 Is to the Sine of half their Sum, 41 deg. 13 min. } 9.8188250
 So is the Tangent of half the Difference of the Sides } 8.7145345
 Z S and Z P, 2 deg. 58 min.
 To the Tangent of half the enquired Side, 21 deg. } 9.5859034
 4 $\frac{1}{2}$ min.
 The Double whereof is 42 deg. 9 min. for the Side S P.

CASE VIII. Two Sides, Z S and Z P, and the Angle P, opposite to the Side Z S, being given; To find the Angle Z, contained between the Two given Sides.

Fig.
XXXI.
Case VIII.

First Operation.

As the Sine of Z S, 30 deg. *Co-Ar.* } 0.3010299
 Is to the Sine of the Angle P, 46 deg. 18 min. } 9.8591186
 So is the Sine of Z P, 24 deg. 4 min. } 9.6104465
 To the Sine of the Angle at S, 36 deg. 8 min. } 9.7705950

Then,

The Sides			The Angles		
Z S	30	00	S	36	08
Z P	24	04	P	46	18
Their Sum	54	04		82	26
Their Difference	5	56		10	10
Half Sum	27	02		41	13
Half Difference	2	58		5	05

Second Operation.

As the Sine of half the Difference of the Sides Z S } 1.2860479
 and Z P, 2 deg. 58 min. *Co-Ar.*
 Is to the Sine of half their Sum, 27 deg. 2 min. } 9.6575423
 So is the Tangent of half the Difference of the } 8.9491675
 Angles S and P, 5 deg. 5 min.
 To the Co-tangent of half Z, 52 deg. } 9.8927577
 The Double whereof, 140 deg. is the Angle at Z required.

N

CASE

Fig. XXXI. *CASE IX. Two Sides Z S and Z P, with the Angle Z, comprehended between them; To find the third Side S P.*

Cafe IX. You must first find the Two other Angles S and P (by the 3d Cafe) and then you may find the Side S P (by the 1st Cafe hereof.)

And so,

	d.	m.
The Angle $\left\{ \begin{matrix} S \\ P \end{matrix} \right\}$ will be found to be $\left\{ \begin{matrix} 36 & 08 \\ 46 & 18 \end{matrix} \right\}$		
And the Enquired Side S P	42	39

This Problem may be otherwise resolved at Two Operations, by a Problem next following the Twelve Cafes.

Fig. XXXI. *CASE X. Two Angles, S and P, with the Side between them, S P being given; To find the third Angle at Z.*

Cafe X.

This Cafe must be resolved in the same manner as the foregoing, it being but the Converse thereof: By finding first the Two Sides, Z S and Z P, (by the 4th Cafe) and then the Angle Z (by the 1st Cafe.) So you shall find

	d.	m.
The Side $\left\{ \begin{matrix} Z S \\ Z P \end{matrix} \right\}$ to contain $\left\{ \begin{matrix} 30 & 00 \\ 24 & 04 \end{matrix} \right\}$		
And the Enquired Angle Z	104	00

Fig. XXXI. *CASE XI. Three Sides, Z S, Z P and S P, being given; To find the Angle at Z, opposite to (the Base) S P.*

Cafe XI.

Note, I call that Side the Base (which ever it be) that is opposite to the Enquired Angle. Then,

First Operation.

As the Radius, or Sine of 90 deg.

Is to the Sine of the Side Z S, 30 deg.

So is the Sine of the Side Z P, 24 deg. 4 min.

To a fourth Sine, viz. 11 deg. 46 min.

10.
9.6989700
9.6104465
12.3094165

Then,

Of Spherical Triangles.

91

Fig.
XXXI.
Case XI.

Then,

	d.	m.
The Sides are { Z P	24	04
{ Z S	30	00
{ S P	42	09
Their Sum	96	13
Half Sum	48	06 $\frac{1}{2}$
The Base S P	42	09 Subst.
Difference	5	57 $\frac{1}{2}$

Second Operation.

As the Sine of the fourth Sine before found, 11 deg. 46 min. Co-Ar. } 0.6905835

Is to the Sine of the half Sum, 48 deg. 6 $\frac{1}{2}$ min. 9.8718115
So is the Sine of Difference 5 deg. 57 $\frac{1}{2}$ 9.0162186

To a seventh Sine, 19.5786136
Half this seventh Sine is 9.7893068

Which is the Sine of 38 deg. whose Complement 52 deg. is half the enquired Angle at Z, viz. 104 deg.

Another Way to resolve this Case.

Take half the Difference between the Two Sides that contain the required Angle; and add it to half the Base, and likewise subtract it from the same, noting the Sum and Difference. Then,

The Arithmetical Complements of the Sines of the Sides comprehending the enquired Angle, added to the Sines of the Sum and Difference before found, half the Sum of them shall be the Sine of half the Angle required.

	d.	m.
The Side { Z S	30	00
{ Z P	24	04
Their Difference is	5	56
The half Difference is	2	58
Which added to half S P	21	04 $\frac{1}{2}$
The Sum is	24	02 $\frac{1}{2}$
And subtracted, the Difference is	18	6

N 2

Then,

Then,

The Sine of the Side Z S, 30 deg. Ar. Co.	0.3013299
The Sine of the Side Z P, 24 deg. 4 min. Ar. Co.	0.3895535
The Sine of the Sum, 24 deg. 2 $\frac{1}{2}$ min.	9.6190219
The Sine of the Difference, 18 deg. 6 $\frac{1}{2}$ min.	9.4924864
The Sum	<u>19.7933917</u>
The half whereof is the Sine of 52 deg. 1 min. half of the Angle at Z.	} 9.8966958

CASE XII. Three Angles Z, P and S, being given; To find any of the Sides.

This is the Converse of the former, and is to be resolved after the same manner: But first, the Angles must be turned into Sides as followeth: For,

The Two Lesser Angles, S and P, are always equal unto Two Sides of another Triangle, comprehended by the Arches of Three Great Circles drawn from their Poles, and the Complement of the Greater Angle Z to a Semicircle (or 180 deg.) must be taken for the third Side. Therefore, in this Triangle thus converted, you shall by the preceding 11th Case find an Angle; that Angle so found, shall be one of the Three Sides enquired.

Fig. As in the Triangle A C D, the Poles of those Arches are H, R and Q; which connected, make the Triangle H R Q the Sides of the former Triangle, being equal to the Angles of the latter; taking for one of them the Complement of the Greater Angle to a Semicircle.

So A D is equal to the Angle at H; whose Measure is the Arch E N.

D C is equal to the Angle Q; whose Measure is the Arch M P. And A C is equal to the Complement of the Angle H R Q; whose Measure is G L.

Therefore, if the Angles A, D and C, be given, the Sides Q R, Q H and R H, are likewise given.

If therefore we resolve the Triangle H R Q by the Directions of the 11th Case beforegoing, the Angle so found shall be the Side required.

C H A P.

C H A P. VIII.

A Problem, and Theorem, by Way of Supplement.

In any Spherical Triangle, Two Sides, D B and B C, with the Angle B, included between them being given; To find the other Angles, B and C, by Two Proportions.

THIS Problem was invented by the Lord *Nepier*, and is celebrated, not less for its Subtlety, than Usefulness, in resolving this Case, without letting fall a Perpendicular, or any Ambiguity. Fig. XXXIV.

A N A L O G I E S.

1. As the Sine of half the Sum of the Sides including the Angle given,
Is to the Sine of half the Difference of those Sides;
So is the Co-tangent of half the Vertical Angle,
To the Tangent of half the Difference of the Two unknown Angles.

Then,

2. As the Co-sine of half the Sum of the Two given Sides,
Is to the Co-Sine of half the Difference of those Sides,
So is the Co-tangent of half the Vertical Angle,
To the Tangent of half the Sum of the Angles.
The half Difference before found, being added to this half Sum now found, gives the Greater, and subtracted therefrom, gives the Lesser of the Two unknown Angles.

D E M O N S T R A T I O N.

Suppose A, E, G, Poles of the Sides D B, D C, B C, therefore the Arch A E is equal to the Angle D; the Arch E G equal to the Angle C; the Arch A G equal to 180 deg. wanting the Angle B. Suppose the Ark E O, equal to the Ark E G, equal to the Ark E P. Then if the Points G A O E P be Stereographically projected, the Right Line A E will be equal to Tangent of half the Angle at D; and A G, equal to the Co-tangent of half B; and A O, equal to the Tangent of half the Difference of the Angles; and A P, equal to the Tangent of half the Sum of the Angles; and O G P will be a Semicircle, described from its Pole E, through G, whose

Fig. XXXIV. whose Center suppose n . Then take $B\lambda$, equal to BC , equal to BL ; and draw the Diameter Bm $A\beta$, and DK , parallel to Bm , parallel to δH . Therefore, DB is equal to δB ; and mK , equal to mH ; and $F\lambda$, equal to DL . Draw $\lambda\omega$ perpendicular to $D\delta$, then the Triangle $\lambda\delta y$ is equiangular to the Triangle DLy ; and DLK , equiangular to the Triangle $\lambda\delta\omega$; therefore, $Ly : \delta y :: DL : \delta\lambda$. And, as $DL : \delta\lambda :: LK : \delta\omega$; and therefore $L\delta$, parallel to $K\omega$. But the Points $\delta\lambda H\omega$ are in a Circle, whose Diameter is $\delta\lambda$: Therefore the Angle $\lambda\omega H$, is equal to (the Angle $\lambda\delta H$, equal to the Angle $L\delta D$, equal to) $K\omega y$. Therefore the Angle $K\omega H$, equal to $y\omega\lambda$, equal to a Right Angle: Therefore, $mH = m\omega = mK$. Also, $\lambda K : LK :: (\omega\lambda : \omega R ::)$ Tangent of the Angle $D\lambda\delta$: the Tangent of the Angle $D\delta L$; that is,

As the Sine of the Sum of the Sides,
Is to the Sine of the Difference of the Sides;
So is the Tangent of half the Sum of the Sides,
To the Tangent of half the Difference of the Sides.

Therefore,

As the Sine of the Sum of the Angles,
Is to the Sine of the Difference of the Angles;
So is the Tangent of half the Sum of the Angles,
To the Tangent of half the Difference of the Angles.

But,

The Sine of the Sum of the Sides,
Is to the Sine of the Difference of the Sides,
(as the Sine of the Sum of the Angles : Sine of the Difference of the Angle;)
So is the Tangent of half the Sum of the Angles,
To the Tangent of half the Difference of the Angles.

That is, As $\lambda K : \lambda H :: AP : AO$.

Therefore,

$$\frac{\lambda K}{\lambda H} \pm \frac{\lambda K}{\lambda H} :: \frac{AP}{AO} \pm \frac{AP}{AO}$$

That is, $\lambda m : m\omega :: An : nG$.

But

Of Spherical Triangles.

95

But also, the Angle $m\lambda\omega$, is equal to the Angle GAn ; therefore (by 7 Pr. 6 Lib. Eucl.) the Triangles $\lambda m\omega$, and AnG , are equiangular; and therefore the Triangles $\lambda H\omega$ and AOG ; and also the Triangles $\lambda K\omega$, and APG , are equiangular. Therefore,

Fig. XXXIV.

$\lambda\omega : \lambda H :: AG : GO$, and $\lambda\omega : \lambda K :: AG : AP$.

But, $D\lambda : \lambda\omega :: DL (LK =) : \lambda H$,

And $\delta\lambda : \lambda H :: AG : AO$, and $\lambda\omega : \lambda K :: AG : AP$.

But $D\lambda : \lambda\omega :: DL (LK =) : \lambda H$; and $\delta\lambda : \lambda\omega :: \delta L (LH) : \lambda K$.

Therefore,

$D\lambda : DL :: (\lambda\omega : \lambda H ::) AG : AO$.

And $\delta\lambda : \delta L :: (\lambda\omega : \lambda K ::) AG : AP$.

That is,

As the Sine of half the *Sum* of the Sides,
Is to the Sine of half the *Difference* of the Sides;
So is the Co-tangent of half the *Vertical Angle*,
To the Tangent of half the *Difference* of the Angles.

And,

As the Co-sine of half the Sum of the Sides,
Is to the Co-sine of half the *Difference* of the Sides;
So is the Co-tangent of half the *Vertical Angle*,
To the Tangent of half the *Sum* of the Angle.

C O R O L L A R I E S.

In any Spherical Triangle,

I. $\text{Tan. } \frac{1}{2} \text{ Base} : \text{Tan. } \frac{1}{2} \text{ Sum of the Sides} :: \text{Tan. } \frac{1}{2} \text{ Differ. of the Sides} : \text{Tan. } \frac{1}{2} \text{ Differ. of the Segments of the Base AG, made by a perpendicular Arch falling thereon from E.}$

$AG : AO :: AP : A\delta$

II. $\text{Tan. } \frac{1}{2} \text{ Base} : \text{Tan. } \frac{1}{2} \text{ Sum Sides} :: \text{Co-sine. } \frac{1}{2} \text{ Sum } \angle\text{'s} : \text{Co-sine } \frac{1}{2} \text{ Diff. } \angle\text{'s. } AG : AP :: \delta\lambda : \delta L$ For the Angle $EAG =$ the Arch DB , and the Angle $AGE =$ to the Arch BC .

III. $\text{Tan. } \frac{1}{2} \text{ Base} : \text{Tan. } \frac{1}{2} \text{ Sum Sides} :: \text{S. } \frac{1}{2} \text{ Sum of Ang.} : \text{S. } \frac{1}{2} \text{ Diff. } \angle\text{'s}$

$AG : AO :: D\lambda : DL$

IV. If

Fig. IV. If the Angle E G A be supposed Right, then $AP \times AO =$
 XXXIV. AG^2 that is, In a Right-angled Spherical Triangle, E G A,
 the Tang. $\frac{1}{2}$ Hypot. $\times \frac{1}{2}$ Per. $\times$ Tan. $\frac{1}{2}$ Hypot. $= \frac{1}{2}$ Per. is =
 the Square of the Tangent of half the Base.

C H A P. IX:

*Some Problems in Plain Triangles, which come not within
 the Limits of the Twelve foregoing Cases.*

P R O B L E M I.

*In the Oblique-angled Triangle C B D; there is given the Angle at
 C, (116.20 deg.) the Side D B, opposite thereunto (1270 Foot :)
 And the Two Sides, D C and C B, in One Sum, (1496 Foot :)
 To find the other Angles D and B, and the Sides severally.*

C O N S T R U C T I O N.

Fig. XXXV. **E**Xtend the Side of the Triangle D C, to A, making C A
 equal to C B; and draw the Line B A, so shall you have
 constituted a new Triangle A O B, in which you have given,
 (1.) The Side A D, equal to the Sum of the Sides D C and C B,
 (1497.) And (2.) the Side D B (1270.) And (3.) being you
 have the Angle D C B, (116 d. 12 m.) you have also the Angle
 B C A in the other Triangle, (63.80 d.) the Complement thereof
 to a Semicircle, or 180 deg.

Then in the Triangle A C B, having the Angle at (63.80)
 you have also the Sum of the other Two Angles, C A B and A B C,
 equal to the given Angle D C B, (116.20) the half whereof is
 the Angle C A B (58.10 d.) to which the Angle A B C is equal,
 because the Sides C A and C B subtending those Angles, are e-
 qual: And now in the Triangle D A B you have given, (1.) the
 Side D A, 1497. (2.) The Side D B, 1270. And (3.) the An-
 gle, D A B (opposite to D B) (58.10 d.) by which you may find
 the whole Angle D B A (by Case I. of Oblique Triangles: For,

As Log. D B : s. C A B :: Log. D A : s. A B D.
 1270 58.10 d. 1497 95.53 d.

From

Figure XXXII.

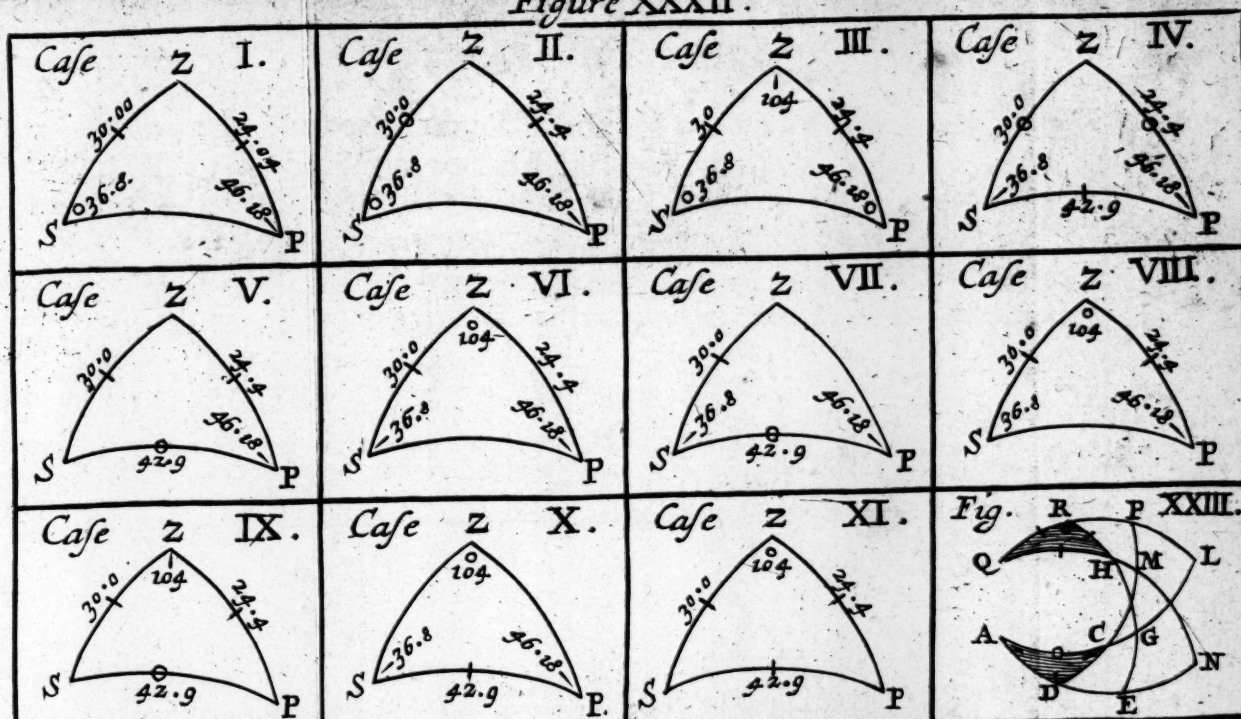
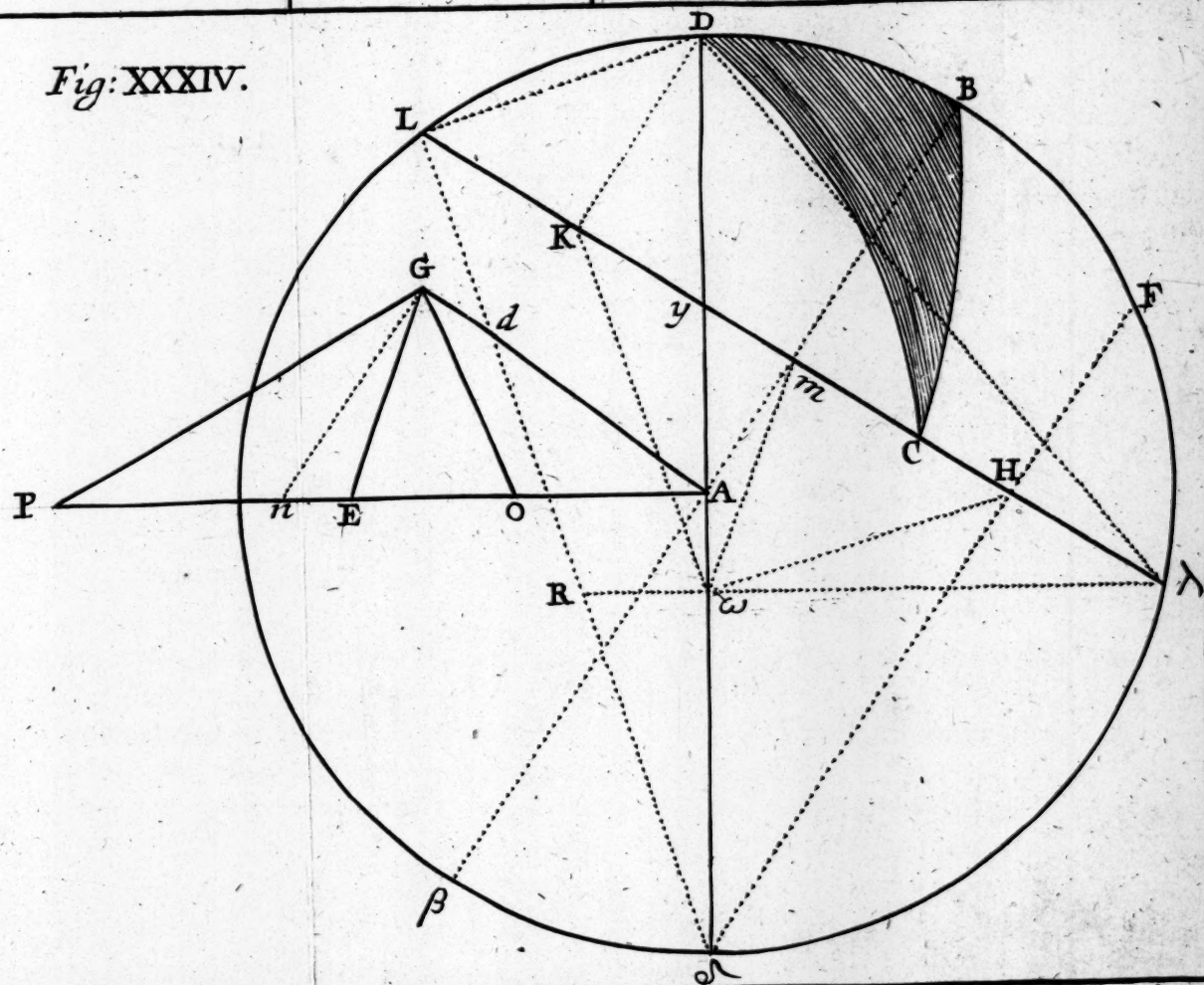


Fig: XXXIV.



Of Problems Extraordinary.

97

From which, the Angle C B A 58.10 d. being substracted, there will remain 37.43 d. for the Angle C B D, and then the Angle C D B will be 26.37 d. the Remainder of the other Two, C and B, to 180 d.

And for the Sides, they may be found by the first Case of Oblique Triangles, thus:

$$\begin{array}{cccc} \text{As } s C : \text{Log. DB} :: s B : \text{Log. DC.} \\ 116.20 \text{ d.} & 1270 & 37.43 \text{ d.} & 865 \end{array}$$

$$\begin{array}{cccc} \text{As } s B : \text{Log. DC} :: s D : \text{Log. CB.} \\ 37.43 \text{ d.} & 865 & 26.37 \text{ d.} & 632 \end{array}$$

PROB. II.

In the Oblique-angled Triangle C D B there is given the Angle at C, (116.20 d.) The Side DB, opposite thereunto (1270 Foot.) And the Difference of the other Two Sides C D and C B, (233 Foot.) To find the Two unknown Sides severally; and the Angles at D and B.

CONSTRUCTION.

IN the Given Triangle, make C A equal to C B, and draw the Line B A; so is the Given Triangle C D B, reduced into Two Oblique-angled Triangles C A B and A D B; in which last is given the Two Sides, A D and D B, but never an Angle; and in the other, only the Angle at C, and never a Side.

But (by Construction) the Side C A being made equal to the Side C B, the Angles C A B and C B A are equal, and the Sum of them, equal to the Complement of the Angle at C, (116.20 d.) to 180 d. that is, to 63.80 d. the half whereof 31.90 d. is equal to the Angle C A B, and the Complement thereof to 180 d. is 148.10 d. equal to the Angle D A B.

And now in the Triangle D A B, there is given, the Sides A D and D B, and the Angle at A, to find the other Angles, by Case I. and II. of Oblique Triangles, Thus,

$$\begin{array}{cccc} \text{As Log. DB} : s A :: \text{Log. AD} : s. A B D. \\ 1270 & 148.10 \text{ d.} & 233 & 5.53 \text{ d.} \end{array}$$

Which added to the Angle C B A, before found, 31.90 d. the Sum will be 37.43 d. for the Angle C B D.

O

Then,

Fig.
XXXVI.

Then,

$$\begin{array}{cccc} \text{As } s D C B : \text{Log. DB} :: s C B D : \text{Log. CD} \\ 116.20 d. & 1270 & 37.43 d. & 865 \end{array}$$

And,

$$\begin{array}{cccc} \text{As } s D C B : \text{Log. DB} :: s C D B : \text{Log. CB} \\ 116.20 d. & 1270 & 26.37 d. & 632 \end{array}$$

P R O B. III.

In the Right-angled Plain Triangle A B C, there is given, the Base A B, (28 Foot.) And the Hypotenuse C B, and Perpendicular C A, in one Sum, (viz. 56 Foot) to find the other Sides and Angles severally.

Fig. XXXVII. **I**N the given Triangle A B C, extend the Perpendicular A C to D, making C D equal to C B; so shall you have constituted a new Triangle D A B; in which there is given, (1.) The Right Angle at A. (2.) The Base A B 28 Foot. (3.) The Side A D, equal to the Sum of the Sides A C and C B, by which you may find the Angles A D B, and A B D, (by Case I. of R. Ang. Tri.)

$$\begin{array}{cccc} \text{As Log. A B} :: \text{Rad.} :: \text{Log. D A} :: \angle D B A \\ 28 & 1.45.00 d. & 56 & 63.43 d. \end{array}$$

Whose Complement 26.57 d. is the Angle A D B, to which the Angle C B D is equal, because the Sides C B and C D, subtending them, are equal by Construction. Then from the Angle D B A 63.43 d. before found, you subtract the Angle D B C, 26.57 d. last found, the Remainder 36.86 d. is equal to the Angle C B A, and the Complement thereof, 53.14 d. to the Angle C A B.

And then,

$$\begin{array}{cccc} \text{As } s A C B : \text{Log. AB} :: s C B A : \text{Log. CA} \\ 53.14 d. & 28 & 36.86 d. & 21 \\ \text{And so} :: \text{Rad.} :: \text{Log. CB} \\ & & 90 d. & 56 \end{array}$$

Other

Of Problems Extraordinary.

99

Otherwise, by this Theorem.

Fig. XXXVII.

If from the Square of the Sum of the Base and Perpendicular D A 56 (3136) you subtract the Square of the Base A B 28, (784) and divide the Remainder (2352) by double the Sum of the Hypotenuse and Perpendicular 56, (viz. 112), the Quotient (21) will give the Perpendicular: Which subtracted from the Sum (56), leaves (35) for the Hypotenuse.

PROB. IV.

The Three Sides of a Right-lined Triangle, being given; To find the Area.

RULE.

ADD the Three Sides of the given Triangle together, and take the half thereof: From which half Sum, subtract each Side of the Triangle severally, and note the several Differences: Then multiply the Half Sum, by any one of the Differences, and that Product multiply by another of the Differences, and that Product multiply by the third Difference: The Square Root of this third Product, shall be the Area of the given Triangle.

Example.

In the Triangle A B C, whose Three Sides are A B 20, A C 34, and B C 42: Their Sum is 96, the half whereof is 48: From which subtract the several Sides, 20, 34, and 42, and there will remain these Differences, 28, 14, 6.—Then, multiply 48 (the half Sum) by 28 (the first Difference) the Product will be 1344: Which multiplied by 14 (the second Difference) the Product will be 18816: And this Product multiplied by 6 (the third Difference) produceth 112896: The Square Root whereof is 336, for the Area of the Triangle A B C.

By Logarithms.

To the Logarithm of half the Sum of the Sides, add the Logarithms of the several Differences of the Sides from the half Sum: Half the Sum of those Logarithms shall be the Logarithm of the Area of the Triangle.

O 2

Example.

Fig.
XXXVIII.

Example.

The half Sum of the Sides	— 48	} whose Loga- rithm is	{ 1.6812412 1.4471580 1.1461280 0.7781512
The Difference of	{ A B — 28 A C — 14 B C — 6		
	Their Sum		5.0526784
	The Half Sum		2.5263392

Which is the Logarithm of 336, the Area of the Triangle.

P R O B. V.

By the Three Sides given; To find the Point in the longer Side, where a Perpendicular shall fall, and the Length of that Perpendicular.

R U L E.

TAKE the Sum and Difference of the Two Sides containing the Angle from whence the Perpendicular is to fall, and multiply them together, and divide the Product by the third Side, upon which the Perpendicular is to fall, the Quotient added to the third Side, or subtracted from it, shall be the Double of the Greater or Lesser Segment on either Side of the Perpendicular.

Example.

Fig.
XXXVIII.

In the former Triangle A B C, the Sum of the Sides A B and A C, is 54, and their Difference is 14; which multiplied together, produce 756; which divided by the Side B C 42, the Quotient is 18; which added to the Side B C 42, gives 60; the half whereof 30 is the Greater Segment of the Side B C; or the Quotient 18, subtracted from the Side B C, leaves 24; the half whereof 12, is the Lesser Segment B D, where the Perpendicular is to fall.

By Logarithms.

To the Sum of the Logarithms of the Sum and Difference of the Sides containing the Angle from whence the Perpendicular is to fall, subtract the Logarithm of the Side upon which it is to fall. The Remainder shall be the Logarithm of a Number; which added to, or subtracted from the Side on which the Perpendicular is to fall, shall be the Double of the Greater or Lesser Segments of the Side on which the Perpendicular is to fall.

Example.

Of Problems Extraordinary.

101

Example.

Fig.
XXXVIII.

The Sum of A B and A C is 54

1.7323937

The Difference of A B and A C is 14

1.1461281

Their Sum

2.8785218

The *Logarithm* of the third Side B C 42, subtract

1.6232493

Remains

1.2552725

Which is the *Logarithm* of 18. Which 18 added to B C 42, makes 60; the half whereof 30, is the *Greater Segment* C D, or 18 subtracted from B C 42, leaves 24; the half whereof 12, is the *Lesser Segment* B D.

For the Length of the *Perpendicular*.

R U L E.

Multiply the Sum of A B and B D 32, by the Difference of A B and B D, 8; the Product will be 256, whose *Square Root* is 16; for the Length of the *Perpendicular* A D.

By *Logarithms*.

Half the Sum of the *Logarithms* of 32 and 8, the Sum and Difference of A B and B D is the *Logarithm* of the *Perpendicular*.

The *Logar.* of the Sum of A B and B D 32,

1.50515

The *Logar.* of the Difference of A B and A D 8.

0.90309

Their Sum

2.40824

The half

1.20412

Which is *Logarithm* of 16, the Length of the *Perpendicular* A D.

P R O B. VI.

The Base B A (or longest Side) of a Plain Triangle B A D, being 47.8 P. and a *Perpendicular* D C, let fall from the Angle opposite to that Side 17.33 P. being given; To find the Area of that Triangle.

TO the *Logarithm* of half the given Side, add the *Logarithm* of the *Perpendicular*; the Sum of those *Logarithms* shall be the *Logarithm* of the Area of that Triangle.

Fig.

XXXIX.

The

Fig.
XXXIX.

The Base B A, is 47.8 —

The half Base 23.9 —

The Perpendicular D C 17.33 —

The Logarithm of 414.2

Log.—1.3783978

Log.—1.2387986

2.6171965

Which is the Area of the Triangle.

After the same manner, if the Base of a Triangle were 42, and the Perpendicular 16, the Area will be found to be 336.

P R O B. VII.

There is an Oblique-angled Plain Triangle A B C, one of whose Angles at the Base B C is 53.48 d. And the Sum of the Two Sides opposite to the Base B C, viz. A B and A C, is 129, and the Base C B, is longer than the longer side A C, by 16.8. The respective Sides and Angles of the Triangle are required.

C O N S T R U C T I O N.

Fig. XL. **D**raw a Right Line B C, at Liberty, for the Base; and upon one end thereof, at B, make an Angle A B C, to contain 53.84 d. Then the Sum of the Two Sides A B and A C being together 120, break it into any Two equal or unequal Parts, as into 43.2 and 76.8, then (by the Proposition) must the Base B C, be 93.6, which is greater than A C by 16.8.—So in the Oblique-angled Triangle A B C, you have given, (1.) The Side A B 43.2. (2.) The Side A C 76.8. (3.) The Angle opposite thereto, A B C 53 d. 50 m. by which you may find the Angle A C B. For,

As A C 76.8 : Is to s A B C 53.84 d.

So is A B 43.2 : To s A C B 27.02 d.

P R O B. VIII.

Of the Mensuration of the Area of a Spherical Triangle.

Lemma I. The Lunary Superficies of the Hemisphericks, are as the Angles of the same Superficies.

Fig. XLI. **T**HE Proof of this Lemma (amongst many other Ways) may be this:

Let the Meridian Semicircle A E D be imagined to be equally moved over the Longitude of the Equator B E C, upon the Poles A and D. Therefore the Angles on the other side of A and D.

(to

(to wit, F and G) shall be as the Times. The Superficies also, Fig. XLI.
K and M, shall be as the Times; therefore F shall be to G, as
the Superficies K, to the Superficies M. And G unto M, as the
Superficies M, to the Superficies K, &c. howsoever they can be
taken. For,

*Those Things that agree to a Third,
Agree among themselves.*

C O R O L L A R Y.

Therefore, by Composition,

$$\begin{aligned} &\text{As } F \div G : G :: K \div M : M, \\ \text{Or, As } F \div G : F :: R \div M : K. \end{aligned}$$

That is,

As the Two Right Angles are unto G,
So is half the Spherical Superficies, to M.
And, As Two Right Angles, are to F;
So is half the Spherical Superficies, to K.

Lemma II. The Triangle G, is equal to the Triangle H, because Fig. XLII.
the Angles and Sides of one, are equal to the Angles and Sides
of the other: To wit, $A = D$, $B = E$, $C = F$. Also,
 $L = O$, $M = P$, $N = Q$. Therefore they are Congruent
and Equal.

T H E O R E M.

The Excess of the Three Angles, over and above Two Right An-
gles, divided by 720, shews what the Area of the Triangle, is, in
respect of the whole Spherick.

For by Lemma I.
$$\begin{cases} 180 : A :: \frac{1}{2} \text{ Sph.} : G \div R \\ 180 : B :: \frac{1}{2} \text{ Sph.} : G \div S \\ 180 : C :: \frac{1}{2} \text{ Sph.} : G \div T = H \div T. \end{cases}$$

(by the Second Lemma.)

Therefore, As $180 : A \div B \div C ::$ half the Spherick : to
 $3 G \div R \div S \div T$, (by 24 El. 5 Ev.)

As $180 : A \div B \div C - 180 ::$
 $3 G \div R \div S \div T - \text{half the Sph.}$
But $G \div R \div S \div T$ is equal to half the Spherick;

Therefore,

Fig. XLII. Therefore, $3 G \div R \div S \div T$ — half the Sph. = 2 G.
 So that, As $180 : A \div B \div C$ — $180 : :$
 $: : \text{So is half the Spherick} : 10 \div 2 G.$

And the Antecedent Terms being Quadropled, it shall be,

As $720 : A \div B \div C$ — $180 : : 2 \text{ Sphericks} : 2 G.$

And so the Spherick to G. Therefore,

$\frac{A \div B \div C - 180}{720}$ } shews what part the Triangle is of the whole Spherick.

These Things are likewise true in all *Spherical Polygons*, of what *Ordinate* Figure soever they be; or *In-ordinate*, so all the *Angles* be given. And the Reason is, because all *Polygons* may be resolved into *Triangles*. Therefore, this *Rule* shall hold in these *Multangles* also.

R U L E.

Multiply 180 d. by the Number of the *Angles*; Subtract the Product out of the Aggregate of all the *Angles* increased by 360 d. The Residue divided by 720 d. gives the *Area* of the *Polygon*.

Of the Completion of a Solid Body.

From the foregoing Mensuration of the *Area* of a *Spherical Triangle*, this Fruit ariseth.

If the Radius of the Sphere be 100000.00 , the Side of an inscribed *Icosaedrum* shall be 105146.22 , equal to the Subtense of 63 deg. 26 min. 10 sec. Therefore the *Plain Equilateral Triangle* F E O (in the *Icosaedrum*) answers to the *Equilateral Spherical Triangle* in the Sphere; whose Three *Spherical Triangles* are connected in the *Plain Angles*, in the same Points, F, E, O.

And the Sides of this *Spherical Triangle* are separately taken 63 deg. 26 min. 10 sec. to wit, because their Subtenses F E, E O, O F, in the *Plain Triangle*, are equal to one another.

Fig. XLIII. Let fall now the Perpendicular E P, the *Spherical Triangle* E P O shall be Rectangled; where, over and above the *Right Angle* at P, are given, E O and P O, equal to half E O; wherefore the *Vertical Angle* P E O shall be 36 deg. just, and the whole *Angle* at E, 72 deg. And the Sum of the Three equal *Angles*, E, F, O, shall be 216 deg. from whence taking Two *Right Angles*,

Of Problems Extraordinary.

105

Angles, equal to 180 Deg. there remains 36 Deg. Therefore, the Triangle E F O is $\frac{1}{7\frac{1}{2}}$ of the whole Spherick, that is $\frac{1}{15}$ Part: And this most truly, for 20 Pyramides F E O C, fill the Solid Place of the Icosaedrum. And so 20 Spherical Bases, (covered over with 20 Triangular Plain Bases,) compleat the whole Spherick.

Fig.
XLIII.

F E O C is One of the 20 Pyramids in the Icosaedrum: The Plain Triangle B, is One of the Hedrae or Bases: C is the Center of the Body, or Sphere, that circumscribes it.

4.77 Icosaedres

4.24 Dodecaedres

9.244 Octaedres

8.000 Cubes

} Fill a Solid Place, as will appear
out of Precedent Practice by Triangles,
and Spherical Polygons.

That is to say, None of the Five Regular Bodies fill a Solid Place, the Cube only excepted.

Contrary to what Potamon, and from him Ramus, and all that have followed Ramus; to wit, Snellius, and others, have delivered.

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SECTION III.

Spherical Trigonometry, Geometrically Performed.

Fig.
XLIV.

AS the *Sides* of Plain Triangles are Three Right Lines, intersecting each other upon a Plain: And a Right Line, is, The Shortest Extension between any Two Points upon a Plain Superficies: So the *Sides* of a Spherical Triangle are Three Arches, of Three Great Circles of the Sphere, intersecting each other upon the Globe. And an Arch of a Great Circle, passing through any Two Points upon a Spherical Superficies, is the nearest Distance between those Two Points.

In Pursuance of the Work in this Chapter intended, I suppose the Reader to be acquainted with the *Circles* of the Sphere; that is, to know their Names and Situations upon the Globe, to what Use each of them serveth; and also, how to project any of them upon a Plain, answerable to any Position of the Globe: For to such as do not, this Section will be but of little Use; and therefore I would advise my Reader, before he enter upon this Geometrical Way of resolving Spherical Triangles, to peruse the Beginnings of the Second and Third Sections of the Third Part hereof, which treat of the *Circles* of the Sphere, and their several Positions and Affections; in which he may receive very much Satisfaction concerning those Particulars. So that it shall suffice, in this Place, that I declare,

1. What a Great Circle-is.
2. How to Project such a Circle of the Sphere upon a Plain, suitable to any Day and Time of the Day, at any time of the Year, and in any Latitude.
3. To discover the Triangle, which is made by the Intersections of those Great Circles so projected.
4. To find the Poles of those Great Circles. And,
5. To Measure the Sides and Angles of the Triangle so laid down: Which to do, is usually called, The Doctrine of the Dimension of Triangles.

I. What

Fig. XXXV.

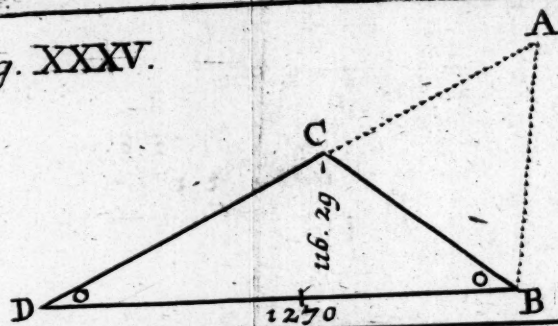


Fig. XXXVI.

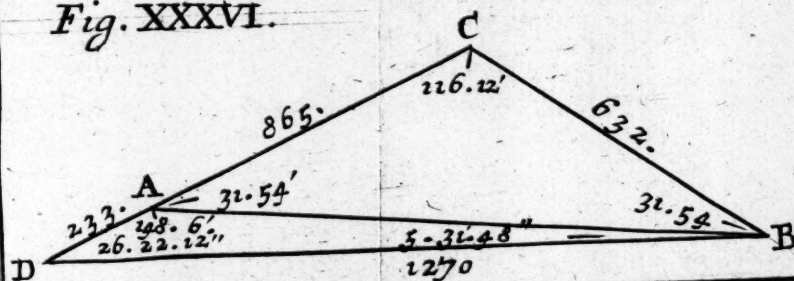


Fig. XXXVII.

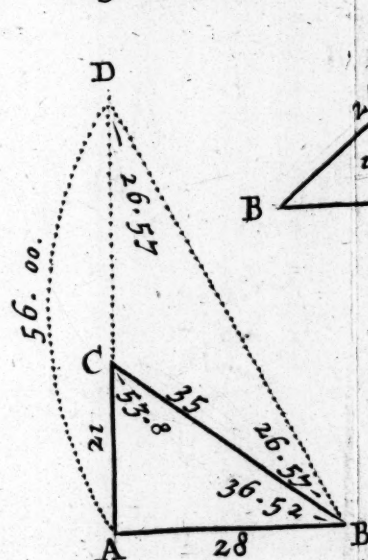


Fig. XXXIX.

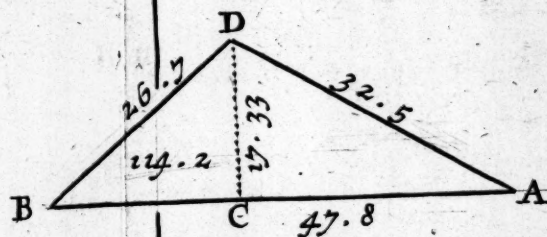


Fig. XL.

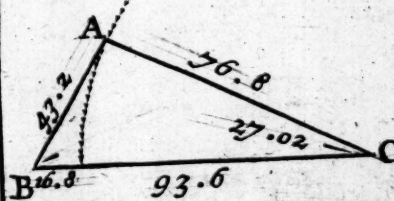


Fig. XXXVIII.

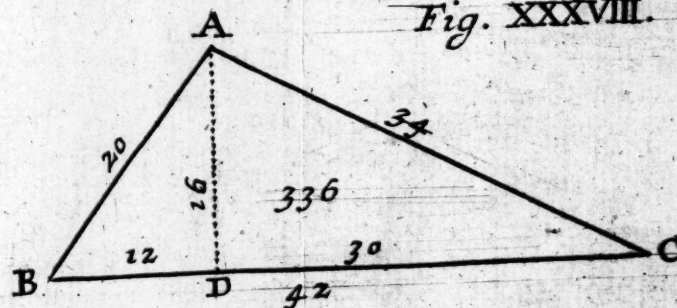


Fig. XLI.

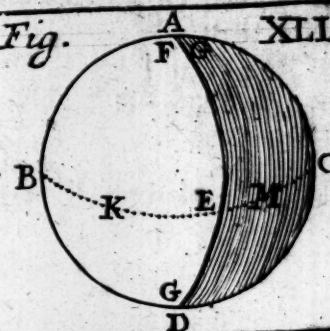


Fig. XLII.

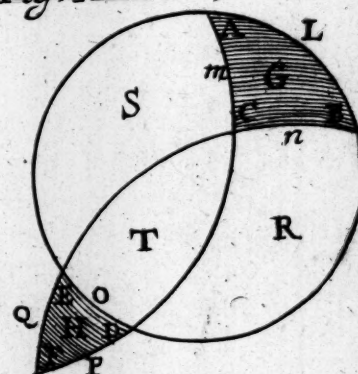
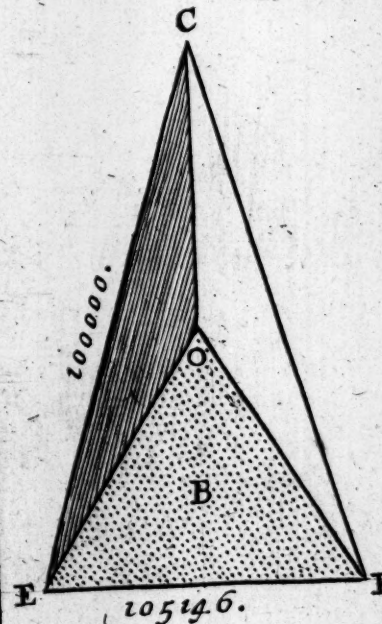
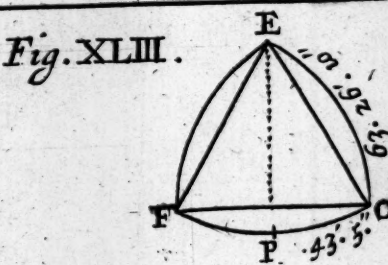


Fig. XLIII.



I. What a Great Circle is.

A *Great Circle* of the *Sphere* is such a *Circle*, as divideth the whole *Sphere* into *Two Equal Parts* or *Hemispheres*: Of which, there are generally accounted Six, viz.

The { Meridian Horizon Æquinoctial }	}	The	{	Æquinoctial Solstitial Ecliptick.	}	Colure.
--	---	-----	---	---	---	---------

Besides these Six Principal *Great Circles* of the *Sphere* there are other *Circles*, which are *Great Circles* also. As are all *Azimuth* or *Vertical Circles*. Also all intermediate *Meridians* or *Hour Circles*. And these are such as we have most occasion to make use of in this Place: And here note, That all *Great Circles* projected upon a *Plain*, *Steriographically* (or *Circular*,) are either, (1.) *Perfect Circles* (as the outward or *Primitive Circle* is. Or, (2.) *Semi-Great Circles*, projected within the *Primitive Circle*. Or, (3.) They are *Straight Lines*, passing through the Centre of the *Primitive Circle*, as afterwards will appear.

II. How to Project the Circles of the Sphere upon a Plain, and answerable to a Prefixed Time and Place.

P R O B L E M.

LET it be required to Project such *Circles* of the *Sphere* in *Plano*, upon the *Plain* of the *Meridian*, in the *Latitude North*, 40 Deg. Upon the 10th of *June*, at the time of the *Sun's Rising* or *Setting*; and also at 10 in the *Morning*, or 2 in the *Afternoon*, the same Day: The *Sun* then having 23 Deg. 30 Min. of *North Declination*.

First, With 60 Deg. of a *Scale of Chords*, upon the Point A, describe the *Primitive Circle* Z H N O, representing the *Meridian* of the Place.

Secondly, Draw the Right Line H A O for the *Horizon* of the Place; and at Right Angles thereto the Line Z A N, for the *Æquinoctial Colure*, Z being the *Zenith*, and N the *Nadir* Points.

Thirdly, Take 40 Deg. (the *Latitude* given) out of your *Scale of Chords*, and set them from O to P, from Z to Æ, from H to S, and from N to æ; and draw the Line P A S for the *Axis* of the World, and *Hour-Circle* of Six; and the Line Æ A æ, for the *Æquinoctial*.

Fourthly, Because the *Sun's Declination* at the time given is 23 Deg. 30 Min. take 23 Deg. 30 Min. and set them from Æ to 6, and from æ to 6: And if you lay a Ruler from Æ or æ

Fig. XLIV. to $\odot$, it will cross the *Axis* of the World PAS , in the Point F ; so have you Three Points, $\odot F \odot$, through which you may draw the Circle $\odot F \odot$; which will cut the *Horizon* HAO , in the Point $\odot$, the Place where the *Sun* will rise that Day.

Fifthly, This Point $\odot$ being found, you have Three other Points given, *viz.* P , $\odot$ and S ; through which you may describe the Circle $P \odot S$, which is the *Hour* at which the *Sun* *Riseth*.

And thus have you *Projected*, so far of the *Problem*, as concerns the *Time* of the *Sun's Rising*. Now for his Place of being at Ten in the Morning, or Two in the Afternoon.

Sixthly, Ten in the Morning, or Two in the Afternoon, are (either of them) Two Hours, or 30 Deg. distant from the *Meridian*: Wherefore, take 30 Deg from your Scale of Chords, and set them upon the *Meridian*, from H to G , then a Ruler laid from P to G , will cross the *Equinoctial* Circle HA , in the Point B , so have you Three Points, S , B and P , by which you may *Project* the Circle SBP , for the *Hour-Circle* of Ten in the Morning, or Two in the Afternoon: And the Circle SBP , will cut the *Tropick* of *Cancer* $\odot A \odot$ (the Line which the *Sun* traces that Day,) in the Point C ; in which Point the *Sun* will be at Ten in the Morning, or at Two in the Afternoon.

Seventhly, And now have you Three other Points, N , C and Z , through which you may draw the *Azimuth* Circle ZCN , for the *Azimuth*, that the *Sun* will be upon at Ten and Two a Clock.

III. Concerning the Spherical Triangles that are made by the Intersections of these Great Circles thus projected.

Many are the *Triangles*, that are made by the Intersections of these few *Circles* upon this *Projection*, but I shall exemplifie only in Two of them; namely, the *Triangle* $P \odot O$, Right-angled at O , which is composed of Three *Arches* of *Great Circles* of the *Sphere*, *viz.* (1.) Of $\odot O$, an *Arch* of the *Horizon*, comprehended between O , the *North* Point of the *Horizon*, and $\odot$, the Point in the *Horizon*, whereon the *Sun* *Riseth* that Day, or his *Amplitude* from the *North*. (2.) Of $P \odot$, an *Arch* of a *Meridian* or *Hour-Circle*, passing through P , the *North Pole* of the World; and $\odot$, the Place of the *Sun's Rising* that Day, equal to 66 Deg. 30 Min. the Complement

plement of the Sun's Declination that Day. And, (3.) Of P O, an Arch of the Meridian of the Place, comprehended between O, the North Point of the Horizon, and P, the North Pole, equal to the Latitude given, 40 Deg.

The other Triangle shall be the Oblique-angled Triangle Z C P, constituted by the Intersection of Three other Great Circles of the Sphere; namely, Of (1.) Z P, an Arch of the Meridian of the Place, comprehended between Z the Zenith, and P, the North Pole, equal to 50 Deg. the Complement of the Latitude of the Place. (2.) Of P C, an Arch of a Meridian, or Hour-Circle of Ten or Two a Clock, equal to 66 Deg. 30 Min. the Complement of the Sun's Declination; or, the Sun's Distance from the elevated Pole P. And (3.) Of Z C, an Arch of an Azimuth or Vertical Circle, passing through the Hour-Circle S C P, in the Point P, where the Sun is at Ten or Two a Clock that Day, and is the Complement of the Sun's Altitude at that time.

IV. To find the Poles of these Great Circles, they being thus Projected.

The Pole of any Great Circle of the Sphere, is, always, 90 Deg. distant in all Places, from the same Circle, upon a Right Line, or Circle, which cuts it at Right Angles: So,

1. The Pole of the General Meridian, or Primitive Circle Z H N O, is A, the Center thereof, in all Places distant 90 Deg.
2. The Poles of the Horizon H O, are Z and N, the Zenith and Nadir Points.
3. The Poles of the Æquinoctial Æ æ, are P and S, the Poles of the World.
4. The Poles of the Æquinoctial Colure, or Axis of the World P S, are Æ and æ.
5. The Poles of the Prime Vertical Circle, or Azimuth of East and West, Z A N, are H and O, the North and South Points of the Horizon.

And thus you see, that all Great Circles of the Sphere, which, when Projected, become Strait Lines, their Poles fall all in the Periferie of the Primitive Circle: But for all other Great Circles, which consist of Circular Arches, as P © S, P B S, and Z D N, the Poles of them fall within the Primitive Circle Z H N O, upon those Great Circles, which cut them at Right Angles: So,

Fig.
XLIV.

$\left. \begin{matrix} R \\ T \\ V \end{matrix} \right\}$ The Pole of the Great Circle. $\left\{ \begin{matrix} P \odot S \\ P B S \\ Z D N \end{matrix} \right\}$ will be upon the Line. $\left\{ \begin{matrix} \overline{AE} A \\ A a \\ A O \end{matrix} \right\}$ at the Point $\left\{ \begin{matrix} R \\ T \\ V \end{matrix} \right\}$.

Which Points or Poles may be thus found.

1. For the Pole of the Hour-Circle $P \odot Q S$, lay a Ruler from P to Q, it will cut the Primitive Circle in a , set 90 Deg. of your Chords, from a to b ; a Ruler laid from P to b , will cut the *Æquinoctial* Circle in R, so is R the Pole of the Circle $P \odot Q S$.
2. For the Pole of the Hour-Circle $P B S$, lay a Ruler from P to B, it will cut the Primitive Circle in G, set 90 Deg. from G to c , a Ruler laid from P to c , will cut the *Æquinoctial* in T, so is T the Pole of the Hour-Circle of 10 and 2, viz. $P B S$.
3. For the Pole of the Azimuth, or Vertical Circle $Z D N$, lay a Ruler from Z to D, it will cut the Primitive Circle in d , set 90 Deg. from d to e , a Ruler laid from Z to e , will cut the Horizon $H A O$ in V, so is V the Pole of the Azimuth Circle $Z D N$.

And in this manner may the Pole of any Great Circle be found. For the Centers of them, they always fall in the same Right Line, in which their Poles are, they being extended, if Need be: But no more of this in this Place: For I intend not here a *Treatise* of Projection, but of the Measuring of Triangles, to which I now proceed.

V. How to Measure the Sides and Angles of Spherical Triangles, they being thus Projected.

You must understand, That the Quantities of the Sides and Angles of Spherical Triangles are measured upon the Periferie of the Primitive Circle.

Then,

1. In the Right angled Spherical Triangle $P \odot O$, Right-angled at O, there is given, (1.) The Perpendicular $P O$ 40 Deg. equal to the Latitude given by the Proposition. (2.) The Hypotenuse $P \odot$, the Complement of the Sun's Declination 66 Deg. 30 Min. And let it be required to find the Base $\odot O$, whose Measure is $\odot O$, the Sun's Amplitude.

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Of Geometrical Trigonometry.

III

Fig.
XLIV.

Lay a Ruler to Z, the Pole of the Horizon, and to the Angular Point $\odot$; it will cut the Primitive Circle in the Point f, so the Distance from f to O, measured upon the Scale of Chords, will be found to contain 58 Deg. 38 Min. and that is the Quantity of the Side $\odot O$; and is the Amplitude of the Sun's Rising or Setting from the North Part of the Horizon at O; and its Complement, 31 Deg. 22 Min. is the Quantity of the Arch A $\odot$; the Sun's Amplitude from A, the East and West Points of the Horizon.

The Canon for Calculation is,

As $c s P O$: Radius :: $c s P \odot$: $c s \odot O$.
50 d. : 90 d. :: 23 d. 30 m. : 31 d. 22 m.

2. By the same Things given to find the Angle at the Perpendicular $\odot P O$, whose Measure is the Arch of the Equinoctial T α , or the Hour from Midnight.

Lay a Ruler to P, the Pole of the World, and T, it will cut the Primitive Circle in a, and the Distance a α measured upon the Scale of Chords, will be found to be 68 Deg. 36 Min. And that is the Quantity of the Angle $\odot P O$: Which 68 Deg. 36 Min. converted into Time, (by allowing 15 Deg. of the Equinoctial for One Hour, and 4 Deg. for One Minute of Time) makes 4 Hours, and 34 Min. Which is the Hour from Midnight, or the Time of the Sun's Rising in the Morning: Whose Complement to 12 Hours, is 7 Hours, 26 Min. the Time of the Sun's Setting.

The Canon for Calculation is,

As Radius : $r. P O$:: $c t. P \odot$: $c s. \odot P O$.
90 d. : 40 d. : 23 d. 30 m. : 21 d. 24 m.

3. By the same Things Given, to find the Angle at the Base $P \odot O$; whose Measure is an Arch of a Great Circle intercepted between the Hour-Circle $P \odot S$, and the Horizon $H \odot O$.

Forasmuch as the Sides $\odot P$, and $\odot O$, of the Triangle $P \odot O$, are less than Quadrants, we must measure the Angle $P \odot O$, by the Angle $H \odot S$, which is equal to it. To perform which,

Lay a Ruler to R, (the Pole of the Hour-Circle $P Q S$) and $\odot$, the Angular Point, and it will cut the Primitive Circle in b; then set 90 Deg. from b to a: A Ruler laid from the Pole R, to a,

f

Fig.
LXIV.

a, will cut the *Hour-Circle* P Q S in *k*: Then lay a Ruler from $\odot$ to *k*, it will cut the *Primitive Circle* in *l*, and the Distance from *l* to H, measured upon the *Scale of Chords*, will be found to contain 44 Deg. 30 Min. for the Quantity of the *Angle* H $\odot$ S, which is equal to the *Angle* P $\odot$ O required: And this is the *Angle* of the *Sun's Position*, (in respect of the *Pole* P, and *North Part* of the *Horizon* O,) at the Time of his *Rising* or *Setting*.

The Canon for Calculation is

As *s. P $\odot$* : *Radius* :: *s. P O* : *s P $\odot$ O*.
66 d. 30 m. : 90 d. :: 40 d. : 44 d. 30 m.

And thus are all the *Sides* and *Angles* of the *Right-angled Spherical Triangle* P $\odot$ O measured: And the like may be done in any other *Right-angled Triangle*; of which, in this *Scheme* there are many.

VI. And now for the *Oblique-angled Spherical Triangle* Z C P.

In this *Triangle* there is given, (1.) The *Side* Z P, 50 Deg. equal to the *Complement* of the *Latitude* given. (2.) The *Side* C P, 66 Deg. 33 Min. equal to the *Complement* of the *Sun's Declination* given. And, (3.) The *Angle* Z P C, 30 Deg. the *Horary Distance* of the *Sun* from the *South Part* of the *Meridian*; namely, at Ten or Two of the *Clock*. And from these *Things* given, let it be required to find the third *Side* Z C, the *Complement* of the *Sun's Altitude* at the Time of the *Question*, viz. at Ten in the *Morning*, or at Two in the *Afternoon*. Which, to perform,

Lay a Ruler to V, the *Pole* of the *Azimuth Circle* Z D N, and to the *Angular Point* C, it will cut the *Primitive Circle* in *m*, and the Distance from Z to *m*, by the *Scale of Chords*, will be found to 30 Deg. 7 Min. whose *Complement* D C, 59 Deg. 53 Min. is the *Sun's Altitude* at Ten or Two of the *Clock*, that Day.

The Canons for Calculation is

(1.) As *c t. P Z* : *Radius* :: *c s. P* : *t P A*.
40 d. : 90 d. :: 60 d. : 45 d. 56 m.

Which subtracted from P C, 66 d. 30 m. there remains
20 Deg. 34 Min.

(2.) As *c s. P A* : *s c. Z P* :: *c s. Rem.* : *c s. Z C*.
44 d. 4 m. : 40 d. :: 69 d. 26 m. : 59 d. 53 m.

To

Of Geometrical Trigonometry.

113

To find the Angle Z C P.

Fig.
XLIV.

This is the *Angle* of the Sun's *Position* at the time of the *Question*, with Reference to the *Pole* and *Zenith*; and may be thus found.

In regard the Two Sides C Z and C P, comprehending the enquired *Angle* Z C P, are both of them less than *Quadrants*, we must make use of the alternate and opposite *Angle* S C N, which is equal to Z C P. And, whose *Measure* is the Arch of a *Great Circle* intercepted between the *Meridian* or *Hour-Circle* S C P, and the *Azimuth-Circle* Z C N, at 90 Deg. Distance from the *Angular Point* C; And to find the Quantity hereof, you must,

Lay a Ruler to V, the *Pole* of the Circle Z C D N, and the Point C, it will cut the *Primitive Circle* in *m*, set 90 Deg. from *m*, and it will reach to *n*: A Ruler laid from V to *n*, will cross the Circle Z D N in the Point X. —Again, Lay a Ruler from T, the *Pole* of the Circle P B S, to C, the *Angular Point*, and it will cut the *Primitive Circle* in *o*, set 90 Deg. from *o* to *p*: Then a Ruler laid from T to *p*, will cross the Circle P B S, in the Point Y. —Lastly, A Ruler laid from C, to X and Y, will cut the *Primitive Circle* in *r* and *s*, so the Distance between *r* and *s*, measured upon the Scale of Chords, will give 49 Deg. 46 Min. for the Quantity of the *Angle* Y C X, which is equal to the *Angle* Z C P enquired. And is the *Angle* of the Sun's *Position* at the time of the *Question*.

The Canon for Calculation.

As s. Z C : s. Z P C :: s. Z P : s. Z C P.
30 d. 7 m. : 30 d. :: 50 d. : 49 d. 46 m.

To find the Vertical Angle C Z P.

This *Angle* is the Sun's *Azimuth* from the North Part of the *Meridian* Z O N; whose *Measure* is the Arch of the *Horizon* D A O; and to find the Quantity of it, lay a Ruler from Z, the *Pole* of the *Horizon*, to D, it will cut the *Primitive Circle* in *d*; so the Quantity of the Arch d N O, measured upon the Scale of Chords, will be found to be 113 Deg. 57 Min. And such is the Sun's *Azimuth* from the North Part of the *Meridian*: The Arch d H, 66 Deg. 3 Min. is the Sun's *Azimuth* from the South; and the Arch d N, 23 Deg. 57 Min. is the Sun's *Azimuth* from the East and West.

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Fig.
XLIV.

The Canon for Calculation.

As $s. ZC : s. ZPC :: s. CP : s. CZP$.
 $30 d. 7 m. : 30 d. :: 66 d. 30 m. 66 d. 3 m.$
 (Or, $113 d. 57 m.$)

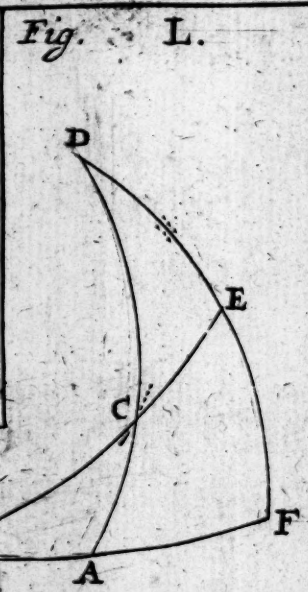
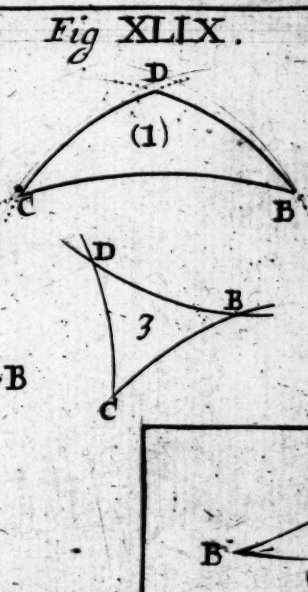
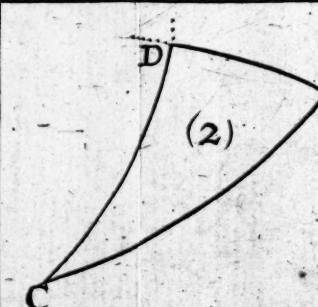
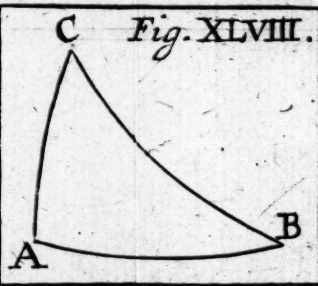
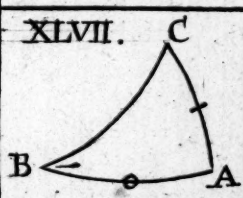
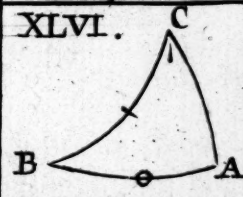
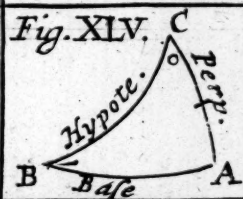
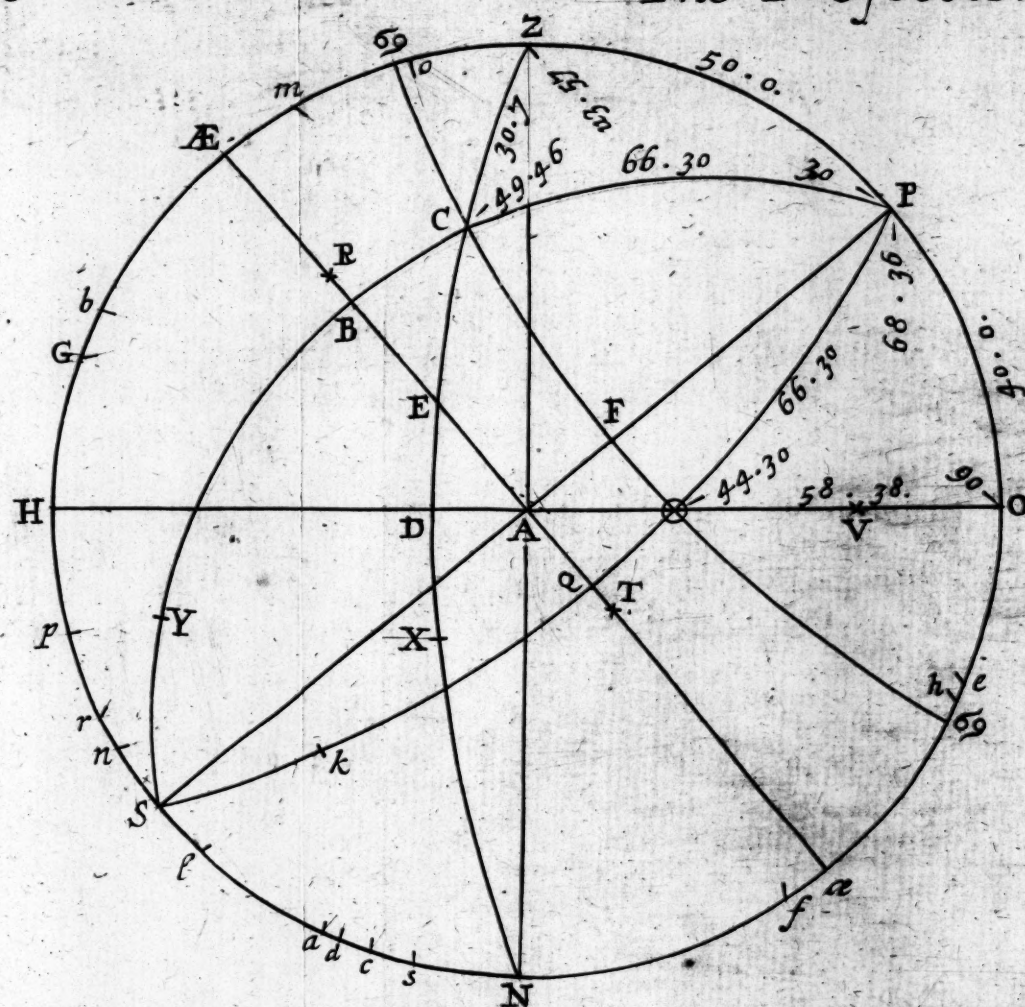
And thus are all the *Sides* and *Angles* of this *Oblique-angled Triangle* also Measured : And so may any other, being thus *Projected*.

And, to conclude, This *Projective Way* will give great *Light* to *Calculation*; for by the true delineating of your *Triangle* upon the *Projection*, you shall thereby discover whether your *Side* or *Angle* be *More* or *Less* than a *Quadrant*, and so be positive in your *Resolution*; which otherwise you must have *Given*, or render a double *Solution*: As in this last *Angle CZP*, whether it were $66 \text{ Deg. } 3 \text{ Min.}$ Or $113 \text{ Deg. } 57 \text{ Min.}$ The same *Sine* in the *Canon* answering to both.

JANUA

Fig. XLIV.

The Projection.



JANUA MATHEMATICA.

SECTION IV.

Of Spherical Trigonometry, *Instrumentally Explained and Performed.*

THE *Explanatory Instrument* here described, is for the better Information of the Fancy, by Speculation; and it is deduced from the *Catholick, or Universal Proposition*, before treated of in *Part II. Sect. II. Chap. III.* of this Book. Notwithstanding, for the Convenience of the Reader, I shall here, again, insert it.

Proposition Universal.

The Sine of the Middle Part, and the Radius, are Reciprocally Proportional, with the Tangents of the Extream Parts Conjunct, and with the Co-sines of the Extreams Disjunct.

In every *Right-angled Spherical Triangle*, there are *Five Parts*, besides the *Right Angle*, and they are called *CIRCULAR PARTS*: Of which, those *Three* which lye most remote from the *Right Angle*, (as the *Hypotenuse*, the *Angle* at the *Perpendicular*, and the *Angle* at the *Base*) are noted by their *COMPLEMENTS*.

Of these *Five Circular Parts*, any *Two* of them (besides the *Right Angle*) being given, a *Third* may be found. And,

Of *Three Parts*, (*Two* given, and *One* required) *One* must (necessarily) be in the *Middle*, and must be called the *MIDDLE PART*.

Of the other *Two Extream Parts*, they must either *Join* to the *Middle Part*, or be *Separate* from it.

If they be $\left\{ \begin{array}{l} \text{Joined to it,} \\ \text{Separate from it,} \end{array} \right\}$ then are they called $\left\{ \begin{array}{l} \text{Conjunct.} \\ \text{Disjunct.} \end{array} \right\}$ *Extreams* If

Q 2

If the *Extreams* be *Conjunct*, the *Proportion* must be performed by *Sines* and *Tangents* jointly. —But,

If the *Extreams* be *Disjunct*, the *Proportion* may be performed by *Sines* only.

And in both *Cases*,

If the *Part Sought* (whether *Side* or *Angle*) fall out to be the *Middle Part*, then the *Rudius* must be the *First* (or leading) *Term* in the *Proportion*. —But,

If one of the *Extreams* (whether *Conjunct* or *Disjunct*) possess the *Middle Part*, then the other *Extream* must be the *First Term* in the *Proportion*.

Of the *Instrument*.

THIS *Instrument* was the Contrivance (many Years since) of the Right Worshipful Sir Charles Scarborough, M. D. and divers of them have been made in *Silver* and *Brass* about the Bigness of a *Crown Piece*, with *Verses* in *Latin* about the Rimb, for the better bringing the *Rules* to be observed, in the Use of it, to Memory, which were to the same Effect with those which I have even now laid down in English.

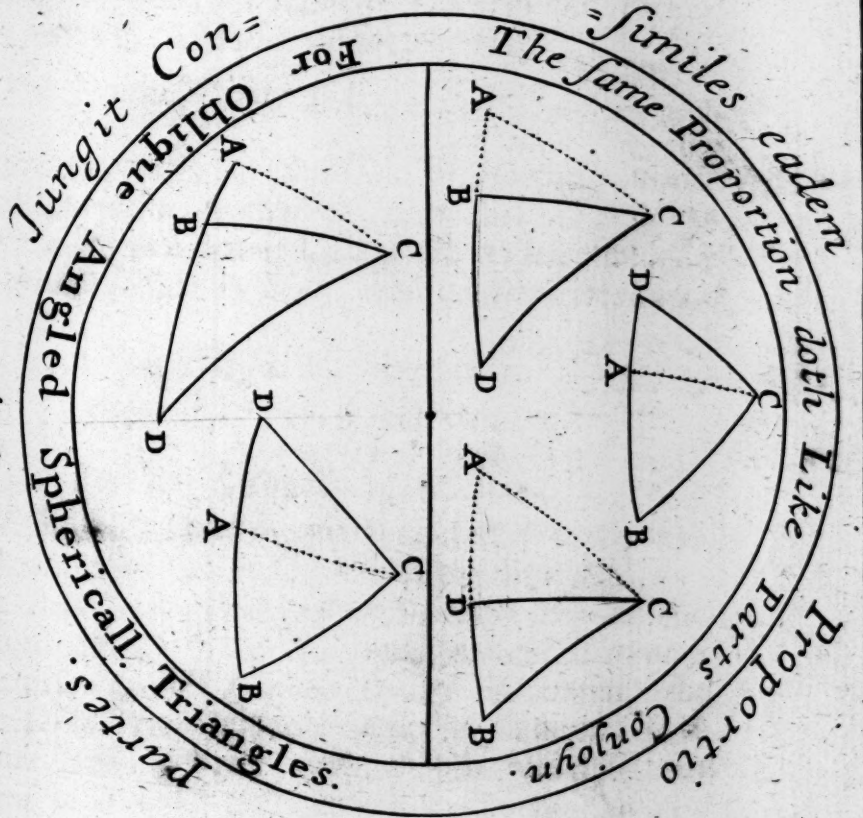
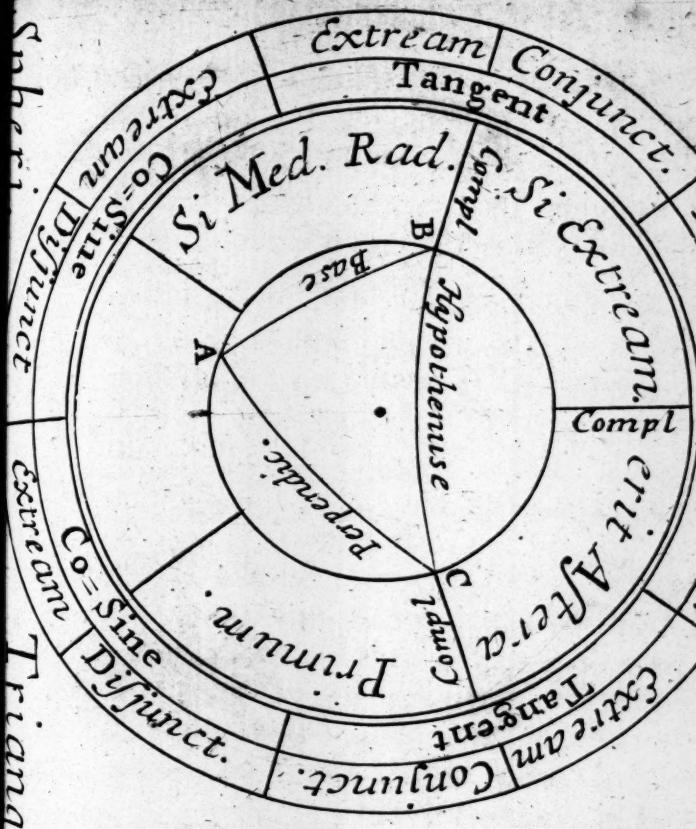
The *Instruments* consists of Two *Parts*, the undermost is a round Piece, divided into Five Equal Parts, which represent the Five Circular Parts before spoken of, it hath Two Circular Margins, near the outer Edge thereof; in One of the Five Parts is engraven *Middle Part*, and under it the Word *Sine*. —On either Side of *Middle Part*, there is written or engraven *Extream Conjunct*; and under them, the Word *Tangent*. —In the Two other of the Five Divisions, which are opposite to *Middle Part*, are engraven *Extream Disjunct*, and under them, *Co-sine*. All which Words answer directly to the Words of the *Universal Proposition*: And this is all that is upon the Under Plate of the *Instrument*.

The Upper Plate of the *Instrument* is a Circle also, divided into Five Equal Parts as the former, and within that another Concentrick Circle, between which are Five Lines drawn from the Centre thereof. —Within the innermost of these Circles, there is described a *Right-angled Spherical Triangle*, the Five Circular Parts whereof do lye directly against the Five Lines, drawn from the Centre between the Two Circles: And upon those Three Lines which lye against the *Hypotenuse*, the *Angle* at the *Perpendicular*, and the *Angle* at the *Base*, is written,

Subiect
Disjunct
Extream
Triana
to the Rule of Tangents of the Difference of the
Segments of the Base.

A SYNOPSIS OF Sphericall Trigonometry.

Right angled.
MIDDLE PART.
FOR



And
As the Tangent of half the Base,
Is to the Tang. of half the Sum of the Sides,
So is the Tang. of half the Difference of y^e Sides
To the Tang. of half the Difference of the
Segments of the Base.

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ten, or engraven, *Complement*; but upon the other Two Lines, which issue from the *Perpendicular*, and the *Base*, there is nothing written. — This Upper Plate is to move upon the Under Plate by a Rivet (or such like) through the Centers of both Plates: And so is your *Instrument* finished.

The Use of the Instrument.

THE Use of the Instrument is principally to give you by Inspection the *Order* of the *Terms* of the *Proportion* in any *Case*.

Example. In the Triangle A B C, suppose there were given the *Perpendicular* C A, and the *Angle* at the *Base* B, to find the *Angle* at the *Perpendicular* C.

Fig.
XLV.

Here it is evident, that the *Angle* at the *Base* B, is the *Middle Part*: Turn the Rundle about till you bring the *Angle* at the *Base* against *Middle Part*: Then shall you find that the *Perpendicular* C A, and the *Angle* at the *Perpendicular* C, will stand against *Extreams Disjunct*; which tells you, that C A and C, are *Extreams Disjunct*: And now (the Rundle thus resting) you see that One of the *Extreams*, as C, is sought; therefore, the other *Extream* C A must be the first *Term* in the *Proportion*, against which you find *Co-sine*. Wherefore say,

As the *Co-sine* of the *Perpendicular* A C,

Is to the *Radius*:

So is the *Middle Part* the *Angle* at the *Base* B, (against which stands *Sine* and *Complement*, that is *Co-sine* B.)

To the *Angle* at the *Perpendicular* C, against which stands *Co-sine* and *Complement*; that is *Sine*; for, *Co-sine Complement* is the *Sine* it self.

And so the *Proportion* in short is this:

As $c s. CA$: to *Radius* :: So $c s. B$: to $s. C$.

Another Example.

In this Triangle A B C let there be given the *Hypotenuse* C B, and the *Angle* at the *Perpendicular* C, to find the *Base* B A.

Fig.
XLVI.

It is evident that B A is the *Middle Part*. Bring B A against the *Middle Part*, then will C B and C stand against *Extreams Disjunct*: And (because the *Middle Part*) B A is sought, the *Radius* must be the *First Term* in the *Proportion*, and the Two *Extreams* the *Second* and *Third*, against both which stands *Comp. Cosine*, (which is *Sine*); and against B A the *Middle Part*

Part fought, there stands *Sine* also: So that your Proportion will run thus:

As Radius,
To the *Sine* of the *Angle* the Perpendicular C:
So is the *Sine* of the *Hypotenuse* C B,
To the *Sine* of the Base B A.

A Third Example.

Fig.
XLVII.

In this *Triangle* A B C, let there be given the Perpendicular C A, and the *Angle* at the Base B, to find the Base B A.

It is here evident that A B is the *Middle Part*: Turn the *Triangle* about till the Base A B lye against the *Middle Part*; then will the Perpendicular C A, and the *Angle* at the Base B, lye against the *Extreams Conjunct*; and seeing the *Middle Part* fought, the Radius therefore must be the *First Term* in the Proportion: And because the *Extreams* are *Disjunct*, the Proportion will be in *Sines* and *Tangents* jointly, as by the *Instrument* appears: And the Proportion will be

As the Radius,
Is to the *Tangent* of the Perpendicular C A:
So is the *Tangent* Complement of B,
To the *Sine* of the Base B A.

But if the same Things were given, and the *Angle* at C, the Perpendicular, had been required, then B had been the *Middle Part*, and C A and B, the *Extreams Disjunct*, and the Proportion in *Sines* only. As thus:

As the Co sine of C A,
To Radius:
So is the Co-sine of B,
To Co-sine Comp. (that is, to *Sine*) of C.

And thus, by this *Instrument*, may the Proportions for the Solution of any *Right-angled Spherical Triangle* be readily set down, and the *Triangle* resolved in any of the XVI. Cases.

On the Back-side of this *Instrument*, there is another Contrivance of the forementioned Gentleman's, with *Latin Verses* for bringing the *Rules* for the Solution of *Oblique-angled Spherical Triangles* to Memory.

I shall not here give you any Account of it more than the *Figure*, and the *Latin Verses* (or *Rules*) in English: Referring you, for the farther understanding of it, to what is delivered in the IVth Chapter hereof, Page 68.

Of the Solution of Spherical Triangles, by
PLANISPHERE.

A *Planisphere*, is a *Projection* of the *Sphere* (or *Globe*) in *Plano*: Of which, there are principally Two; One called *Stereographical*, projected by *Circles*; The other *Orthographical*, projected by *Ellipses*, and generally known by the Name of *Analemma*. Fig. XLVIII.

To either of which *Planispheres*, there belongs proper *Indexes*, to move upon the Centres of the *Planispheres*. As to the *Stereographical*, an *Index* having Two *Legs* as a *Sector*; both which are to be divided, according to the *Tangents* of *half Arks*, as the *Semidiameter* of the *Planisphere* it self is divided, and must be numbred by 10, 20, 30, &c. to 90 Deg. both Ways, and in both *Legs*.

To the *Analemma*, or *Orthographical Projection*, there must be an *Index* of the whole Length of the *Diameter* thereof, to move about upon the Centre of the *Projection*, which must be divided as the *Diameter* of the *Planisphere* is; namely, as a *Scale* of *Natural Sines*, and must be numbred both Ways, from the Centre, by 10, 20, 30, &c. to 90 Deg. — Upon this *Index*, (by help of a *Groove* made through the former *Index*) another *Index* is to be made to move upon, and with, the former, and always keeping at *Right Angles* with it: And this *Index* is to be divided as a *Scale* of *Sines*, as one half of the other; and so numbred, by 10, 20, 30, &c. to 90, both Ways: And this second *Index* (in the Use of this *Planisphere*) I shall call the *Cursor*. And thus much for the Descriptions.

Concerning their Use, I shall only lay down such *General Rules* as are necessary for the counting of the Quantities of the *Sides* and *Angles* of *Spherical Triangles* upon the several *Planispheres* and their *Indexes*; in all Cases, both of *Right* and *Oblique Triangles*. Not insisting upon particular *Examples*, for that throughout all this Book there are such Variety. All which (by these few *General Rules* here delivered) may be brought upon either of these Two *Projections*.

Fig.
LXVIII.

The Solution of Spherical Triangles, by the
Stereographical Projection.

I. Of Right-angled Spherical Triangles.

1. **T**O retain the Method before observed in the 16 Cases of *Right-angled Spherical Triangles*; I will here also follow the same Order, wherein I shall note the *Triangle* to be resolved by the Letters A B C, setting A at the *Right Angle*, and B and C at the other Two *Acute Angles*. So shall the *Base* be A B; The *Cathetus*, (or *Perpendicular*) C A; The *Hypotenuse* B C. —The *Angle* at the *Base* B; and the *Angle* at the *Cathetus* C.

2. There are therefore, besides the *Right Angle*, Five *Circular Parts*; namely, Three *Sides*, and Two *Angles*: Of which Four come into the *Account* at once; Two of them are given and the other Two found out.

3. If the Four Parts, which at once come into the *Account*, be the Three *Sides*, and One *Angle*. —Let that *Acute Angle* be evermore noted with the Letter B; and the *Triangle* may be resolved thus:

Set the *Angle* B at the Centre, reckoning it upon the *Limb* from the *Diameter*; and the *Base* B A upon the *Diameter* from the Centre; and the *Cathetus* C A upon the *Great Circle* from the *Diameter*, by help of the *Parallels*; and the *Hypotenuse* B C upon the *Index* from the Centre. —Note, That if any of those *Accounts*, fall not just upon some Line in the *Instrument*, either of the *Great Circles* or *Parallels*, the Excess is to be estimated in *Minutes* or *Parts* of a Degree.

Example. The *Perpendicular* C A, and the *Angle* at the *Base* B, given; To find, (1.) The *Base* B A. (2.) The *Hypotenuse* C B. (3.) The *Angle* at the *Cathetus* C.

1. Reckon on the *Limb*, from the *Diameter*, the Quantity of the given *Angle* B; and to the End thereof, set either of the *Legs* of the *Index*.

2. Upon the *Limb*, from the *Diameter*, reckon the *Cathetus* C A; and from the end of that Arch, estimate reasonably, a *Parallel Circle*, till it meet with the *Index*: For that Point of Intersection shall shew both the *Hypotenuse* B C, upon the *Index*: And the *Base* B A, upon the *Great Circle*, meeting also in that Point. —Then, to find the *Angle* at C: Take C A for

the *Base*, and *C A* for the *Cathetus*; and to the end of that *Cathetus* apply the *Index*; so shall you have the *Angle* sought for *C*, upon the *Limb*. Fig. XLVIII.

3. Otherwise, you may set the *Angle B* at the *Pole*, reckoned upon the *Diameter* from the *Limb*; and *B A* upon the *Limb* from the *Pole*; and *C A* upon the *Index* from the *Limb*; and *B C* upon a *Great Circle* from the *Pole*.

4. But if the *Four Parts* which at once come into the Account, be the *Two Oblique Angles*, *C* and *B*, and the *Two Right Sides* *C A* and *B A*, then the *Triangle* may be resolved thus.

1. Reckon the *Lesser Angle* on the *Index* from the *Centre*, and the *Greater Angle* on a *Great Circle* from the *Pole*: So both of these, with the *Axis*, shall include a *Quadrantal Triangle*: And the *Greater Right Side* shall be on the *Limb* from the *Pole*: And the *Lesser Right Side* shall be upon the *Diameter* from the *Centre*.

Lastly, Two of the *Four Parts* being had, there will be no Difficulty in finding out of the *Hypotenuse*.

II. Of Oblique-angled Spherical Triangles.

1. In an *Oblique-angled Spherical Triangle B C D*, Five of the *Circular Parts* come into the Account at once; namely, the *Side B D*; and the *Two Oblique Angles B* and *D*, and the *Two other Sides D C* and *B C*: The third *Angle C*, opposite to *B D*, is not here enquired, but only the *Place* of the *Angular Point C*, both for *D C* and *B C*.

In the first of the *Three Schemes*, the inner *Angle D* is *Obtuse*, and *B Acute*: In the second the inner *Angle D* is *Acute*, and *B Obtuse*: In the third, both the inner *Angles D* and *B* are *Acute*. Fig. XLIX.

2. The *Side B D*, is evermore understood to be *Given*: And in every *Operation*, the first thing to be done, is to open the *Legs* of *Index* to the *Wideness* of *B D*, and thereto screw them fast: Also I call the *Two Legs* of the *Index*, the *Leg* or *Index B*, and the *Leg* or *Index D*, as they are noted with those Letters.

The *Angles D* and *B*, of every *Triangle* proposed, are reckoned upon the *Diameter* from the *Limb*, by the *Great Circles*; namely, the *Angle B*, and the *Conjunct* of the *Angle D*, from *s*, but the *Angle D*, and the *Conjunct* of the *Angle B*, from *w*: And the *Sides D C* and *B C*, are to be reckoned by the

Fig. the *Parallels*; namely, the *Sides* themselves, from the *Pole*
 XLIX. or their *Complements* and *Excesses*, from the *Diameter*.

Note, That of a *Triangle*, the *Conjunct Angle*, or the *Outer Angle*, is all one.

4. Two *Sides* of an *Oblique-angled Spherical Triangle*, where of one is *B D*, with the *Angle intercepted*, being given: To find out, at one *Work*, the *Third Side*; and the other *Angle* at *B D*.

There are these Two *Varieties*

Given, $\left\{ \begin{array}{l} D C \text{ and } D \\ B C \text{ and } B \end{array} \right\}$ to find out $\left\{ \begin{array}{l} B C \text{ and } B \\ D C \text{ and } D \end{array} \right\}$

5. The *Rule*. If the *Angles D* and *B*, be reckoned in the *Upper Semicircle* of the *Planisphere*, at the *Pole N*, and the *Angle D* be *Obtuse*, or *B Acute*; the *Point C* shall be taken in the former *Semicircle* towards $\odot$: But, if the *Angle D* be *Acute*, and *B Obtuse*, the *Point C* shall be taken in the latter *Semicircle* towards ω ; and contrariwise, for the lower *Pole S*. —Then, upon the *Diameter*, count the *Angle* given, *D* or *B*, either by it self, or by its *Conjunct*, according as was taught in *Sect. 3*. And from the end of that reckoning, imagine an *Arch* of a *Great Circle* aptly traced, until it meet with the *Parallel* of the *Complement* of the respective *Side* given, *D C* or *B C*, for that *Point of Concurrence* shall be the *Point C*, for the given *Side*, either *D C* or *B C*. To this *Point C*, thus found, apply the *Index* noted with the proper *Letter* of the *Angle* given, *D* or *B*; and mark the *Distance* of it from the *Centre*; for this *Distance* being exactly taken on the other *Index*, shall in the *Planisphere* give the true *Place* of the *Point* sought: Which being aptly estimated, will, upon the *Diameter*, shew the *Angle* sought; either *B*, or the *Conjunct D*, from $\odot$; or else, the *Conjunct* of *B*, or the *Angle D*, from ω . And, it will also shew upon the *Limb*, the *Complement* of the *Side* sought.

By what hath been before shewed, (that the *Point C*, both of the *Sides D C* and *B C*, are evermore equally distant from the *Centre*,) there is opened a *Way*, whereby *Any Two* of *Five Circular Parts*; together with the *Side D B*, being given, the other *Two* may be found at one *Operation*.

Herein

Of Instrumental Trigonometry.

125

Fig.
XLXI.

Herein are Four Varieties.

Given, $\left\{ \begin{array}{l} DC \text{ and } BC \\ DC \text{ and } B \\ D \text{ and } BC \\ D \text{ and } B \end{array} \right\}$ to find out $\left\{ \begin{array}{l} D \text{ and } B. \\ D \text{ and } BC. \\ DC \text{ and } B. \\ DC \text{ and } BC. \end{array} \right.$

The RULE.

In the *Upper Semicircle* of the *Planisphere*, if the *Angle D* be *Obtuse*: Or if the *Side DC* be much *Lesser* than *BC*, the Point *C* shall be taken in the *former Quadrant* towards $\odot$: But, if *D* be *Acute*, or *DC* much *Greater* than *BC*; the Point *C* shall be taken in the *latter Quadrant* towards ω . And, contrariwise, in the *Under Semicircle*. —And in both, if the *Angles D* and *B* be *Acute*, or if the *Sides DC* and *BC*, differ but little in length, the true Point sought, shall fall near the *Axis*, towards ω , within a *Space*, equal to the *Index* opened at the *Wideness* of *BD*.

Set, therefore, the *Index* thus opened, so that the Two Points *C*, may fall within it. And among the *Great Circles* and *Parallels*, or both (according as Reason shall direct) reckon the Two *Circular Parts* given, tracing them proportionally, with a *Pin* in each *Hand*, until they shall both exactly meet with their proper *Legs* of the *Index*, at an equal *Distance* from the *Centre*, and as near to it as possibly may be: So have you upon the *Planisphere*, the true *Places* of both the Points *C*; and thereby, the Two *Circular Parts* sought, respectively, to each Point *C*; both of *DC*, and *BC*.

And note farther, That if any Point *C*, falleth, either among the *Parallels*, near the *Diameter*; or among the *Great Circles*, near the *Limb*; where they be almost equidistant, there may be some *Uncertainty* in finding the true Points exactly: The remedying whereof requireth the more *Diligence* in the *Computation*; but this may be remedied, by reducing the *Oblique Triangle* into Two *Right-angled*.

Other *Inconveniencies* may arise in the *Use* of this, as in all other *Instrumental Operations*; for the remedying whereof, *Ufus optimus Magister*.

The

Herein

Fig.
XLIX.

The Solution of Spherical Triangles, by the Orthographical Projection, or Analemma.

I. Of Right-angled Spherical Triangles.

1. **T**HE Hypotenuse is, always, represented upon the Index.

2. The Right Angle, at the Section of any Meridian, and the Æquinoctial.

3. One Leg in the Meridian, and the other in the Æquinoctial.

4. One Angle only entering the Question, is represented at the Centre, between the Æquinoctial and the Index, and is numbered in the Limb.

Fig. L.

5. But the Two Angles entering the Question, you must turn the Case into an Angle, and its opposite Leg. For in the Triangle A B C, whose Right Angle is A; the other Two Angles A B C and A C B, both entering the Question, you must lengthen the Hypotenuse to a full Quadrant to E; as also, the Legs adjacent to that Extremity of the Hypotenuse, from which it was lengthned. Thus in the Scheme, the Leg A C, adjacent to C from which Extremity the Hypotenuse was lengthned: And so, in the Triangle C D E, Right-angled at E: The Angle D C E is equal the Angle B C A; and the Leg D E, equal to the Complement of the other Angle C B A.

II. Of Oblique-angled Spherical Triangles.

1. Three Sides being given; To find an Angle opposite to any of them.

Reckon the Greatest Leg from the Pole upon the Limb, and where it endeth, apply the Index; upon the Index reckon, from the Limb, the Base; (that is, the Side opposite to the Angle sought: And to the Point of the Index, where this Base endeth; apply (or bring to) the Cursor: Then look where the Cursor cutteth the Parallel of the Lesser Leg, to be reckoned from the Pole, and observe what Meridian passeth by this Section of the Cursor and Parallel; for, where this Meridian cutteth the Æquinoctial, there you have the Measure of the Angle sought; to be accounted from thence to the Limb.

2. Three Angles given, to find a Side.

Turn the Angles into Sides, and deal with them, as with the Sides.

3. The

3. The Parts Given and Sought, being altogether Opposite.

Fig. L.

Reckon the *Greater* of the Two first *Terms* upon the *Index*, and where this Number endeth upon the *Index*; apply that Point to the lesser of the said Two first *Terms*; which Parallel is to be reckoned from the *Æquinoctial*; then order the *Terms*, reckoning the *First* and *Third* both upon the *Index*; both upon the *Parallel*; and so likewise do with the *Second* and *Fourth* *Terms*.

4. Two Sides, with an Angle between them Given: To find the Third Side.

Reckon the *Greater Side* given, from the *Pole* upon the *Limb*; and to the end of it, apply the *Index*. Reckon the other *Side Given* upon the *Parallels*, from the *Pole*: And the *Angle* given, upon the *Æquinoctial*, from the *Limb*; and the *Meridian* it comes to, pursue till it comes to the *Parallel* of the *Lesser Leg*: And to this *Section* of the said *Meridian* and *Parallel*, apply the *Cursor*, so it may justly lye on the *Index*; and mark what Point of the *Index* it pointeth out: For, this Point of the *Index*, counted from the *Limb*; is the *Third Side* required. And after the *Third Side* is found, you may find either of the Two unknown *Angles*, by the *Rule* for *Opposite* *Parts*.

5. Two Sides, and an Angle opposite to the Lesser of them; Given: To find the Third Side.

Reckon the *Difference* of the given *Sides*, and the *Sum* of them (one and the same Way) from the *Pole*; and mark the Points where both of them do end. Count also, the *Angle* given, upon the *Æquinoctial* from the *Limb*, and mark what *Meridian* it cometh to. Then extending a strait Line (or applying a strait Ruler) between the Two Points marked in the *Limb*; it will cut the said *Meridian* in Two Places: So you are to observe the *Parallels*, where the strait Line (or Ruler) cutteth the said *Meridian*; for, these being reckoned from the *Pole*, will give you the *Quantity* of the *Third Side* required: For, this same *Third Side*, may be of a *Twofold Quantity*: The *Lesser* it is, when the *Angle* opposite to the *Greater* given *Side* is *Obtuse*; and the *Greater* it is, when the said *Angle* opposite to the *Greater* given *Side* is *Acute*.

6. Two

Fig. L.

6. Two Sides, and an Angle opposite to the Greater of the Given; To find the third Side.

Reckon the *Difference* of the given *Sides* from the *Pole*, one way; and the *Sum* of the same, the other way. Count also the given *Angle* upon the *Æquinoctial*: And extend the straight Line (or lay a Ruler) cutting the *Meridian*, as in the last; for now it will cut but once; and so the *Third Side* will admit only of one single Answer.

The End of the Second Part.

POSTSCRIPT.

Many are the Ways by which Plain and Spherical Triangles may be both Geometrically and Instrumentally performed. As by Scales of Natural Sines, Tangents, Secants, and Equidistant Parts, by Protraction: Also, by Scales of Artificial Numbers, Sines, Tangents, and Versed Sines, as Mr. Gunter long time since contrived them, to be used with Compasses: And I have now lately contrived an Instrument, which I call TRISSOMETRAS, which printed on a large Sheet of Paper, and pasted upon a Board, all Triangles, both Plain and Spherical may be Resolved by Inspection, without Pen, Compasses, opening Joints, or other Moveable; save only the Extension of a Thread or thin Streight Ruler, upon the Instrument; the Description and Use whereof I may hereafter publish by it self. But, the best and most absolute Way of Resolving Triangles; and to what Use soever they be applied, is by the Canons of Artificial Sines, Tangents and Logarithms; both Decimal and Sexaginary: And such a Canon was intended to be joined to this Book, at this time, but must be referred till farther Opportunity, which may be shortly.

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ANCILLA MATHEMATICA.

VEL,

Trigonometria Practica.

SECTION I.

OF

GEOMETRY.

IN this *Section* of GEOMETRY, I shall treat only of such *Practical Parts* thereof, as the *Doctrine* of Plain (or Right-lined) Triangles, (both for their *Illustration* and *demonstration*) becomes subservient: As,

In *ALTIMETRIA*: By which the *Height* of any *Object* (accessible or inaccessible) may be obtained; As of *Towers*, *Steeple*s, *Trees*, &c.

In *LONGIMETRIA*: By which the *Distance* of one *Object* from any Place, or of many *Objects* one from another, (whether approachable, or inapproachable) may be known, their true Positions laid down, and a Map made of them.

In *PLANOMETRIA*: By which all Kinds of *Superficies*, (Regular or Irregular) as *Plains*, *Land*, &c. may be *Measured*.

CHAP. I.

Of ALTIMETRIA.

I. Of an Altitude that is Accessible.

Fig. I. Suppose A C to be a Tower, Steeple, or other upright Object and you standing at B, were required to tell the Height thereof. — First, Measure the Distance from B, to the Foot of the Object A, which suppose to be 432.5 Feet. — Secondly, At B, (by a Quadrant, or other Graduated Instrument) look to the top of the Object at C, where we will suppose the Degrees found by the Instrument to be 32.25 deg. — By this Observation, the Distance Measured, and the Object, you have a Right-angled Triangle constituted; in which, there is Given, (1.) A B the Distance Measured 432.5 Foot. (2.) The Angle at B, observed by your Instrument, 32.25 deg. And, (3.) The Right Angle at A: To find the Leg. C A, which will be the Height of the Object: (By CASE II. of R, A, P, T,) thus:

As Radius, Tangent 45 deg.

Is to the Distance Measured A B, 432.5 Feet;

So is the Tan. of the Angle observed by Instrument at B, 32.25 d.

To the Height of the Object C A, 272.89 Feet.

And thus having found the Height of the Object to be 272.89 F. you may find the Length of the Visual Line (which is the Hypotenuse) C B, (by CASE V. of R, S, P, T.) For,

As the Sine of the Angle Observed, B, 32.25 deg.

Is to the Altitude of the Object, 272.89 Foot;

So is the Radius, Sine 90 deg.

To the Length of the Visual Line C B, 511.39 Foot.

II. Of an Altitude Un-accessible.

Fig. II. Suppose DE to be an Object, as Steeple, Tower, or the like: And that you standing at G, were required to know the Height thereof; but (by reason of some broad Moat, or other Impediment, you cannot come to measure from G to E. In this Case, — First measure from G, towards E, as far as conveniently you can, suppose to F, 95.25 F. — Then making Observation at G,

you find the Degrees of your *Quadrant* to be 52.50 d. and Observing at F, you find the Degrees to be 63.25. Now from this *Distance Measured*, and the Two *Observations* by the *Instrument* made, the *Altitude* D E may be attained unto by *Trigonometrical Calculation*, thus:

First, Upon a Sheet of Paper, draw a Line at Pleasure, as H K, upon which assume a Point for the Place of your first standing, as at G, and upon G, protract the *Angle* observed 52.50 d. drawing a Line through those Degrees at Liberty.

Secondly, By help of a Scale, set your *Measured Distance* 95.25 from G to F; and upon F, by a Scale of *Chords*, protract an *Angle* of 63.25 d. as you observed them to be; and through them draw another Line at Pleasure, which will cut the former Line drawn, in the Point D, which will represent the top of the *Object* to be measured; and a *Perpendicular* let fall from D, upon the Ground-line H K, will fall in the Point E, and so will the Line D E represent the *Object* it self.

Thirdly, By these Lines thus drawn, you will have constituted Two *Triangles*; one D E F, *Right-angled* at E; and the other D F G, *Obtuse-angled* at F; by the resolving of which, the height of the *Inaccessible Object* D E, will be found. For,

1. In the *Oblique-angled Triangle* D F G, you have given, the Side F G (which was the *Measured Distance*) 95.25 F. and the observed *Angle* D G F, 52.50 d. — And the *Angle* observed at F being 63.25 d. the Complement thereof to 180 d. viz. 116.75 d. the Quantity of the *Obtuse Angle* D F G, so have you in the *Oblique-angled Triangle* D F G, Two *Angles* given; the Sum of which, viz. 169.25 d. taken from 180 d. there remains 10.75 d. for the *Angle* F D G: And now in the *Triangle* D F G you have given Two *Angles* F D G, and D G F, with the Side F G, opposite to F D G; whereby, you may find the Side D F, (by *Ax.* 2.) For,

As the *Sine* of F D G, 10.75 d.
Is to the Side F G, 95.25 Foot.
So is the *Sine* of F G D, 52.50 d.
To the Side D F, 405.14.

2. In the *Right-angled Triangle* D E F, (having found the *Hypotenuse* as before) you have given, (1.) The *Observed Angle* at F, 63.25 d. (2.) The *Hypotenuse*, last found, D F 405.14 Foot; by which you may find D E, the *Height* of the *Object*, (by *Case IV.* R, A, P, T,) thus: As

Fig. II.

As the Radius, *Sine* 90 d.Is to the *Hypotenuse* D F, 405.14 F.So is the *Sine* of D F E, (the *Angle* observed at F) 63.25 d.

To the Leg. D E, 367.77 Foot: Which is the Height of the Object.

And now (if you please) you may find the *Visual* Line G D, (by *Case* V. of R, A, P, T,) thus:As the *Sine* of the *Angle* at G, 52.50 d.

Is to the height of the Object D E, 361.77 F.

So is Radius, *Sine* 90 d.To the *Length* of the *Visual* Line D G 456.00 F.And also, the Distance E F, (by *Case* II. of R, A, P, T,) thus:As the *Sine* of the *Angle* observed at F. 63.25 d.Is to the *Height* of the Object, D E, 361.77 F.So is the *Co-sine* of the *Angle* at F, 63.75 d.To the *Distance* E F, 182.35 Foot.Unto which, if you add the *Distance* F G, 95.25 Foot, their *Sum* will be 277.6 Foot. And that is the whole *Distance* from G to E, the Foot of the Object.

III. Of the Altitude of an Object standing upon a Hill, Un-accessible.

Fig. III.

Suppose M O to be such an Object; and you standing at L, were required to tell the *Height* thereof.First, Upon Paper, or the like, draw a Right-line at Pleasure, as Q R; and therein, assume any Point, at Pleasure, for the Place of your standing, as L; where, with your Instrument directed to the top of the Object, you find the Degrees cut, to be 40.52 d. and directed to the bottom of the Object at O, the Degrees cut, to be 22.25 d. Wherefore upon L, protract an *Angle* of 40.52 d. and draw a Line L, at Pleasure: And also b, an *Angle* of 22.25 d. and draw the Line L c at Pleasure.Secondly, Go forwards, in a Right-line towards the Object, some considerable Distance, as to N, 212.5 Foot; and there, by your Instrument directed to the top of the Object at M, you find the Degrees cut, to be 61.82 d. through which draw a Line at Pleasure, as N a, crossing the Line L b in the Point M, which is the top of the Object: From whence, a *Perpendicular* let fall upon the Ground.

Ground-line Q R, as M P, that Line shall be equal to the *Altitude*, of the *Object*, and the Hill together.

Now, by the Intersections of these Four *Visual Lines*, L M, L O, N M, and N O, there are constituted Four *Right-lined Triangles*, viz. L M P, and N O P, both *Right-angled* at P: And L M N, and N M O, *Oblique-angled*: By the resolving of which, from the *Distance Measured*, L N, and the several *Angles observed*, at L and N, the required *Altitude* may be obtained. For,

1. In the *Oblique-angled Triangle* L M N, there is given, (1.) The *Angle* M L N, 40.52 d. (2.) The *Angle* L N M, 118.18 d. (it being the Complement of the *Angle* O N P 61.82 d. to 180 d.) (3.) The *Side Measured* L N, 212.5 Foot: And having the *Angles* at L and N, the Sum of them 158.70 d. taken from 180 d. there will remain 21.30 d. for the *Angle* L M N. From which Things given, the Two other *Sides*, L M and M N, may be found, by *Axiom II.* thus:

As the *Sine* of L M N, 21.30 d.

Is to the *Side* L N, 212.5 Foot;

So is the *Sine* of M L N, 40.52 d.

To the *Side* M N, 380.08 Foot:

And so is the *Sine* of 61.82 d. (the Complement of M N L, to 180 d.)

To the *Side* L M. 515.66

In the *Right-angled Triangle* N M P, there is given, (1.) The *Hypotenuse* M N, 380.08 Foot. (2.) The *Angle* M N P, 61.82 d. whereby you may find N P, (by *Case IV.* of R, A, P, T,) thus:

As the *Radius*, Sine 90 d.

Is to the *Hypotenuse* M N, 380.08 Foot;

So is the *Sine* of the *Angle* M N P, 61.82 d.

To the *Side* M P, 335.03 Foot:

Which is the *Height* of the *Object* and the Hill together.

And so is the *Co-sine* of M N P, viz. N M P, 28.18 d.

To to *Leg.* or *Side* N P, 179.49 Foot.

To which, if you add the measured *Distance* L N, 212.05, their Sum will be 391.54, for the whole length L P. Then,

3. In the *Triangle* L O P, (*Right-angled* at P) you have given,

(1.) The *Side* L P, 391.54 Foot. (2.) The *Angle* O L P, 22.25 d.

whereby

Fig. III. whereby you may find O P, (By Case II. of R, A, P, T,) thus:

As the Radius, Tang. 45 d.

Is to the Side L P, 391.54 Foot;

So is the Tangent of the Angle O L P, 22.25 d.

To the Height of the Hill O P, 160.18 Foot.

Which subtracted from M P (the whole Height) 335.93, there remains 175.75 Foot, for the Altitude of the Object M O.

IV. How the Altitude of the Sun may be taken, by the Shadow of a Staff, or other Object, of a known Length.

Fig. IV. Upon A B, being a level Plain, let there be erected a straight Staff, or the like, of any Length, suppose 60.00 Inches, or 5 Foot) as C D; and the Sun shining, suppose it casts the Shadow thereof upon the plain Ground to E; which measured, suppose to contain 108 Inches, or 9 Foot.

Draw a Line at Pleasure, as A B; upon any Point thereof, as D, erect a Perpendicular, upon which set 60 Inches, the length of your Staff, from D to C; and the length of the Shadow thereof 108 Inches, from D to E; drawing the Line of Umbrago C E, and thus have you constituted a Right-angled Plain Triangle C D E, in which there is given, (1.) The Leg. C D, 60 Inches. (2.) The Leg. D E 108 Inches, by which you may find the Angle C E D, (by Case I. of R, A, S, T,) thus:

As the Length of the Shadow C D 108.00 Inches,

Is to the Radius, Tang. 45 d.

So is the Length of the Staff C D, 60.00 Inches,

To the Tangent of the C E D, 29.05 d.

And such is the Sun's Altitude at that time.

V. How the Height of an Accessible Object may be obtained, by the Length of the Shadow of it.

Fig. V. Suppose that the Sun should cast the Shadow of some upright Object, as F G, 84.5 Foot, from G to L, and at the same time, your Staff of 5 Foot, does cast its Shadow from L to K, 6.32 Foot: And from hence I would compute the Altitude of the Object F G.

The

The Two Lines of the *Object* F G, and *Staff* H L, together with the Two Lines of *Shadow*, G L and L K, being laid down, do constitute Two Right-angled Plain Triangles, viz. F G L, and H L R, Equiangled, and their Sides Parallel, and therefore Proportional: by Theorem VIII. Lib. I.) And therefore,

As the Length of the *Shadow* of the *Staff* L K, 6.32 Foot,
Is to the Height of the *Staff* H L, 5.00 Foot;
So is the Length of the *Shadow* of the *Object* G L 84.5 Foot,
To the *Altitude* of the *Object* F G, 66.93 Foot.

CHAP. II.

Of LONGIMETRIA.

How (standing upon an Object of a known Height) to find the Distance from thence, to some other remote Object.

Suppose C A to be the Side of a Fort or Bulwark 22.5 Foot high, and being upon the Platform at C, you see a Tree, or other Object at B, whose Distance you would know, from the Foot of the Wall at A.

The Lines A B and A C being drawn, and the Height of the Wall 22.5 Foot, set from A to C, where by your Instrument directed to B, you find the Degrees cut to be 71.25, which Angle lay down, so have you the Right-angled Triangle C A B, in which there is given, (1.) C A, the Height of the Wall 22.5 Foot. (2.) The Angle observed at C, 71.25 Deg. by which you may find the Distance A B, (by Case I. of R, A, P, T.) thus:

As Radius, Tangent 45 Deg.

To C A, the Height of the Wall 22.5 Foot;

Is the Tangent of A C B, the Angle observed, 71.25 Deg.

To the Distance A B, 66.28 Foot.

And if you would find the Length of the Visual Line C B, you may (by Case V. of R, A, P, T.) thus:

As the Sine of the Angle observed at C, 71.25 Deg.

Is to the Distance B A, 66.28 Foot;

Is the Radius, Sine 90 Deg.

To the Visual Line C B, 69.98 Foot.

B b

II. To

II. To take a Distance, (*Accessible or In-accessible*) at Two Stations.

Fig. VII. Being in a *Field* at E, there is a *Wind-mill* (or other *Object*) in another *Field* at F, (separated by a *River* or other *Impediment*) whose *Distance* is required.

In any other Part of the *Field*, remote from E, cause a *Mark* to be set up, as at D, and measure the *Distance* between E and D, which let be 115.00 Foot. — Then by your Instrument at E, find the Quantity of the *Angle* F E D, which suppose to be 106.50 Deg. And going from E to D, make Observation of the *Angle* F D E, which suppose to be 57.10 Deg.

By these Two *Angles*, and the *Measured Distance*, you have constituted an *Oblique Triangle* D E F, in which there is given, (1.) The *Measured Distance* E D, 115.00 Foot. (2.) The Two *Angles* at D and E, 106.50 Deg. and 57.10 Deg. And, (3.) The *Angle* at F, (the *Complements* of the other Two to 180 Deg.) 16.40 Deg. Whereby you may find the *Sides* E F and D F, (by *Axiom*. II.) thus:

As the *Sine* of the *Angle* at F, 16.40 Deg.

Is to the *Measured Distance* D E, 115.00 Foot;

So is the *Sine* of the *Angle* at E, 57.10 Deg.

To the *Distance* E F, 341.98 Foot.

And so is the *Angle* at D, 106.50 Deg. (or 73.50.)

To the *Distance* D F, 390.54 Deg.

III. If there were Three (or more) Ships on the Sea, and you being upon the Land, desire to know how far those Ships are from you; and also, how far they are distant one from the other.

Fig. VIII. LET the Three Ships be A, B, C; and you being upon the Shoar at M, are required to tell how far those Ships are from you, and also, how far from each other.

First, Being at M, make choice of some other Place upon the Shoar, at some considerable Distance, as at O 130.00 Fathom.

Secondly, Being at M, observe the Quantity of the *Angle* A M O, which we will suppose to contain 104.50 Deg. and then removing to O, and observing the *Angle* M O A, you find it to be 37.82 Deg. through which Degrees, Lines being drawn from M and O, will cross each other in A, which is the Place of the first Ship; which, with the Line of Distance M O, will form the *Oblique*

Oblique-angled Triangle A M O: In which there is given, (1.) The Side $M O$ 130.00 Fathom. (2.) The *Angle* $A M O$, 104.50 Deg. And, (3.) The *Angle* $A O M$, 37.82 Deg. And having Two *Angles* given, you have the third also given, viz. $M A O$, 37.68 Deg. whereby you may find the other Two Sides $M A$ and $O A$ (by *Axiom II.*) thus:

As the Sine of $M A O$, 37.68 Deg.

Is to the Side $M O$, 130.00 Fathom:

So is the Sine of $A O M$, 37.82 Deg.

To the Side $M A$, 130.41 Fathom, the Distance of the Ship at A , from M :

And so is the Sine of $A M O$, 104.50 (or 75.50 Deg.)

To the Side $O A$, 205.91 Fathom, the Distance of the Ship at A , from O .

Again,

Observing from M and O , to B , you found the *Angle* $B M O$ Fig. VIII. to contain 65.50 Deg. and $M O B$ 68.00 Deg. wherefore, if upon M and O , you lay down those *Angles*, you shall have another *Oblique-angled Triangle M B O*: In which you will have given as in the former, (1.) The *Angle* $B M O$, 65.50 Deg. (2.) $B O M$ 68.00 Deg. And consequently $M B O$, 46.50 Deg. together with the measured Distance $M O$ 130 Fathom, by which you may find the other Two Sides $M B$ and $O B$, by *Axiom II.* as in the former. For,

As the Sine of $M B O$, 46.50 Degrees,

Is to $M O$, 130.00 Fathom:

So is to the Sine of $B M O$, 65.50 Degrees,

To the Side $B O$, 163.08 Fathom:

And so is the Sine of the *Angle* $B O M$ 68.00 Degrees,

To the Side $B M$, 166.18 Fathom.

Lastly,

Observing again from M and O , to C , you find the *Angle* CMO to be 11.75 Deg. and $M O C$ 132.75 Deg. the which *Angles* laid down, you have a third *Oblique-angled Triangle C M O*, wherein there is given, as before, all the Three *Angles*, and One Side, $M O$, whereby you may find the other Two, $M C$ and $O C$, (by *Axiom II.*) as in the former.

So will $M C$ be 164.39 Fathom, and $O C$ 45.62 Fathom.

B b 2

And

Fig. VIII.

And thus have you the *Distances* of all the *Ships*, from the Two places, M and O, on the *Shoar*.

Now for their *Distances* one from another:

And First, For the *Distance* A B.

In the *Oblique Triangle* A B M, there is given, the *Sides* A M 130.41 Fathom, and B M 168.18 Fathom, and the *Angle* contained by them, A M B 39.00 Deg. whereby the third *Side* A B may be found, (By *Case* II. of O, A, P, T,) thus:

As the Sum of the *Sides*, A M and B M, 296.59 Fathom,
Is to the Difference of those *Sides*, 35.77 Fathom:

So is the Tangent of half the *Angles* at A and B, 70.50 Deg.

To the Tangent of half the Difference of those *Angles*, 18.81 Deg.

Which added to 70.52 Deg. gives 89.31 Deg. for the greater *Angle* B A M; and subtracted therefrom, leaves 51.69 Deg. for the lesser *Angle* A B M.

Then say, (By *Axiom* II.)

As the Sine of the *Angle* A B M, 51.69 Degrees,

Is to the *Side* M A, 130.41 Fathom:

So is the Sine of the *Angle* A M B, 39.00 Degrees,

To the *Side* A B, 104.59 F.

And in the same manner may the *Distance* from B to C, and from C to A, be found.

According to this Method may the *Distances* of many *Places* upon the *Land*, one from another be obtained, by making of *Observation*, by a *Theodolite*, *Semi Circle*, (or other *Graduated Instrument*) from Two *Places*, from whence all the other may be seen: An *Example* whereof I shall give, with the manner of making the *Observations*; and protracting of them, whereby the *Triangles* to be resolved for the Performance will be conspicuous: And for the *resolving* of them (it being altogether the same with that foregoing) I shall leave to the Ingenuity of the Practitioner.

Wherefore,

Let A B C D E be several *Places*, as *Churches* in a *Town* or *City*, or such like *Objects*.

Fig. IX.

1. Make Choice of Two such *Places*, from either of which, you may see all the *Places* whose *Distances* you require; which *Places* let be F and G, distant from each other 1000 Foot, more or less.

2. Set-

2. Set up your *Instrument* at F, and direct the *Sights* on Fig. IX. the *Diameter* thereof, to the other Place at G, and there fix it: Which done, direct the *Sights* to the several Places, A, B, C, D and E, noting what *Degrees* of the *Instrument* are cut by the *Index* at every *Observation*.

3. Then removing your *Instrument* to the second Place G, direct the *Sights* which are upon the *Diameter* thereof, back towards the first Place at F, where fix it: Then, turning the *Index* about, direct it to the several Places, A, B, C, D and E, as before; noting the *Degrees* cut by the *Index*. Which we will suppose to be such as are noted in this *Table*.

From F to	$\left\{ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \right\}$	the Degrees cut were	$\left\{ \begin{array}{c} 70.00 \\ 98.50 \\ 140.00 \\ 200.40 \\ 290.60 \end{array} \right\}$	From A to G	$\left\{ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \right\}$	the Degr. cut were	$\left\{ \begin{array}{c} 55.00 \\ 64.50 \\ 96.00 \\ 185.00 \\ 282.00 \end{array} \right\}$
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And the *Distance* from F to G 1000.

From these *Observations*, to make a *Plot* or *Map* of the *Situation* of the several Places.

1. Upon a Sheet of Paper, draw a *Right Line*, as F G, to contain 1000 Foot, of any *Scale*, which represents the *Line* of the *Distance* of the *Two Places*, where you *Observed* at F and G.

2. Place the *Center* of a *Protractor* upon F, and the *Diameter* thereof upon the *Line* F G; and there holding it fast, make *Marks* against the several *Degrees* that were cut by the *Index*, when the *Sights* were directed to the several *Objects* at A, B, C, &c. when you *Observed* at F, and through those *Points* draw *Right Lines*, at Pleasure; as F A, F B, F C, F D, F E.

3. Lay the *Center* of the *Protractor* to the Point G, and the *Diameter* thereof upon the *Line* F G, and there holding it fast, make *Marks* against those *Degrees* of the *Protractor*, as the *Index* did cut upon the *Instrument*, when you made *Observation* at G; and through those *Points*, and the Point G, draw *Lines*, as G A, G B, G C, G D, and G E, crossing the former *Lines* (drawn from F,) in the respective *Points*, A, B, C, D and E. Which *Points* will lye upon your *Paper*, in the same *Position* as the *Places* you took notice of, were situate on the *Ground* on which they stood: And being thus laid down, if you take with your *Compasses*

Fig. IX. passes the *Distance* between any Two of them, and measure it on the same *Scale* you laid down the *Line* F G by, it will give you the *Distance* between those Two Places.

But, their *Distances* may be more exact and accurately attained unto by *Trigonometrical Calculation*.

For, in every *Triangle*, as in F G A, F G B, F G C, F G D, and F G E, there is given, (1.) The *Angle* A F G, observed at F; (2.) The *Angle* A G F, observed at G; and, (3.) The *Side* F G (the *Stationary Distance*) included between them, to find the other *Sides*, A F and A G. The *Practice* whereof I commit to the Ingenuity of the *Practitioner*.

C H A P. III.

Of PLANOMETRIA.

O R,

Of the Mensuration of Plain Superficial Figures.

I. Of the Geometrical Square A B C D, whose Side A B is 27.32 Foot. By Logarithms.

Fig. X. As 1 F : to 27.32 F : : 27.32 F : to 746.38 F.

Logar. of 27.32

1.4364

Logar. of 27.32

1.4364

Logar. of 746.38

2.8729

The *Superficial Content* of the *Square* in *Feet*.

II. Of the Parallelogram (or Long Square) E F G H, whose Length E F is 27.25 Yards, and Breadth E G, 6.29 Yards.

Fig. XI. As 1 : 27.25 : : 6.29 : 17.03

Logar. of 27.25

1.43536

Logar. of 6.25

0.79588

Logar. of 17.03

2.23125

III.

Of the Triangle K L M, whose Longest Side L M is 267.26
Perches; and its Perpendicular L N 160.25 Perches.

$$\begin{array}{r} \text{As } P : \frac{1}{2} L M \text{ } 133.63 P :: L N \text{ } 160.25 P : 214.14 P. \\ \text{ar. } 160.25 \quad \quad \quad 2.204798 \\ \text{ar. } 133.63 \quad \quad \quad 2.125904 \\ \hline \text{ar. of } 214.14 \quad \quad \quad 4.330702 \end{array}$$

Fig. XII.

Of the Trapezia (or Figure of Four unequal Sides) O P Q R.
The Diagonal, whereof O Q is 27.32 Perches, the Perpendicular
P S 9.53 P. and the Perpendicular R T, 21.06 P.

$$\begin{array}{r} \text{As } 1 : \frac{1}{2} O P \text{ } 13.66 :: P S \text{ } 9.53 : 417.85 \\ \text{ar. of half } O Q \text{ } 13.66 \quad \quad \quad 1.135451 \\ \text{ar. of } P S \text{ and } R T, \text{ } 30.59 \quad \quad \quad 1.485579 \\ \hline \text{ar. of } 417.85 \text{ Perches} \quad \quad \quad 2.621030 \end{array}$$

Fig. XIII.

Which is the Area, or Content in Perches.

Of an Irregular Plot, consisting of many unequal Sides and An-
gles: As the Figure A B C D E F G.

Before such Irregular Figures can be measured, they must be re-
duced in Triangles or Trapezia's, by drawing of Lines from An-
gle to Angle at the best Advantage, as in this, by the Lines F C
and F G, which Two Lines divides the whole Plot into the Two
Trapezia's A B C F, F C D E, and One Triangle F E G. And
the Bases and Perpendiculars being such as are expressed in
the Figure, they may be measured, as is shewed in the Two fore-
going Sections hereof; and according to the following Operations.

I. For the Trapezia A B C F.

$$\begin{array}{r} \text{Arithm of } \left\{ \begin{array}{l} \text{half } A C, \text{ } 12.16 \\ B H \text{ and } H F, \text{ } 8.13 \\ \text{the Area of } A B C F, \text{ } 98.86 \end{array} \right. \quad \begin{array}{r} 1.084933 \\ 0.910090 \\ \hline 1.995023 \end{array} \end{array}$$

II. For the Trapezia F C D E.

$$\begin{array}{r} \text{Arithm of } \left\{ \begin{array}{l} \text{half } F D, \text{ } 12.96 \\ C L \text{ and } E M, \text{ } 23.11 \\ \text{the Area of } F C D E \text{ } 299.51 \end{array} \right. \quad \begin{array}{r} 1.112605 \\ 1.363800 \\ \hline 2.476405 \end{array} \end{array}$$

HL. For

III. For the Triangle F E G.

Logarithm of $\left\{ \begin{array}{l} \text{half EF, } 9.50 \\ \text{GN, } 5.82 \\ \text{the Area of FEG, } 55.29 \end{array} \right.$ $\begin{array}{r} 0.97772 \\ 0.76492 \\ \hline 1.74264 \end{array}$

The Area of $\left\{ \begin{array}{l} \text{A B C F} \\ \text{F C D E} \\ \text{F E G} \end{array} \right.$ is $\left\{ \begin{array}{l} 98.86 \\ 299.51 \\ 55.29 \end{array} \right.$
 The Area of A B C D E F G $\underline{453.66}$

V. Of a Circle A B C D, whose Diameter B C let be 14.00.

Fig. XV. The Proportion of the Diameter of any Circle, to the Circumference thereof, is (in the least Terms) as 7 to 22: But in greater Numbers, as 113 to 355; and of these I shall make use.

I. By the Diameter, to find the Circumference.

As 113 : 355 :: 14 Diam. : 43.98 Cir.

Logar. of 113

2.053078

Logar. of 355

2.550228

Logar. of 14.00 the Diameter

1.146128

The Logar. of 43.98 the Circumference

3.696356

1.643278

II. By the Circumference, to find the Diameter.

As 355 : 113 :: 43.98 Circum. : 14.00 Diam.

Logar. of 355

2.550228

Logar. of 113

2.053078

Logar. of 43.98 Circ.

1.643278

Logar. of 14.00, the Diam.

3.696356

1.146128

III. By

Of Geodæcia.

155

III. By the Diameter, to find the Area.

Fig. XVI.

113 in 4 (<i>viz.</i> 452.)	2.655138
Is to 355	2.550228
is the Diam. 14 in 14 (<i>viz.</i> 196)	2.292256
	4.842484
To the Logar. of 159.94, the Area	2.187346

IV. By the Circumference, to find the Area.

355 $\div$ 4 (<i>viz.</i> 113)	3.152288
Is to 113	2.053078
is 43.98 $\div$ 43.98, Area Squa.	{ 1.643255
The same again	
	5.339588
To the Logar. of the Area 153.49	2.187300

V. By the Diameter, to find the Area.

As 10 : 8.862 :: 14 Di. : 12.41 :: 12.41 : 154.00	
g. 8.862	0.947532
g. 14	1.146128
gar. 12.41	1.093660
the double of it	2.187320 Log. of 154 the Area.

CHAP. IV.

Of GEODÆCIA: Or, Land Measuring.

How (by any Graduated Instrument) to take the true Plot of any large Piece of Ground, as Common Field, Park, Wood, &c.

IN going round about a Field to Survey it, there are Two ways; for in going round about it, you must either go on the *inside* or the *outside*; and sometimes you may be constrained to go sometimes *within*, and sometimes *without*.

Let A B C D E F, be such a Field to be surveyed and plotted. Begin at any Angle thereof, as at A, and there setting up your Instrument, lay the Index and Sights upon the Diameter thereof; and turning it about, direct the Sights to B, and there

C c

screw

Fig. XVII.

Fig.
XVII.

screw it fast; and turn the *Index* about, till through the *Sights* you see the *Angle* at F; and there note what *Degrees* of the *Instrument* were cut by the *Index*, which we will suppose to be 300 *Deg.* for the *Quantity* of the *Exterior Angle* F A B, without the *Field*: Or, 60 *Deg.* for the *Quantity* of the *Interior Angle* within the *Field*: Then measure the *Side* A F, which suppose to be 41.00 *Pole* or *Perches*, and the *Side* A B 30.00 *Perches*. These *Distances*, with the *Quantity* of the *Angle* before observed 300, or 60 *Deg.* set down in a *Book* or *Paper*, as you see in the following *Table*.

2. Remove your *Instrument* to B, and laying the *Index* upon the *Diameter*, turn the *Instrument* about, till through the *Sights* you see the *Place* where the *Instrument* last stood at A, and then screw it fast; and turn the *Index* about, till, through the *Sights*, you see the *Angle* C, and note the *Degrees* which the *Index* cut, which are here 145 *Deg.* for the *Exterior*, or 215 *Deg.* for the *Interior Angle* A B C; and the *Distance* B C measured is 28.00 *Perches*: Both which, note down as before.

3. Remove your *Instrument* to C, and placing the *Index* on the *Diameter*, look back to B, and there screw it fast; and turn the *Index* about, till you see the *Angle* at D; where the *Index* cut 270.25 *Deg.* for the *Exterior*, or 89.75 *Deg.* for the *Interior Angle* B C D, and the measured *Distance* C D is 23.40 *Deg.*

4. Remove the *Instrument* to D, the *Index* on the *Diameter*, look back to C, and then screw it fast; then turn the *Index* about, till by the *Sights* you see the *Angle* at E; the *Index* cutting 220 *Deg.* for the *Exterior*, or 97 *Deg.* for the *Interior Angle* C D E, the *Measured Distance* D E being 28.00 *Perches*: Which note down as the other.

5. Place your *Instrument* at E, the *Index* on the *Diameter*, look back to D, and then screw it fast, and turn the *Index* about, till through the *Sights* you see the *Angle* at F, the *Index* cutting 220 *Deg.* for the *Exterior*, or 140 *Deg.* for the *Interior Angle* D E F; the *Measured Distance* E F being 42.80 *Perches*.

6. Place the *Instrument* at F, and the *Index* lying on the *Diameter*, look back to E, and then screw it; and turn the *Index* about, till through the *Sights* you see the *Angle* at A, (or the *Place* where you first placed your *Instrument*) where the *Index* cut 210 *Deg.* for the *Exterior*, or 150 *Deg.* for the *Interior Angle* E F A, the *Measured Distance* F A being 41.00 *Perches*, as before. All which being noted down according to these *Observations*, the *Measured*, will stand as in this *Table*.

Of Geodæcia.

157

Fig.
XVII.

	Exterior.	Inferior.		Perches.
The Angle at	A 300.00	60.00	The Distance from	A to B—30.00
	B 145.00	215.00		B to C—28.40
	C 270.25	89.75		C to D—23.40
	D 263.00	97.00		D to E—28.00
	E 220.00	140.00		E to F—42.80
	F 210.00	150.00		F to A—41.00

These *Observations* of *Sides* and *Angles*, as they were taken in the *Field*, being noted down in a *Book* or *Paper*, as in this *Table*; a *Plot* of the *Field* may be drawn upon *Paper* or *Parchment*, by the following *Directions*.

To *Protract* the former *Work*.

1. Upon a Sheet of *Paper* or *Parchment* draw a Line A B, to contain 30.00 *Perches*, of any *Scale*: And upon the end thereof A, place the Centre of a *Protractor*, laying the Diameter of it upon the Line A B: —Then, the *Exterior Angle* at A, being 300.00 *Deg.* make a Mark against 300.00 *Deg.* of your *Protractor*; and through that Point, and the Point A, draw a Right Line, downwards, as A F, to contain 41.00 *Perches*, of the same *Scale*.
2. Apply the Centre of the *Protractor* to the Point B, and the Diameter upon the Line A B; then against 145.00 *Deg.* (the *Angle* at B) make the Mark, and through that Mark, and B, draw a Right Line B C, to contain 28.40 *Perches*.
3. Lay the Centre of the *Protractor* upon C, and the Diameter upon B C, and against 270.25 *Deg.* make a Mark; through which, and the Point C, draw the Line C D, to contain 23.40 *Perches*.
4. Apply the Centre of the *Protractor* to the Point D, and its Diameter to the Line C D; making a Mark against 263.00 *Deg.* through which, and the Point D, draw the Line D E, to contain 28.00 *Perches*.
5. Lay the Center of the *Protractor* upon the Point E, and its Diameter upon D E, and against 220.00 *Deg.* make a Mark; through which, and the Point E, draw a Right Line; which (if you have committed no former Error) will cut the Line, first drawn, in the Point F: And, by this Means, you will have upon your *Paper*, the exact *Figure* of your *Piece* of *Ground*: Which you may cast up, and find the Quantity thereof in *Perches*, (by the *Directions* of the Third Chapter hereof) which being divided by 160, will give the Content in *Acres* and odd *Perches*.

Fig. XVII. In this *Example*, we have wrought by the *Exterior*, (or *Outward*) *Angles*: But if you would *Protract* the same by the *Interior* (or *Inward*) *Angles*; it is done, by subtracting the *Exterior Angle* from 360 Deg. and the Remainder will be the *Interior Angle*: So, the *Angle* at A, being 300 Deg. that subtracted from 360 Deg. leaves 60 Deg. for the *Interior Angle* at A: And so of all the rest, as they are set down in the *Table*.

C H A P. V.

Of STEREOMETRIA:

O R,

Mensuration of Solids.

Fig. XVIII.

I. Of the Cube A B C D E F, whose Side is 5.20 Foot.

As 1 : 5.20 :: 5.20 27.04 :: 27.04 : 140.61 Foot.

Logar. 5.20	0.716003
Logar. 5.20	0.716003
Logar. 27.04	1.432006
Logar. 5.20	0.716003
Log. of 140.61	2.148009

Equal to the *Solidity* of the *Cube* in *Feet*.**Fig. XIX.**This might have been done more easily, for the *Logar.* of the *Side* 5.20, which is 0.716003, multiplied by 3, would have produced 2.148009, the *Logar.* of 140.61, the *Solidity*.

II. Of a Long Cube or Parallelipipedon, G H I K L, whose Side of the Square at the End, K I, is 1.27 Foot, and of its Length G H, 5.32 Foot.

As 1 : 1.27 :: 1.27 : 1.61 :: 5.32 : 8.58 Foot.

Log. 1.27	0.103804
Log. 1.27	0.103804
Log. 5.32	0.725911
Log. 8.58	0.933519

The *Solid Content*.

III. Of

III. Of an Oblong Parallelipedon M N O P Q R, whose Breadth Fig. XX: at the End N O is 3.40 Foot, and Depth M N 6.50 Foot, and the Length thereof M P, 16.25 Foot.

As 1 : 3.40 :: 6.50 : 22.10 :: 16.25 : 159.12 Foot.

Logar. 3.40	0.531479
Logar. 6.50	0.812913
Logar. 16.25	1.210853
Logar. 359.12	2.555245

The Solidity.

IV. Of a Pyramis S T V X N, whose Side of its Base S T, &c. is Fig. XXI. 1.12 Foot, and Altitude N X 12.90 Foot.

As 1 : 1.12 :: 1.12 : 1.25 : $\frac{1}{3}$ N X : 4.30 : 5.39 the Sol. Con.

Logar. 1.12	0.049218
Logar. 1.12	0.049218
Logar. 4.30	0.633468
Logar. 5.39	0.731904

the Solid Content.

V. Of a Cone A B C D, whose Diameter B C is 6.12, and Altitude A D 12.90 Foot.

Fig. XXII.

As 7 in 4 (28) : 22.

So 6.12 in 6.12 (37.45) : 29.30

The Area of the Circle of the Base.

As 1 : 29.30 :: So 4.30 : 126.00

The Solidity.

Logar. 28	1.447058
Logar. 22	1.340422
Logar. 37.45	1.573452
	2.913874
Logar. 29.30	1.466816
Logar. 4.30	0.633468
Logar. 126.00	2.100284

the Area.
the Solidity.

VI. Of the Frustum of a Pyramis or Cone.

Fig.

Let A B C H G F be the Frustum of a Square Pyramid, the Side of the Square at the Lesser end A B, is 1.60 Foot, and therefore

XXIII.
XXIV.

fore the *Area* of that *Square* A B C D is 2.56 Foot: — The *Side* of the *Square* at the *Greater End* F G, is 2.30 Foot; and therefore the *Area* thereof is 5.29 Foot, and the *Altitude* D E 39.3 Foot.

Then,

The Logarithm of A B C D 2.56 is	0.408240
The Logarithm of E F G H 5.29 is	0.723455
Their Sum	1.131695
Their half Sum	0.565847

Which is the Logarithm of	3.68
The Sum of the Three Products	11.53
The Logar. of the Sum	11.53
The Log. of $\frac{1}{2}$ of the Altitude	11.31
The Log. of	151.04
Which is the <i>Solidity</i> of the <i>Frustum</i> A B C D E F G H.	2.179100

In the same manner the *Frustum* of a *Cone* of the same *Altitude*, and the *Area's* of the *Circles*, at both *Ends*, the same with the *Area's* of the *Squares*, the *Solidity* of such a *Frustum Cone*, will be found to be the same.

Fig.
XXV.

VII. Of a *Globe* or *Bullet*, whose *Diameter* is 1.75 Foot.

The Logar. of the Diameter 1.75	0.243038
Multiplied by 3	0.729114
The Logar. of 355	2.550228
Their Sum	3.279342
Logar. of [678 always] Subtract	2.831229
Refts the Logar. of 2.806 Foot	0.448113
Which is the <i>Solidity</i> of the <i>Bullet</i> .	

Fig. X.

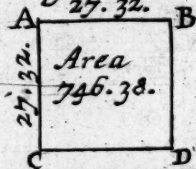


Fig. XI.

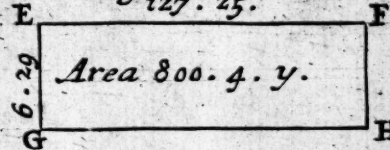


Fig. XII.

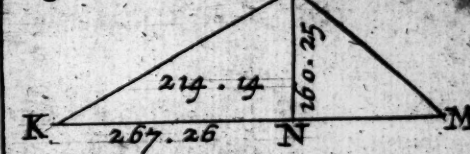


Fig. XIII.

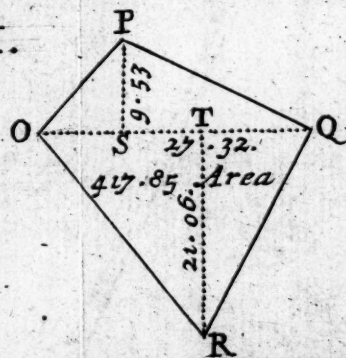


Fig. XIV.

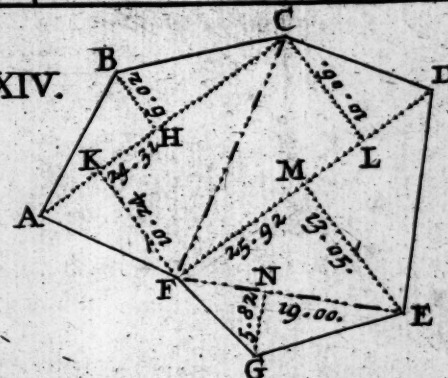


Fig. XV.

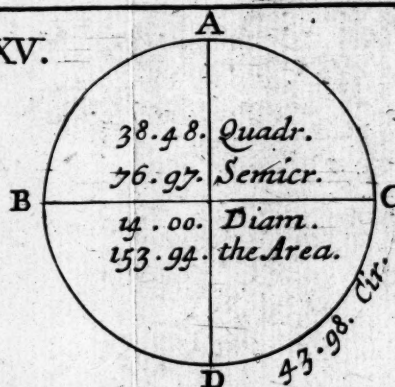


Fig. XVII.

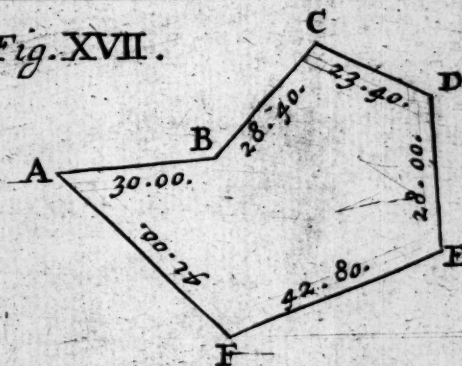


Fig. XVIII.

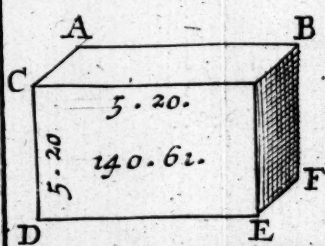


Fig. XIX.

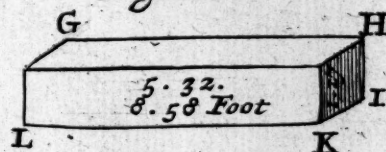


Fig. XX.

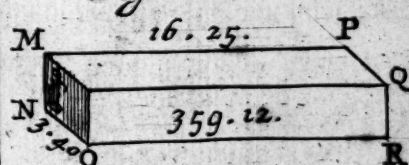


Fig. XXI.

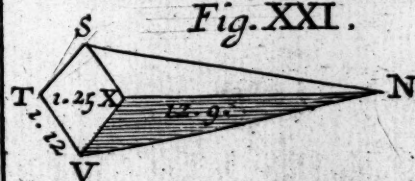


Fig. XXII.



Fig. XXIII.

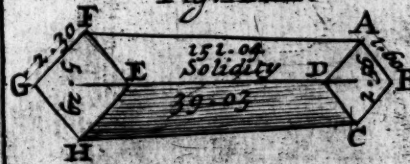


Fig. XXIV.



ANCILLA MATHEMATICA.

VEL,

Trigonometria Practica.

SECTION II.

OF

COSMOGRAPHY.

COSMOGRAPHY, in Greek Κοσμογραφία, is derived α Κοσμος; i. e. *Mundus*; & γραφή; i. e. *Scriptum*, the Description: So that COSMOGRAPHY is a Description of the World; consider'd under the Names of the Heavens, as the *Celestial*; and of the Earth and Water, as the *Terrestrial* Parts thereof. A true and perfect Representation of both which, may be (and usually is) described upon Two Round Bodies, called *GLOBES*: And therefore we shall first treat of them.

CHAP. I.

Of a material Sphere or Globe; and of such Circles, Lines and Points, as are described upon it, and that are appendant to it. And how to rectifie the Globe, fitting it for the resolving of such Problems as are to be performed by it.

I. Of the Circles of the Sphere.

A Globe or Sphere is an Artificial Representation of the *Starry Heaven*, or of the *Earth and Water*, under the Form or Figure

Fig.
XXV.

Fig. XXV. gure of Roundness, which they are supposed to have: Shewing in a just Proportion every particular Constellation in the Heavens: And such a Globe is called [A Cælestial Globe] and every particular Region or Country on the Earth, and Island in the Sea: And such a Globe is called [A Terrestrial Globe.]

Upon the Convex Superficies of either of these Globes (besides the Constellations of the Stars, upon the Cælestial; and the Countries, Kingdoms, Sea-Coasts, Islands, &c. upon the Terrestrial) there are described several Circles; some Great Circles, which divide the whole Body of the Globe into Two Equal Parts; and some Lesser Circles, which divide the Body of the Globe into Two Unequal Parts.

The Great Circles described upon the Body of either Globe are,

The *Æquinoctial* } { The *Æquinoctial* }
The *Ecliptick* } { The *Solstitial* } Colure.

The Lesser Circles are,

The Two Tropicks; and, The Two Polar Circles.

I. Of the *ÆQUINOCTIAL*.

The *Æquinoctial* is a Great Circle, dividing the Body of the Globe into Two Equal Parts, passing through $\mathcal{A}$ and α ; which Points are, each of them, equidistant from P and S, the Two Poles of the World; and so the Line or Axis P A S passing through the Body of the Globe, cutteth this Circle always at Right Angles.

II. Of the *ECLIPTICK* and *ZODIACK*.

The *Zodiack* is a Great Circle, crossing both the *Horizon* and *Æquinoctial* in A, the East and West Points of the *Horizon* at Oblique Angles, viz. the *Horizon* at an Angle equal to the Sun's greatest Meridian Altitude in any Latitude, but the *Æquinoctial*, always, at an Angle, equal to (23.5 Deg.) the Sun's greatest Declination. This Circle is supposed to have Breadth; for, through the middle of it, there passeth a Line, noted in the Figure with the Twelve Signs of the *Zodiack*, γ δ Π ϵ Ω ϖ on the upper Part, and with $\varnothing$ $\cap$ $\dagger$ $\sphericalangle$ $\approx$ $\times$ on the lower Part; which Line is called the *Ecliptick*, or *Via Solis*, for that the Sun always keepeth in this Line.

III. Of the Two COLURES.

These are Two *Great Circles*, which pass through the *Poles* of the *World*, where they cut each other at *Right Angles*: One of these cutteth the *Horizon* in A, and the *Æquinoctial* also in the Point $\gamma \equiv$, and is therefore called the *Æquinoctial Colure*. The other *Colure* cutteth the *Horizon* in H and O, the *North* and *South* Points thereof, and also the *Ecliptick* in the Points $\ominus$ and $\wp$, and is therefore called the *Solstitial Colure*. In the Figure the *Æquinoctial Colure* is noted with P A S (and is the same with the *Sixth Hour Circle*.) the *Solstitial Colure*, with H P O S, and is the same with the *Meridian*.

Fig.
XXV.

IV. Of the TROPICK of CANCER.

This is a *Lesser Circle* of the *Sphere*, described upon the *Superficies* of the *Globe*, parallel to the *Æquinoctial*, on the *North Side* thereof, at 23.5 Deg. distant therefrom, equal to the *Sun's* greatest *Declination*. It is called the *Northern Tropick*, for that it lyeth towards the *North Pole*, and is represented in the Figure by the *Small Circle*, $\ominus 6 \ominus$.

V. Of the TROPICK of CAPRICORN.

This is a *Lesser Circle* also, described upon the *Globe*, parallel to the *Æquinoctial* on the *South Side* thereof, at the Distance of 23.5 Deg. towards the *South Pole*, and is called the *Southern Tropick*: It is noted in the Figure with $\wp 6 \wp$.

VI. Of the Two POLAR CIRCLES.

These are Two *Lesser Circles* described about the *Poles* of the *World*, parallel to the *Æquinoctial* and *Tropicks*, and so far distant from either *Pole*, (*viz.* 23.5 Deg.) as the *Tropicks* were from the *Æquinoctial*: That described about the *North Pole*, noted with B 6 C, is called the *Artick Circle*; and the other, noted with G 6 E, described about the *South Pole*, is called the *Antartick Circle*.

Besides these *Circles*, which are described upon the *Convex-Superficies* of either *Globe*, there are others, which are framed without the *Body* of the *Globe*; and those are, principally, Two,
D d of

Fig. XXV. of the *Meridian*, and the *Horizon*; the one of *Brass*, the other of *Wood*. — *Circles*, indeed they are not so properly called; for, in the rigorous Sense, no *Line* is supposed to have any *Breadth*; but both these have *Breadth* allowed them; that such Things might be written, or engraven upon them, as might render them more useful in all Positions of the *Globe*: And therefore, (they being of a *Circular Form*) notwithstanding the Impropriety of Speech, we will have it so; and we must call them, The *Meridian* and *Horizontal Circles*. The *Meridian* in the Figure, is noted with the Letters Z Æ N æ, and the *Horizon* with H A O.

Unto this *Brazen Meridian*, there belong Two other Appendants, viz. An *Hour Circle*, with its *Index*; and a *Quadrant of Altitude*.

Through the Body of either *Globe*, there runs a strong *Wyre*; the Ends whereof are so fixed in the *Brazen Meridian*, that the *Body* of the *Globe* may turn about together with the *Wyre*. — This *Wyre* is called the *Axis* of the *World*, and the Ends thereof, the *Poles* of the *World*, one the *Artick* or *North Pole*, noted with P; the other, the *Antartick*, or *South Pole*, noted with S.

Unto this *Brass Meridian* also, there belongs another Appendant, called A *Quadrant of Altitude*; which is a thin Plate, divided into 90 equal Parts or Degrees, and fitted with a *Nut* and a *Screw*, to move to any Degree upon the *Meridian*; but generally in the Vertical Point Z, the *Zenith* of any Place.

II. Of the several Positions, that a *Globe* or *Sphere*, may be posited in its *Horizon*.

There are but Three Positions, in which a *Globe* may be seated in its *Horizon*; viz. (1.) *Direct*. (2.) *Parallel*. (3.) *Oblique*. Of which, the Two first are *Particular*, the third more *General*.

I. Of *Direct* Position.

The *Globe* may be so placed in the *Frame*, that both the *Poles* thereof may rest upon (or lye directly in) the *North* and *South* Points of the *Horizon*, neither *Pole* having any *Elevation*: The *Zenith* Point being in the *Æquinoctial Circle*, and the *Axis* of the *World* in the Plain of the *Horizon*. And this is called *Direct Position*.

II. Of

II. Of Parallel Position.

The *Globe* may be so placed in the *Frame*, that one of the *Poles* shall be in the *Zenith*, and the other in the *Nadir Points*; the *Poles* having 90 Deg. of *Elevation* above the *Horizon*: And in this *Situation*, the *Æquinoctial Circle* will be in the *Horizon*. This *Position* is called *Parallel Position*.

III. Of Oblique Position.

The third *Position* of the *Globe* is more *General*; for it hath relation to all *People* living between the *Æquinoctial* and either *Pole*: And according as the *Poles* are *Elevated* or *Depressed*, accordingly are the *People* said to be *situate*: That is, if the *Globe* be placed in the *Horizontal Frame*; so that the *Pole* be elevated above the *Horizon* 10 Deg. then is the *Globe* elevated, or fixed, to resolve such *Questions* *Geographical* or *Astronomical*, as relate to such *Places* and *People*, who have the *Pole* elevated (or who live in the *Latitude* of) 10 Deg. And this *Position* of the *Globe* is called *Oblique*, because the *Axis* of the *World*, the *Æquinoctial*, and all the *Parallels* of the *Sun's Declination*, are cut by the *Horizon* at *Oblique Angles*; whereas in the Two former *Positions*, they cut one the other *Right Angles*.

III. How to Rectifie the Globes, fitting them for Use in any Latitude or Part of the World.

Being provided of a *Pair of Globes*, the *Meridian*, *Horizon*, and *Hour-Circle*, truly turned and divided; also the *Balls* truly hung or poized upon the *Axis*, and the *Meridian* and *Horizon* (in all *Positions*) cutting each other at *Right Angles*; the *Papers* truly joined in their pasting upon the *Bodies*, &c. All which are to be performed by the *Workman*, (yet the *Buyer* ought also to have *Inspection* thereunto) you may proceed to *Rectifie* them for *Use* in this manner.

1. Put the *Brass Meridian* into the Two *Notches* that are cut in the *North* and *South* Parts of the *Horizon*; the *Graduated*, or *Divided*, Part thereof, towards the *East Part* of the *Horizon*; and the plain, or undivided *Side* thereof, towards the *West*, and let the *Meridian* also rest in the *Notch* which is in the *Foot*, or bottom, of the *Frame* of the *Horizon*.

Fig. XXV. 2. Place the *Brass Hour-Circle*, or *Wheel*, about the Pole; so that the *Hour-Lines* of 12 and 12 do lye directly over the *East*, or *Graduated*, Side of the *Meridian*; and that the Point of the *Axis* of the *Globe* do pass directly through the *Centre* of the *Hour-Wheel*; then shall the Two *Twelves* represent the Two Hours of 12 and 12: That towards the South Part of the *Meridian* 12 at Noon, and the other towards the North Part, 12 at Midnight: And the Two Sixes shall represent the Two Hours of Six a Clock: That towards the *East*, 6 in the *Morning*, and the other 6 at *Night*. Then put the *Index* (or *Pointer*) upon the end of the *Axis*; so that as the *Globe* being turned, which way soever, the *Pointer* may move with it; and so is your *Hour-Circle* rectified.

3. Elevate the Pole of your *Globe* (whether *North* or *South*) according to the *Latitude* of the Place of that what part of the World you are in: As suppose *London*, which hath 51 Deg. 30 Min. of *North Latitude*: The *Meridian* being in the Notches of the *Horizon*, and also in the Notch at the bottom of the Frame, as is before directed. Move the *Meridian* upwards or downwards in the Notches, till you find 51 Deg. 30 Min. of the *Meridian*, justly to touch the upper part of the *Horizon*, on the *North* part thereof: And so is your *Globe* rectified to the *Latitude* of 51 Deg. 30 Min.

4. The next thing to be rectified is the *Quadrant of Altitude*; which must be done, by having respect to the *Latitude* also: Wherefore, the *Latitude* being 51 Deg. 30 Min. Count 51 Deg. 30 Min. *North*, upon the *South* part of the *Meridian*, from the *Equinoctial Circle*, towards the *North* (or elevated) Pole, and there put on the *Nut*, which is at the end of the *Quadrant*; so that the edge of the *Divisions* of the *Quadrant*, may be directly under the Degrees of the *Latitude*, viz. 51 Deg. 30 Min. and there screw the *Nut* fast. And thus is your *Globe* Rectified for the Solution of all such Questions Cosmographical, as are to be wrought thereby in that *Latitude* of 51 Deg. 30 Min.

C H A P. II.

Cosmographical ELEMENTS,

Necessary to be known.

There are Two kind of *Motions* in the Heavens; the first is called the *Common Motion* of the *fixed Stars* and *Planets* together; by which they go all about in 24 Hours from *East* to *West*. The second Motion is called the proper *Motion*; by which they go about, every one in his own *Time* or *Period*, from *West* to *East*.

Fig.
XXV.

These Two *Motions* are the Original of Two *Circles*, the *Æquinoctial* and the *Ecliptick*; for the *Diurnal Motion* is done about the Pole of the *Æquinoctial* either in the *Æquinoctial* it self, or in a *Lesser Circle*, parallel unto it: But the proper *Motion*, is about the Poles of the *Ecliptick*, either in the *Ecliptick* it self, or in a *Lesser Circle*, parallel unto it.

I. The Sun's Center keepeth always upon the *Ecliptick Line*, but the other *Planets* do go from the *Ecliptick* on both Sides 8 Deg. Hence the broad *Circle*, whose Middle is the *Ecliptick*, doth arise, and is called the *Zodiack*.

II. The *Æquinoctial* is in the Heavens about that Streak, which the Sun doth make by his *Diurnal Motion* on the Days of the Two *Æquinoxes*, viz. the 10th of *March*, and the 12th of *September*.

III. The *Zodiack* is known by the Twelve *Asterisms* of *fixed Stars*, called, The *Twelve Signs*.

IV. The Two *Luminaries* are the Sun ☉, and the Moon ☾, the Moon cometh round by her proper *Motion*, in a *Month*, the Sun in a *Year*.

V. The other *Planets* are either the *Superior*, as Saturn ♄, coming about in his proper *Motion*, once in 30 Years. *Jupi-*
ter

Fig.
XXV.

ter ♃ in 12, Mars ♂ in 2 Years. The *Inferior Planets* are Venus ♀ and Mercury ☿; Venus is 9 Months *Morning Star* and other 9 Months *Evening Star*. These Two Planets keep always near to the Sun, so that Mercury ☿ is for the most part covered with its Beams.

VIII. The *fixed Stars* move also from West to East, either in the *Ecliptick*, or in a Parallel to the *Ecliptick*, but very slowly viz. One Degree in 70 Years. Hence the Signs are distinguished in *Starred* and *Un-starred*. The *Starred Signs* are the Twelve *Asterisms* of the *Zodiack*; but the *Un-starred* are every one a Twelfth part of the *Ecliptick*. Now the *Starred Signs* leave their former Places, and are preceded in some 1800 Years almost One whole Sign; so the *Starred Aries* ♈, stands now in the Place of the *Un-starred Taurus* ♉; and the *Starred Taurus* ♉, in the Place of the *Un-starred Gemini* ♊, &c.

IX. The *Æquinoctial* and *Ecliptick* are immutable, for there is never but One *Æquinoctial*, and One *Ecliptick*: But the *Horizon* and *Meridian* are mutable: For every Body walking upon the Superficies of the *Earth*, doth carry along with him his *Horizon*; So this Circle is as manifold as there are divers Points upon the Surface of the *Earth*. The *Horizon* is determined by the Eye of the Man turning about in an even open Field, where the Heaven seemeth to join with the *Earth*; and its Office is to shew the *Rising* and *Setting* of all *Heavenly Bodies*.

X. The *Meridian* is not alter'd by going on straight towards South or North, but only when you walk never so little towards the East or West, you have presently another *Meridian*. It is observable in the Heaven, by letting fall a Plummets or Perpendicular from the *Vertex*, by the Sun (or any Star) being at its highest.

XI. Every one of these Four Circles hath its Poles, which the Circle is just between, and every way equally distant from it, exactly dividing the Sphere into Two equal Hemispheres, and they divide each other into Two equal Semicircles: And by the Poles of each, there are described Secondary Circles (the Meridian only excepted) which Secondary Circles do cut the Principal Circle into Two Equal Parts, and at Right Angles.

XII. The

Fig.
XXV.

II. The *Poles* of the *Æquinoctial* are the same with the *Poles* of the *World*; the one of which is called the *Arctic Pole*, because it is near to the Two *Arktos* or *Bears*: The other opposite to it, is called *Antarctic*: And the straight Line, which passeth between, through the Centre of the *Sphere* (from one *Pole* to the other) is called the *Axis* of the *World*. The *Æquinoctial* divideth the *Ecliptick* into Six *North*, and Six *South*, *Signs*: The *Secondary Circles* of the *Æquinoctial*, are called in the *Heavenly Sphere* *Circles* of *Declination*. Amongst these is one of chiefest Note, the *Meridian*; and besides it, Eleven *Hour-Circles*, passing by every 15th Degree of the *Æquinoctial*, to be reckoned from the *Meridian*, and so they divide the whole *Æquinoctial* into 24 equal *Hours*. There are also Two chief *Secondary Circles* of the *Æquinoctial*, which are called *Colures*; the one passing by the *Vernal* and *Autumnal* *Section*, is called the *Colure* of the *Æquinoxes*; the other passing by the Two *Solstitial Points*, viz. the beginning of *Cancer* ☊ and *Capricorn* ☋, is called the *Colure* of the *Solstice*. This latter, divideth the *Ecliptick* in *Ascending* and *Descending* *Signs*; because in the first the Sun doth ascend to our *Zenith* in *Capricorn*, viz. in ☊ ☋ ☌ ☍ ☎ ☏, which are also called the *Signs* of *Short Ascension*, because they rise in a short time equal to the *Shortest Day* of the *Year*: But in the *Descending* *Signs*, the Sun doth descend every *Day* more and more from our *Zenith*, and those are ☐ ☑ ☒ ☓ ☔ ☕: These are called also *Signs* of *Long Ascension*, because they Rise in a time, equal to the *Longest Day* in the *Year*. Both *Colures* together divide the *Ecliptick* into Four *Quadrants*; the *Vernal* containing *Aries* ♈, *Taurus* ♉, *Gemini* ♊; The *Summer* Quadrant, *Cancer* ☊, *Leo* ♌, *Virgo* ♍; The *Autumnal*, *Libra* ♎, *Scorpio* ♏, *Sagittarius* ♐; the *Winter* Quadrant, *Capricorn* ☋, *Aquaries* ♒, *Pisces* ♓.

III. The *Secondaries* of the *Terrestrial Æquinoctial* are called *Meridians*, and they are 18, passing by every 10th Degree of the *Æquinoctial*, and in some *Globes*, through every 15th Degree of the *Æquinoctial*, which is equal to One *Hour*, in *Time*. The first of these doth pass by the *Islands* of *Azores*, in some *Globes*. But by the newest, and best made *English*, *Globes*, at *St. Michael's Island* in the *Azores*.

XIV. The

Fig.
XXV.

XIV. The *Parallels* to the *Æquinoctial*; are *Lesser Circles*, dividing the *Sphere* into Two *unequal* Parts: They are Two *Tropicks* which the Sun describeth, the one in the *Longest Day*, the other in the *Shortest Day*. Their Name *Tropicks*, signifieth a returning, viz. of the Sun. And Two other *Lesser Circles* are described by the *Diurnal Revolution* of the *Poles* of the *Ecliptick*; the one called *Artick*, the other *Antartick*.

XV. The Two *Tropicks*, together with the Two *Polar Circles*, distinguisheth the whole Surface of the *Earth* into Five *Zones*: the *Hot* or *Torrid Zone* is between the Two *Tropicks*, but from the *Tropick of Cancer* to the *Artick Circle* is the *North Temperate Zone*, and from the *Tropick of Capricorn* to the *Antartick*, is the *South Temperate Zone*. What lyeth within the *Artick Circle*, is called the *North Cold* or *Frigid Zone*; and that which lyeth within the *Antartick*, the *South Frigid Zone*.

XVI. The *Inhabitants* of the *Torrid Zone* are called *Amphiscii* because their *Mid-days Shadow* falleth now towards the *South* and then towards the *North*: But the *Inhabitants* of the Two *Temperate Zones* are called *Heteroscii*, because their *Mid-day Shadow* falleth only towards the *North* in our *Hemisphere* and only towards the *South* in the other *Hemisphere*: And lastly, the *Inhabitants* of the Two *Frigid Zones* are called *Perscii*, because their *Shadow* goeth round about them in 24 Hours.

XVII. There are *Parallels* also which distinguish the *Climates* which *Climates* are, as it were, *Little Zones*: They are greatest near the *Æquinoctial*, and from thence they grow smaller and smaller towards both the *Poles*. Their Distance and Largeness is determined by the length of the *Longest Day*; for as often as the *Longest Day* gaineth half an Hour above 12 Hours there is produced a new *Climate*: So the first natural *Climate* ought to have its midst where the *Longest Day* is 12 Hours and a half long: But the Ancients (perhaps knowing nothing of those Parts of the World so near to the *Æquinoctial*) left out this first natural *Climate*, and made their first *Climate*, where indeed the Second should have been, viz. where the *Longest Day* was 13 Hours; and because the midst of this did pass by the

the Island which the *Nilus* maketh, and is called *Meroe*; they named their first *Climate* *Dia Meries*, the second *Dia Syenes*, *Syene* being a Town lying under the Tropick of *Cancer*; the third they called *Dia Alexandria*, this being a Town situated upon the Mouth *Nilus*; the fourth, *Dia Rhodia*; the fifth, *Dia Romes*; the sixth, *Dia Ponta*; the seventh, *Dia Boristhenis*, where the Longest Day was 16 Hours; and here the Ancients left it: But the Modern have continued their *Climates* to the *Artick Circle*, where the Longest Day is 24 Hours, because the Sun doth not set there in the beginning of *Cancer*, but only toucheth the *Horizon* with his *Circle*. And now, there are as many *Climates* on the other Side of the *Æquinoctial*. The *Circle of Perpetual Apparition* is described by a Point touching the *North Cardinal*, being carried about by the daily Motion; and all the Stars that are within this *Circle of Perpetual Apparition* are always seen above our *Horizon*. Another *Circle of Perpetual Occultation* there is described by a Point touching the *South Cardinal*, and being carried about by the daily Motion; so all the Stars that are within this, do never Rise with us.

VIII. The *Secondaries* of the *Ecliptick*, are called *Circles of Latitude*; there are Six of them upon the *Cælestial Globe*, dividing the Signs of the *Ecliptick*, and also the whole *Sphere* into Twelve equal Parts; by which Division all Stars are referred to that Sign which is between the Two next *Circles of Latitude*. Now Six Signs of the *Ecliptick* are always above the *Horizon*, and Six are beneath.

IX. The *Poles* of the *Horizon*, are the *Zenith* and the *Nadir*; the *Secondary Circles* are called *Vertical* or *Aximuth*, or *Circles of Altitude*, amongst which, the chiefeft are, (1.) The *Meridian*. (2.) The *Circle of the 90th Degree of the Ecliptick*, passing by the *Poles* of the *Ecliptick*, the *Zenith*, and the 90th Degree of the *Ecliptick*, being counted from the *Horizon*, either from the *East* or *West*. (3.) The *Vertical*, passing by the *East* and *West Cardinals*; where the Intersection is of the *Æquinoctial* and the *Horizon*; which Sections also, be *Poles* of the *Meridian*: And the *Poles* of this *Vertical* passing by *East* and *West*, are in the *South* and *North Cardinals*, where the *Meridian* doth divide the *Horizon* in the *Ortive* and *Occidental Semicircles*. Now the *Secondaries* of this said *Vertical*, which

Fig.
XXV.

passeth by the true East and West, are the *Circles of Position*, passing by every 30th Degree of the *Æquinoctial*, reckoning from the *Meridian* or *Horizon*, and dividing the whole *Sphere* into *Twelve Houses*. The first *House* is called *Horoscope*, and is that which is next under the *Ortive Horizon*; from thence the other *Houses* do succeed under the *Earth* after the Succession of the *Signs*. Where every *Arch of Position* cutteth the *Ecliptick*, there is the *Cuspides* of the *Houses*. There are besides these *Circles of Position*, an infinite Number passing by every Point of the *Sphere*.

XX. The *Parallels* of the *Horizon* are called *Almacantarath*, and are described upon the *Astrolabe*, to shew the *Altitude* of the *Sun*, or of the *Stars* above the *Horizon*.

XXI. The *Meridian* is the *Original* of *Winds*; there are Four *Cardinals*, and Four *Mean*: The *North* is known by the *Flower-de-Luce*, and the *East* by the *Cross*: The *Mean* do compound their *Names* from the next adjacent *Cardinals*; being *North-East*, *North-West*, *South-East*, *South-West*: Now every one of the *Cardinals* and *Means* hath Two *Laterals*, bearing the same Name with their *Principals*; so they are called *North by East*, *North by West*; *North-East by East*; *North-East by North*, &c. Now these *Laterals* being 16, make, together with the 8 *Principals*, 24; and just in the midst, between every Two *Principals*, there are the Eight *Residual Winds*, bearing the same Name with the *Means* they are next unto; but taking a fore Name from the next *Cardinal*, so they are called *North-North-East*, *East-North-East*, &c.

XXII. These 32 *Winds* being continued upon the Surface of the *Earth*, do make as many *Rumbs*; that *Rumb* which passeth by the *South* or *North*, is always a *Meridian*; and that which passeth by *East* and *West*, is always either the *Æquinoctial*, or a *Parallel* unto it. The other *Rumbs* are crooked *Lines*, neither *Circular* nor *Elliptical*, and are *Seven* in every *Quadrant*, to be numbred both Ways from the *Meridian*. The general Propriety of all *Rumbs* is to cut all the *Meridians* they pass by into equal *Angles*. There may also, besides the said 32 *Rumbs*, pass one by every Point of the *Horizon*.

XXIII. The

Fig.
XXV.

XIII. The Effects of all the foresaid *Circles*, both *Principal* and *Secondary*, are *Angles* and *Arks*. The *Angles* which they make are *Right*, when a *Secondary* meeteth with its own *Principal*, and the other *Angles* are altogether *Oblique*: So the *Angle* which the *Æquinoctial* maketh with the *Ecliptick* is always 23 Deg. 30 Min. as much as the greatest *Declination* of the *Sun*.

XIV. The general Way of *Measuring Angles* upon the *Sphere*, is to set one Foot of a Pair of *Callipers* in the Point where the *Angle* is made, and extend the other 90 Deg. from thence, and so describe an *Ark* between the Two *Legs*, (or *Circles* which make the *Angle*) for as many Degrees as that *Ark* containeth, so many Degrees also is the *Angle*. So the *Measure* of the *Angle* made by the *Æquinoctial* and *Ecliptick* is taken in the *Solstitial Colure*, between the beginning of ♄, and the 90th Degree of the *Æquinoctial*. Likewise the *Angle* which the *Æquinoctial* makes with the *Horizon*, is measured 90 Deg. from thence in the *Meridian*, between the *Horizon* and *Æquinoctial*; and that which is farther in the *Meridian*, from the *Æquinoctial* to the *Zenith*, is the *Latitude* of one *Place*; into which is, always, equal to the *Poles Height*, to be reckoned from the *North Cardinal* to the *Pole*. Sailing freight towards *South* or *North* One Mile, the *Æquinoctial* is raised or fallen One Minute; going 60 Miles, it riseth or falleth One Degree: So at length, coming to the middle Line of the World, the *Æquinoctial* will be raised to the very *Zenith*, both *Poles* lying in the *Horizon*, and all the *Parallels* to the *Æquinoctial* cut the *Horizon* at *Right Angles*; whence this *Position* of the *Sphere* is called *Sphæra Recta*; where all the *Stars* do rise, and abide as long above the *Horizon* as beneath; so there is a perpetual *Æquinoctial* all the Year: But as soon as one of the *Poles* doth rise above the *Horizon*, and the other cometh to be under the *Æquinoctial*, and all its *Parallels* make *Oblique Angles* with the *Horizon*: And for this Reason, such a *Position* of the *Sphere* is called *Sphæra Obliqua*; where, not all the *Stars* do rise, but some are always above, and some always below, the *Horizon*. When the *Pole* cometh to unite with the *Zenith*, then the *Æquinoctial* falleth wholly in the *Horizon*, and the *Parallels* of it, are also parallel to the *Horizon*; and this *Position* of the *Sphere*

Fig.
XXV.

is called *Sphæra Parallela*. The Sun moving in that part of the *Ecliptick*, which is above the *Horizon*, that is on one side the *Æquinoctial*, doth never set, but is turned continually round about, and maketh a Day of Six Months: So likewise, running through the other Six Signs that are under the *Horizon*, it doth never rise, but maketh a Night of Six Months also.

XXV. The *Angle* which any Degree of the *Ecliptick* maketh with the *Right Horizon*, (that is, in *Sphæra Recta*) is equal to the *Angle* which the same Degree of the *Ecliptick*, maketh with the *Meridian*: But whether the *Ecliptick* make such an *Angle* with the *Right* or *Oblique Horizon*, the same *Angle* is always called the *Angle Orient*; that is, of the rising Degree of the *Ecliptick*; and its *Measure* is in the *Circle* of the 90th Degree between the said 90th Degree and the *Horizon*.

XXVI. The *Angle* which the *Meridian* maketh in the *Pole* of the *World* with any *Circle* of *Declination*, taketh its *Measure* in the *Æquinoctial*, between the *Meridian* and the said *Circle* of *Declination*, and this they call, The *Distance of the Star from the Meridian*. So likewise the *Angle* which the *Meridian* maketh with the *Vertical Circle* at the *Zenith*, taketh its *Measure* in the *Horizon*, between the *Meridian* and the said *Vertical*, and this they call, The *Azimuth of the Sun or Star*.

XXVII. The *Arks* to be measured in every *Principal Great Circle*, or its *Secondaries*, have also their proper *Appellations*. So the *Ark* of the *Æquinoctial*, which is comprehended between the beginning of γ , and the *Circle* of *Declination* passing by any *Star*, is called the *Right Ascension* of that *Star*: And the *Ark* which in the said *Circle* of *Declination*, is between the *Æquinoctial* and the *Star*, is called, The *Declination* of that *Star*. But the *Oblique Ascension* of a *Star*, is an *Ark* of the *Æquinoctial*, reckoned from the beginning of γ , to the Point of the *Æquinoctial* Rising with that *Star*: So likewise, the *Oblique Descension* of a *Star* is, the *Ark* of the *Æquinoctial* reckoned from the beginning of γ , to that Point of the *Æquinoctial*, which is setting together with the *Star*. Now the *Difference* that is between the *Right* and *Oblique Ascension* of the *Sun*, or any *Star*, is called, The *Ascensional Difference*. Moreover, the *Ark* of the *Ecliptick*, which is between the beginning of

Direct Position

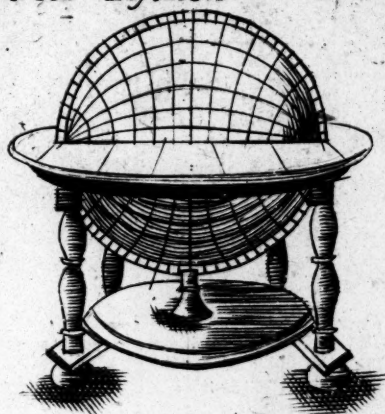
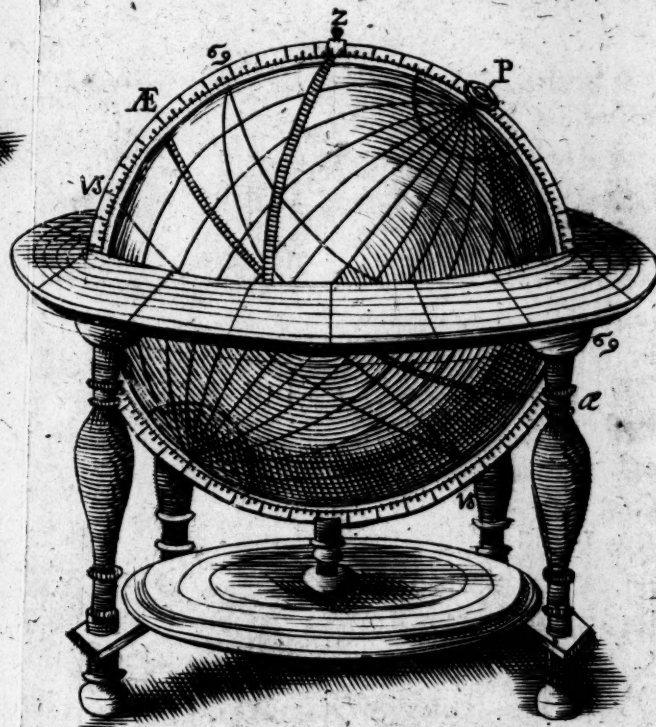
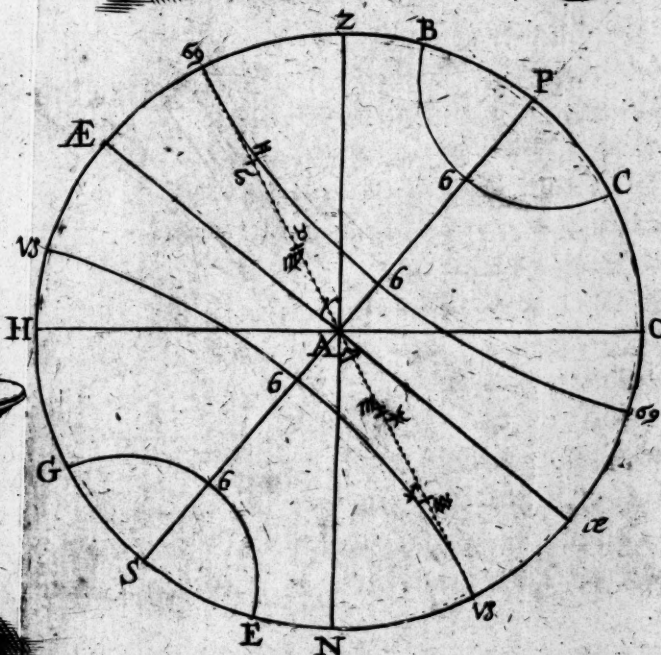
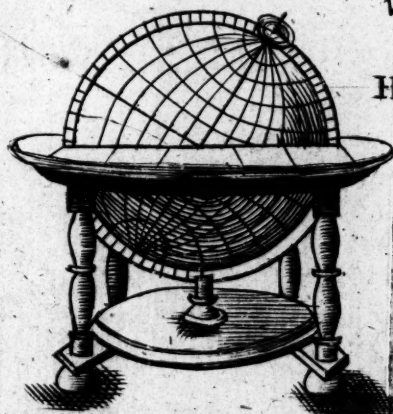


Fig. XXV.

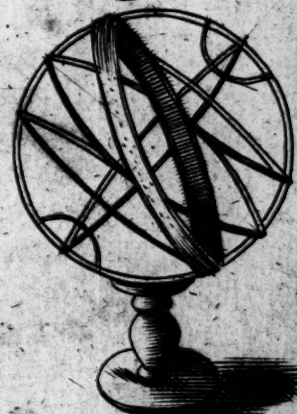
Parallel Position



Oblique Position



A Sphere



and Circle of *Latitude* passing by any *Star*, is called, The *Longitude* of that *Star*: And the *Ark*, which in the said Circle of *Latitude*, is between the *Ecliptick* and the *Star*, is the *Stars Latitude*: And all this is to be understood of the *Cælestial Globe*. But upon the *Terrestrial Globe*, the *Longitude* and *Latitude* of any *Place* are referred to the *Æquinoctial* and *Meridian*: So the *Longitude* of an *Earthly Place* is an *Ark* of the *Æquinoctial*, intercepted between the *First Meridian*, and the *Meridian* passing by the same *Place*. And the *Latitude* of the same *Place* is, an *Ark* of the *Meridian*, to be reckoned from the *Æquinoctial* to the *Place* upon the *Globe*.

Fig.
XXV.

XXVIII. In the *Horizon* we reckon the *Amplitude* of the *Sun* or any *Star*, between the true *East* or *West* Points, and that Point where the *Sun* or *Star* doth Rise or Set: And the said *Amplitude* is either *North* or *South*, according to the Beaming of the *Sun* or *Star*, in respect of the true *East* or *West* Points. The *Altitude* of the *Sun* or a *Star*, is taken in the *Vertical Circle*, passing by the same, between the *Horizon* and the said *Star*: So the *Depression* of the *Star* is, An *Arch* of the *Vertical Circle*, between the *Horizon* and the said *Star*.

ANCILLA MATHEMATICA. VEL, Trigonometria Practica.

SECTION III.

OF GEOGRAPHY.

THE following *Geographical Problems* being first to be performed upon the *Terrestrial Globe*; upon which the *Spherical Triangle*, that resolves any *Question* is discovered, in order to the *Trigonometrical Calculation*: I conceive it necessary, in the first place, to insert this *General*

PROBLEM.

How to Measure the Sides and Angles, of all Spherical Triangles, upon the Convex Superficies of the Globe.

Fig.
XXVI.

THE *Sides* of all *Spherical Triangles* upon the *Globe*, are Measured by the *Degrees* of those *Great Circles*, that make (or constitute) the *Triangle*, contained between the *Two Angular Points*.

1. If the *Side*, or *Sides*, of the *Triangle* to be measured, do consist of such *Great Circles* as are actually divided into *Degrees* upon the *Globe*, or its *Appendants*; as the *Equinoctial*, the *Colures*, the *Ecliptick*, the general *Meridian* or *Horizon*: Then, the number of *Degrees* contained in that *Great Circle*, contained between the *Two Angular Points*, is the Quantity of that *Side* of that *Triangle*, in *Degrees*. But,

2. If

Of Geography.

177

Fig.
XXVI.

2. If the *Side* or *Sides* of the *Triangle* be composed of *Arches* of such *Great Circles* as are not actually divided (as all *Circles* of *Longitude*, and other *Oblique Great Circles*) then, take the Length of such *Side* in a Pair of *Calliper Compasses*, and apply it to any of the forementioned *Great Circles* (as the *Æquinoctial*, &c.) It shall thereupon shew you the Quantity of that *Side* in *Degrees*. — Or, the *Quadrant of Altitude* (but rather, a thin *Plate of Brass* longer than the *Quadrant of Altitude*, divided into *Degrees*, as the *Quadrant* is) applied to the *Side* to be *Measured*, between the *Two Angular Points*, shall give you the Quantity of the *Degrees* of that *Side* of the *Triangle*.

II. For the Angles.

The *Angles* of *Spherical Triangles* are *Measured* upon the *Superficies* of the *Globe*; by counting (or setting off) 90 *Deg.* from the *Angular Point*, of the *Angle* to be *Measured*, upon both the *Sides* which contains the *Angle* to be *Measured*: And at the *Terminations* of those 90 *Deg.* on both the *Sides*, make *Two* small *Marks* upon the *Globe*. Unto these *Two Marks*, apply the *Quadrant of Altitude*, or thin *Plate of Brass*; so the Number of the *Degrees* thereof, contained between the *Two Marks*, is the Quantity of that *Angle*.

Geographical Problems.

P R O B. I.

To find the *Longitude* of any *Place*, described upon the *Terrestrial Globe*.

Longitude is the Distance of a *Place* from the first *Meridian* reckoned in the *Degrees* of the *Equator*, beginning, as was said, in the *New Terrestrial Globe*, (made by Mr. Morden) as St. Michael's Island in the *Azores*.

Practice.] Bring the *Place*, (that is, the *Mark* of the *Place*) suppose *London*, to the *Brazen Meridian*; then count how many *Degrees* of the *Equator* are contained between the first *Meridian*, and that of *London* cut by the *Brazen Meridian*, which you will find to be 28 *Deg.* and that is the *Longitude* required. And in this manner you find *London*

Fig.
XXVI.

		D.	M.
London	} to be distant from the first <i>Meridi- an</i> by these <i>New Globes.</i>	28	0
<i>Ferusalem</i>		66	30
<i>Fedo in Japan</i>		167	0
<i>Rio de la plata</i>		32	0
<i>Mexico</i>		75	0
<i>Charleton Isle</i>		51	30

P R O B. II.

To find the Latitude of any Place.

THE Latitude of a Place, is the Distance of the *Equator* from the Parallel of that Place, reckoned in the Degrees of the *Brass Meridian*; and is either *North* or *South*, according as it lyes between the *North* or *South Poles* of the *Equator*.

To find the *Latitude*, bring the Mark of the Place, suppose *London*, to be the *Brazen Meridian*; then count the Number of Degrees upon the *Meridian*, contained between the *Equator* and the Place ☉. Thus you shall find the *Latitude*, by this new *Globe*, of *London*, to be 51 Deg. 30 Min. and of

	D.	M.		D.	M.
<i>Labor in the Mogul's Country to be</i>	31	30	} By other <i>Globes</i> and <i>Maps.</i>	23	30
<i>The South Part of the Caspian Sea to be</i>	37	0		41	0
<i>Astracan on the North Part of the Caspian Sea to be</i>	46	0		49	0
<i>The North Part of China to be</i>	42	0		52	0
<i>Delli in India to be</i>	28	0		21	0

P R O B. III.

Two Places, which differ only in Latitude, to find their Distance.

IN this there are Two Varieties of Position.

1. If both the *Places* lye under the same *Meridian*, and on one and the same Side of the *Æquinoctial*: *Subtract the Lesser Latitude from the Greater, the Difference (or Remainder) reduced into Miles, (by allowing 60 Deg. to One Mile) shall give you the Distance.*

Exam-

Example. London and Ribadio lye both under the same Meridian, but differ in Latitude: For London hath 51 Deg. 30 Min. of Latitude, at L, and Ribadio hath 34 Deg. of Latitude, at R, both North; the Difference of Latitude is 17 Deg. 30 Min. equal to the Arch LR: And that Reduced into Miles, makes 1050 for their Distance.

2. If the Two Places lye under the same Meridian, but in different Hemispheres, i. e. one on the North, and the other on the South Side of the *Æquinoctial*: Then, Add both the Latitudes together, and the Sum of them is their Distance.

Example. London, and the Island Tristian Dacunbu, lye both under One Meridian, but London hath 51 Deg. 30 Min. of North Latitude, at L, and the Island hath 34 Deg. of South Latitude, at D; the Sum of these Two Latitudes is 85 Deg. 30 Min. equal to the Arch of the Meridian L Æ D; the which reduced into Miles, (by multiplying the Degrees by 60, and allowing for every Minute One Mile) makes 5130 Miles, for their Distance.

P R O B. IV.

Two Places, which differ in Longitude only; To find their Distance.

IN this there are Two Varieties of Position.

1. If the Two Places lye both under the *Æquinoctial*, and have no Latitude; in this Case, Their Difference of Longitude (if it be less than 180 Deg.) is their Distance: But, if the Difference exceed 180 Deg. Subtract it from 360 Deg. and the Remainder is their Distance, in Degrees.

Example. The Island Samatra, and Island St. Thomas, lye both under the *Æquinoctial*: St. Thomas having 22 Deg. 10 Min. of Longitude at T, and the Island Samatra 82 Deg. 10 Min. at S. Now, the Lesser Longitude 22 Deg. 10 Min. subtracted from the Greater 82 Deg. 10 Min. leaves 60 Deg. equal to the Arch S T, for their Difference in Degrees: Which converted into Miles, makes 3600, and so many Miles are the Two Islands distant from each other.

2. But if the Two Places differ only in Longitude, and lye not in the *Æquinoctial*, but under some other intermediate Parallel of Latitude: As Hierusalem at H, and Baldo at B, both in the Parallel of 31 Deg. 40 Min. of North Latitude, but differing in Longitude 60 Deg. 15 Min. equal to the Angle H P B, to find the Distance of these Two Places.

F f

I. By

Fig.
XXVI.

I. By the Globe.

Apply the *Quadrant of Altitude*, or *Brass Plate*, to the Two Places, and the Number of Degrees thereof contained between the Two Places, is their Distance, which will be found to be 50 Deg. 32 Min.

II. By Trigonometrical Calculation.

The *Quadrant of Altitude* (or *Brass Plate*) applied to the Two Places, is represented by the *Arch* H B, and the *Arches* of the Two *Meridians*, which pass through the Two Places, are P B N and P H M; and P B and P H, are equal to the Complement of of the *Latitude* of both the Places, viz. 58 Deg. 20 Min. So that now you have constituted upon the *Globe* an *Oblique Spherical Triangle* P B H; in which you have given, (1.) The Two Sides P B and P H, both equal to 58 Deg. 20 Min. the Complement of the *Latitude*. (2.) The *Angle* B P H 60 Deg. 15 Min. the *Difference* of the *Longitude* of the Two given Places. To find the Side B H, their *Distance*. For which this is

The Canon for Calculation. By CASE I. of R. A. S. T.

As the Radius, Sine 90 Deg.

Is to the Co-sine of the Common *Latitude* (P H or P B) 58 D. 20 M.

So is the Sine of half the *Difference of Longitude*, (half B P H) 30 Deg. 07 $\frac{1}{2}$ Min.

To to Sine of half the *Distance* (half B H) 25 Deg. 16 Min.

The Double whereof, 50 Deg. 32 Min. is the *Distance* B H, which in Miles is 3032 Miles.

P R O B. V.

Two Places, which differ both in *Longitude* and *Latitude*; to find their *Distance*.

IN this there are Three various *Positions*.

1. If one of the Places lye under the *Æquinoctial*, and so have no *Latitude*; and the other under some *Parallel of Latitude* between the *Æquinoctial*, and one of the *Poles*: As *London*, in 51 Deg. 30 Min. of North *Latitude* at L; and *St. Thomas* Island under the *Æquinoctial* at T, but differ in *Longitude* 18 Deg. For finding the *Distance* of these Two Places.

I. Upon

I. Upon the Terrestrial Globe.

Bring *London* to the *Brass Meridian*, and over it, fix the *Quadrant of Altitude*: The *Globe* being in this Position, bring the *Quadrant of Altitude* to lye just over *St. Thomas Island*, and you will find it cut the *Quadrant of Altitude*, in 54 Deg. 45 Min. for the *Distance* of the Two Places.

Fig.
XXVI.

II. By Trigonometrical Calculation.

The *Globe* resting in the former Position, you will find constituted upon it a *Right-angled Spherical Triangle* $L \text{ } \text{Æ} \text{ } T$, composed of, (1.) $L \text{ } \text{Æ}$, an *Arch* of the *Brass Meridian*. (2.) $\text{Æ} \text{ } T$, an *Arch* of the *Æquinoctial*. And, (3.) $L \text{ } T$, an *Arch* of a *Great Circle* (made by the *Quadrant of Altitude*) passing through both the Places: And in this *Triangle*, you have given, (besides the *Right Angle* at Æ) (1.) The *Perpendicular* $L \text{ } \text{Æ}$, the *Latitude* of *London*, 51 Deg. 30 Min. (2.) The *Angle* at L , the *Difference* of *Longitude* 18 Deg. 10 Min. To find the *Hypotenuse* $L \text{ } T$, the *Distance*.

The *Canon* for Calculation. By CASE XIV. of R. A. S. T.

As Tang. of the *Latitude* $L \text{ } \text{Æ}$, 51 Deg. 30 Min.

Is to the *Radius*;

So is the *Co-sine* of $\text{Æ} \text{ } L \text{ } T$, 18 Deg. 10 Min.

To the *Co-tangent* of $L \text{ } T$, 52 Deg. 55 Min.

Which reduced into Miles, makes 3175 Miles, for *Distance* between *London*, and *St. Thomas Island*.

2. If both the Places proposed shall be without the *Æquinoctial*, but both of them, either on the *North* or *South Side* thereof: As *London* in 31 Deg. 30 Min. at L , and *Hierusalem* in 31 Deg. 40 Min. at H , both on the *North Side* of the *Æquinoctial*; and their *Difference* of *Longitude* 46 Deg: To find their *Distance*.

Upon the Terrestrial Globe.

Bring one of the Places, as *London*, to the *Brass Meridian*, and over it screw the *Quadrant of Altitude*, and keep the *Globe* there fixed, then move the *Quadrant of Altitude*, till it lye over *Hierusalem*, and you shall find it to lye under 39 Deg. 32 Min. of the *Quadrant*: And that is the *Distance* of the Two Places.

Fig.
XXVI.

By Trigonometrical Calculation.

The Globe being in the former Position, you will find upon it an *Oblique-angled Spherical Triangle*, composed of P L, an Arch of the *Brass Meridian*: Of P H, an Arch of the *Meridian* that passeth over *Hierusalem*; and of L H, an Arch of the *Quadrant of Altitude*. In which *Triangle* you have given, (1.) P L, the Complement of the *Latitude* of *London*, 38 Deg. 30 Min. (2.) The Side P H (the Complement of the *Latitude* of *Hierusalem* 59 Deg. 20 Min. (3.) The Angle L P H, the *Difference of Longitude* of the Two Places, 46 Deg. To find the Two Angles of *Position* at L and H: And, (2.) The Side L H, the *Distance* of the Two Places.

The Canons for Calculation.

By CASE III. and CASE IX. of A.O. S. T.

	D.	M.
The Side { P L } is {	38	30
{ P H } is {	59	20
Their Sum is	96	50
Their Difference is	19	50
The Half Sum is	48	25
The Half Difference is	9	55

Being thus prepared, I say,

(1.) As the Sine of half the Sum of the Sides P L and P H, 48 Deg. 25 Min.

Is to the Sine of half their *Difference*, 9 Deg. 55 Min.

So is the Co-tangent of half the *Difference of Longitude* (i. e.) half the Angle L P H, 23 Deg.

To the Tangent of 28 Deg. 29 Min.

Which is the half *Difference* of the Angles P L H and P H L.

(2.) As the Co-sine of half the Sum of the Sides P L and P H, 41 Deg. 35 Min.

Is to the Co-sine of half their *Difference*, 80 Deg. 2 Min.

So is the Co-tangent of half the *Difference of Longitude*, i. e. half the Angle L P H 23 Deg.

To the Tangent of 74 Deg. 2 Min.

Which is the Tangent of half the Sum of the Two Angles P L H and P H L.

The

	D.	M.	
The Half Sum is	74	02	
The Half Difference is	28	29	
Their Sum	102	31	equal to $\angle$ P L H.
Their Difference	45	32	equal to $\angle$ P H L.

Fig.
XXVI.

(3.) For the Side L H, which is the *Distance*,
 As the Sine of P H L, 45 Deg. 33 Min.
 to the Sine of P L, 38 Deg. 30 Min.
 So is the Sine of L P H, the Differ. of *Longitude*, 46 Deg.
 to the Sine of L H, 30 Deg. 51 Min.
 Which 38 Deg. 31 Min. reduced in Miles, makes 2331 Miles
 for the *Distance* of the Two Places.

3. If the Two Places proposed should be so situate, that one have
North, and the other *South*, *Latitude*, and under different *Meri-*
ans. As suppose *Constantinople*, lying in *North Latitude* 47 D.
 C. and the *Cape of Good Hope*, lying in 35 Deg. of *South La-*
titude, at V; and differing in *Longitude* 59 Deg. To find the *Di-*
stance of these Two Places

Upon the Terrestrial Glöbe.

Bring one of the Places (as *Constantinople*) to the *Brass Meridi-*
n, and there keep the *Globe*; then apply the *Quadrant of Alti-*
tude (or rather a thin Plate of Brass divided as that is) to the Two
Places, and you shall find 97 Deg. 42 Min. of the *Quadrant* (or
Brass Plate) to be contained between them, and that is their
Distance.

By Trigonometrical Calculation.

The *Globe* resting in its former Position, you will discover up-
 on it an *Oblique-angled Spherical Triangle*, composed of P C, an
 Arch of the *Brass Meridian*: P V, an Arch of a *Meridian*, passing
 through the Place in *South Latitude*: And of C V, the *Quadrant*
 (or *Brass Plate*) representing the Arch of a *Great Circle* passing
 through both the *Places*: —In which *Triangle* you have given,
 (1.) The Side P C, the *Complement* of the *Latitude* of *Constanti-*
nople, 43 Deg. (2.) The Side P V, the *South Latitude* of the
Cape of Good Hope, with 90 Deg. added, which make 125 Deg.
 And, (3.) The Angle V P C, the *Difference* of *Longitude*, 59 Deg.
 To find, (1.) The Angles of *Position* P V C and P C V: And,
 (2.) The third Side V C, the *Distance* of the Two Places.

The

Fig.
XXVI.The Canons for Calculations, as in the last, by CASE III.
IX. of A. O. S. T.(1.) As the Sine of the *Half Sum* of the Sides P V and P C, 84
Is to the Sine of half their *Difference*, 41 Deg.So is the Co-tangent of half the *Difference of Longitude*, i.
(half the Angle V P C) 29 Deg. 30 Min.

To the Tangent of 49 Deg. 23 Min.

Which is the *Half Difference* of the Angles P V C and P C V(2.) As the Co-sine of half the *Sum* of the Sides, P V and P C
6 Deg.Is to the Co-sine of half their *Difference*, 49 Deg.So is the Co-tangent of half the *Difference of Longitude*, 61 Deg.

To the Tangent of 85 Deg. 32 Min.

Which is half the *Sum* of the Two Angles, P C V and P V C

	D.	M.	
The half Sum is	85	32	
The half Difference is	49	23	
Their Sum	134	55	equal to $\angle$ P C V.
Their Difference	36	09	equal to $\angle$ P V C.

The Two Angles of Position.

(3.) For the Side V C, which is the *Distance* of the Two Places.

As the Sine of P V C, 36 Deg. 9 Min.

Is to the Sine of P C, 43 Deg.

So is the Sine of V P C, 59 Deg.

To the Sine of 82 Deg. 18 Min.

Whose Complement to 180 Deg. is 97 Deg. 42 Min. for the

Side V C, which is the *Distance* of the Two Places. Which
in Miles is 5862.And these are all the Varieties of Positions that any Two
Places upon the *Globe* can be situate.

ANCILLA MATHEMATICA.

V E L,

Trigonometria Practica.

SECTION IV.

O F

ASTRONOMY.

ADVERTISEMENT.

W Hereas, for the resolving of the following *Astronomical Problems* by *Trigonometrical Calculation*, it is absolutely necessary, that the true *Place* (or *Longitude*) of the *Sun* in the *Ecliptick* be first known; I have therefore inserted *Astronomical Tables* of the *Sun's Mean Longitude* and *Anomaly* in *Years, Months, Days* and *Hours*; and also of *Æquation*, whereby the *Sun's* true *Place* in the *Ecliptick* may be found at any time. It is also to be noted, That the several *Problems* in this *Section*, being wrought out by the *Cælestial Globe*, in order to their *Trigonometrical Calculation*; I have, therefore, to inform the *Fancy*, and ease the *Memory*, of the *Reader* (in all the *Schemes* relating to them, which consist of *Great Circles* of the *Sphere*, and represent the *Triangle* to the *Eye*, which the *Problem* is to be resolved by) noted the *Circles, Lines* and *Points*, in all the *Schemes*, with the same *Letters* or *Characters*; and prefixed *Them*, and their *Significations*, next before the *Problems*.

Fig.
XXVII.

TABLE

Fig.
XXVII.

TABLE I.

The Sun's Mean Motion in Years.

Longit.							Anomal.						
☉.							☉.						
Years.	S.	D.	M.	S.	D.	M.	Years.	S.	D.	M.	S.	D.	M.
B 1700	9	19	58	6	12	33	1731	9	20	27	6	12	30
1701	9	20	42	6	13	16	B 1732	9	20	12	6	12	14
1702	9	20	29	6	13	1	1733	9	20	57	6	12	58
1703	9	20	14	6	12	46	1734	9	20	43	6	12	43
B 1704	9	20	0	6	12	31	1735	9	20	29	6	12	27
1705	9	20	45	6	13	14	B 1736	9	20	14	6	12	12
1706	9	20	30	6	12	59	1737	9	20	59	6	12	56
1707	9	20	16	6	12	43	1738	9	20	45	6	12	40
B 1708	9	20	2	6	12	28	1739	9	20	30	6	12	25
1709	9	20	46	6	13	12	B 1740	9	20	16	6	12	10
1710	9	20	32	6	12	56	1741	9	21	1	6	12	53
1711	9	20	18	6	12	41	1742	9	20	46	6	12	38
B 1712	9	20	3	6	12	26	1743	9	20	32	6	12	23
1713	9	20	48	6	13	10	B 1744	9	20	18	6	12	7
1714	9	20	24	6	12	54	1745	9	21	2	6	12	51
1715	9	20	30	6	12	39	1746	9	20	48	6	12	36
B 1716	9	20	5	6	12	23	1747	9	20	34	6	12	20
1717	9	20	50	6	13	7	B 1748	9	20	20	6	12	5
1718	9	20	36	6	12	52	1749	9	21	4	6	12	49
1719	9	20	21	6	12	37	1750	9	20	50	6	12	33
B 1720	9	20	7	6	12	21	1751	9	20	36	6	12	18
1721	9	20	52	6	13	5	B 1752	9	20	21	6	12	2
1722	9	20	37	6	12	50	1753	9	21	6	6	12	46
1723	9	20	23	6	12	34	1754	9	20	52	6	12	31
B 1724	9	20	9	6	12	19	1755	9	20	38	6	12	16
1725	9	20	54	6	13	2	B 1756	9	20	23	6	12	0
1726	9	20	39	6	12	47	1757	9	21	8	6	12	44
1727	9	20	25	6	12	32	1758	9	20	54	6	12	29
B 1728	9	20	11	6	12	16	1759	9	20	39	6	12	13
1729	9	20	55	6	13	C	B 1760	9	20	25	6	11	58
1730	9	20	41	6	12	45	1761	9	21	10	6	12	42

TABLE

TABLE II.

The Sun's Mean Motion in Months and Days.

Longit. ☉			Days in Years.		JANUARY.						Days in Years.		FEBRUARY.					
					Longit. ☉			Anom. ☉					Longit. ☉			Anom. ☉		
D.	M.		Com.	By.	S.	D.	M.	S.	D.	M.	Com.	By.	S.	D.	M.	S.	D.	M.
12	30		1	0	0	59	0	0	59		1	0	1	1	32	1	1	32
12	14		2	1	1	58	0	1	58		2	1	1	2	32	1	2	31
12	58		3	2	2	57	0	2	57		3	2	1	3	31	1	3	31
12	43		4	3	3	56	0	3	56		4	3	1	4	30	1	4	30
12	27		5	4	4	56	0	4	56		5	4	1	5	29	1	5	29
12	12		6	5	5	55	0	5	55		6	5	1	6	28	1	6	28
12	56		7	6	6	54	0	6	54		7	6	1	7	27	1	7	27
12	40		8	7	7	53	0	7	53		8	7	1	8	26	1	8	26
12	25		9	8	8	52	0	8	52		9	8	1	9	25	1	9	25
12	10		10	9	9	51	0	0	51		10	9	1	10	25	1	10	24
12	53		11	10	10	50	0	10	50		11	10	1	11	24	1	11	24
12	38		12	11	11	50	0	11	50		12	11	1	12	23	1	12	23
12	23		13	12	12	49	0	12	49		13	12	1	13	22	1	13	22
12	7		14	13	13	48	0	13	47		14	13	1	14	21	1	14	21
12	51		15	14	14	47	0	14	47		15	14	1	15	20	1	15	20
12	36		16	15	15	46	0	15	46		16	15	1	16	19	1	16	19
12	20		17	16	16	45	0	16	45		17	16	1	17	19	1	17	18
12	5		18	17	17	44	0	17	44		18	17	1	18	18	1	18	18
12	49		19	18	18	44	0	18	43		19	18	1	19	17	1	19	17
12	33		20	19	19	43	0	19	43		20	19	1	20	16	1	20	16
12	18		21	20	20	42	0	20	42		21	20	1	21	15	1	21	15
12	2		22	21	21	41	0	21	41		22	21	1	22	14	1	22	14
12	46		23	22	22	40	0	22	40		23	22	1	23	13	1	23	13
12	31		24	23	23	39	0	23	39		24	23	1	24	13	1	24	12
12	16		25	24	24	38	0	24	38		25	24	1	25	12	1	25	12
12	0		26	25	25	38	0	25	37		26	25	1	26	11	1	26	11
12	44		27	26	26	37	0	26	37		27	26	1	27	10	1	27	10
12	29		28	27	27	36	0	27	35		28	27	1	28	9	1	28	9
12	13		29	28	28	35	0	28	35		29	28	1	29	8	1	29	8
11	58		30	29	29	34	0	29	34									
12	42		31	30	1	0	33	1	0	33								

TABLE II.

The Sun's Mean Motion in Months and Days.

March.								April.							
		Longit. ☉			Anom. ☉					Longit. ☉			Anom. ☉		
Com.	Bij.	S.	D.	M.	S.	D.	M.	Com.	Bij.	S.	D.	M.	S.	D.	M.
1	0	1	29	8	1	29	8	1	0	3	29	42	2	29	41
2	1	2	0	7	2	0	7	2	1	3	0	41	3	0	40
3	2	2	1	6	2	1	6	3	2	3	1	40	3	1	40
4	3	2	2	6	2	2	6	4	3	3	2	39	3	2	39
5	4	2	3	5	2	3	5	5	4	3	3	28	3	3	38
6	5	2	4	4	2	4	4	6	5	3	4	37	3	4	37
7	6	2	5	3	2	5	3	7	6	3	5	36	3	5	36
8	7	2	6	2	2	6	2	8	7	3	6	36	3	6	35
9	8	2	7	1	2	7	1	9	8	3	7	35	3	7	34
10	9	2	8	0	2	8	0	10	9	3	8	34	3	8	34
11	10	2	9	0	2	8	59	11	10	3	9	33	3	9	33
12	11	2	9	59	2	9	58	12	11	3	10	32	3	10	33
13	12	2	10	58	2	10	58	13	12	3	11	31	3	11	33
14	13	2	11	57	2	11	57	14	13	3	12	30	3	12	33
15	14	2	12	56	2	12	56	15	14	3	12	20	3	13	23
16	15	2	13	55	2	13	55	16	15	3	14	29	3	14	23
17	16	2	14	54	2	14	54	17	16	3	15	28	3	15	23
18	17	2	15	54	2	15	53	18	17	3	16	27	3	16	23
19	18	2	16	53	2	16	53	19	18	3	17	26	3	17	23
20	19	2	17	52	2	17	52	20	19	3	18	25	3	18	23
21	20	2	18	51	2	18	51	21	20	3	19	24	3	19	23
22	21	2	19	50	2	19	50	22	21	3	20	23	3	20	23
23	22	2	20	49	2	20	49	23	22	3	21	23	3	21	23
24	23	2	21	48	2	21	48	24	23	3	22	22	3	22	23
25	24	2	22	49	2	22	47	25	24	3	23	21	3	23	23
26	25	2	23	47	2	23	46	26	25	3	24	20	3	24	23
27	26	2	24	46	2	24	46	27	26	3	25	19	3	25	19
28	27	2	25	45	2	25	45	28	27	3	26	18	3	26	19
29	28	2	26	44	2	26	44	29	28	3	27	17	3	27	17
30	29	2	27	43	2	27	43	30	29	3	28	17	3	28	16
31	30	2	28	42	2	28	42	31	30	3	29	16	3	29	17

TABL

TABLE II.

The Sun's Mean Motion in Months and Days.

		M A Y.									J U N E.						
		Longit. ☉				Anom. ☉					Longit. ☉				Anom. ☉		
Com.	Bis.	S.	D.	M.	S.	D.	M.		Com.	Bis.	S.	D.	M.	S.	D.	M.	
1	0	3	29	16	3	29	15		1	C	4	29	49	4	29	49	
2	1	4	0	15	4	0	14		2	1	5	0	48	5	0	48	
3	2	4	1	14	4	1	14		3	2	5	1	47	5	1	47	
4	3	4	2	13	4	2	13		4	3	5	2	46	5	2	46	
5	4	4	3	12	4	3	12		5	4	5	3	46	5	3	45	
6	5	4	4	11	4	4	11		6	5	5	4	45	5	4	44	
7	6	4	5	10	4	5	10		7	6	5	5	44	5	5	43	
8	7	4	6	10	4	6	9		8	7	5	6	43	5	6	43	
9	8	4	7	9	4	7	8		9	8	5	7	42	5	7	42	
10	9	4	8	8	4	8	8		10	9	5	8	41	5	8	41	
11	10	4	9	7	4	9	7		11	10	5	9	40	5	9	40	
12	11	4	10	6	4	10	6		12	11	5	10	40	5	10	39	
13	12	4	11	5	4	11	5		13	12	5	11	39	5	11	38	
14	13	4	12	4	4	12	4		14	13	5	12	38	5	12	37	
15	14	4	13	4	4	13	3		15	14	5	13	37	5	13	36	
16	15	4	14	3	4	14	2		16	15	5	14	36	5	14	36	
17	16	4	15	2	4	15	2		17	16	5	15	35	5	15	35	
18	17	4	16	1	4	16	1		18	17	5	16	34	5	16	34	
19	18	4	17	C	4	17	0		19	18	5	17	33	5	17	33	
20	19	4	17	59	4	17	59		20	19	5	18	33	5	18	32	
21	20	4	18	58	4	18	58		21	20	5	19	32	5	19	31	
22	21	4	19	58	4	19	57		22	21	5	20	31	5	20	30	
23	22	4	20	57	4	20	56		23	22	5	21	30	5	21	29	
24	23	4	21	56	4	21	55		24	23	5	22	29	5	22	28	
25	24	4	22	55	4	22	55		25	24	5	23	28	5	23	28	
26	25	4	23	54	4	23	54		26	25	5	24	28	5	24	27	
27	26	4	24	53	4	24	53		27	26	5	25	27	5	25	26	
28	27	4	25	52	4	25	52		28	27	5	26	26	5	26	25	
29	28	4	26	51	4	26	51		29	28	5	27	25	5	27	24	
30	29	4	27	51	4	27	50		30	29	5	28	24	5	28	24	
31	30	4	28	50	4	28	49		31	30	5	29	23	5	29	23	

TABLE II.

The Sun's Mean Motion in Months and Days.

JULY.								AUGUST.							
		Longit. ☉			Anom. ☉					Longit. ☉			Anom. ☉		
Com.	Bif.	S.	D.	M.	S.	D.	M.	Com.	Bif.	S.	D.	M.	S.	D.	M.
1	C	5	29	23	5	29	23	1	C	6	29	56	6	29	56
2	1	6	0	22	6	0	22	2	1	7	0	56	7	0	55
3	2	6	1	21	6	1	21	3	2	7	1	55	7	1	54
4	3	6	2	21	6	2	20	4	3	7	2	54	7	2	53
5	4	6	2	20	6	3	19	5	4	7	3	53	7	3	52
6	5	6	4	19	6	4	18	6	5	7	4	52	7	4	52
7	6	6	5	18	6	5	17	7	6	7	5	51	7	5	51
8	7	6	6	17	6	6	17	8	7	7	6	50	7	6	50
9	8	6	7	18	6	7	16	9	8	7	7	50	7	7	49
10	9	6	8	16	6	8	15	10	9	7	8	49	7	8	48
11	10	6	9	15	6	9	14	11	10	7	9	48	7	9	47
12	11	6	10	14	6	10	13	12	11	7	10	47	7	10	46
13	12	6	11	13	6	11	12	13	12	7	11	46	7	11	45
14	13	6	12	12	6	12	11	14	13	7	12	45	7	12	45
15	14	6	12	11	6	13	11	15	14	7	13	44	7	13	44
16	15	6	14	10	6	14	10	16	15	7	14	44	7	14	43
17	16	6	15	9	6	15	9	17	16	7	15	43	7	15	42
18	17	6	16	9	6	16	8	18	17	7	16	42	7	16	41
19	18	6	17	8	6	17	7	19	18	7	17	41	7	17	40
20	19	6	18	7	6	18	6	20	19	7	18	40	7	18	39
21	20	6	19	6	6	19	5	21	20	7	19	39	7	19	39
22	21	6	20	5	6	20	5	22	21	7	20	38	7	20	38
23	22	6	21	4	6	21	4	23	22	7	21	38	7	21	37
24	23	6	22	3	6	22	3	24	23	7	22	37	7	22	36
25	24	6	23	3	6	23	2	25	24	7	23	36	7	23	35
26	25	6	24	2	6	24	1	26	25	7	24	35	7	24	34
27	26	6	25	1	6	25	0	27	26	7	25	34	7	25	33
28	27	6	26	C	6	25	59	28	27	7	26	33	7	26	33
29	28	6	26	59	6	26	59	29	28	7	27	32	7	27	32
30	29	6	27	58	6	27	58	30	29	7	28	31	7	28	31
31	30	6	28	57	6	28	57	31	30	7	29	31	7	29	30

TABLE

TABLE II.

The Sun's Mean Motion in Months and Days.

T.		SEPTEMBER.							OCTOBER.								
nom. ☉		Longit. ☉			Anom. ☉						Longit. ☉			Anom. ☉			
D.	M.	Com.	Bif.	S.	D.	M.	S.	D.	M.	Com.	Bif.	S.	D.	M.	S.	D.	M.
29	56	1	C	3	0	30	8	0	25	1	0	9	0	4	9	0	3
0	55	2	1	3	1	29	8	1	28	2	1	9	1	5	9	1	2
1	54	3	2	8	2	28	8	2	28	3	2	9	2	2	9	2	2
2	53	4	3	8	3	27	8	3	27	4	3	9	3	1	9	3	1
3	52	5	4	8	4	26	8	4	26	5	4	9	4	1	9	4	0
4	52	6	5	8	5	25	8	5	25	6	5	9	5	0	9	4	59
5	51	7	6	8	6	25	8	6	24	7	6	9	5	5	9	5	58
6	50	8	7	8	7	24	8	7	23	8	7	9	6	5	9	6	57
7	49	9	8	8	8	23	8	8	22	9	8	9	7	5	9	7	56
8	48	10	9	8	9	22	8	9	21	10	9	9	8	5	9	8	55
9	47	11	10	8	10	21	8	10	21	11	10	9	9	5	9	9	54
10	46	12	11	8	11	20	8	11	20	12	11	9	10	5	9	10	54
11	45	13	12	8	12	19	8	12	19	13	12	9	11	5	9	11	53
12	45	14	13	8	13	19	8	13	18	14	13	9	12	5	9	12	52
13	44	15	14	8	14	18	8	14	17	15	14	9	13	5	9	13	51
14	43	16	15	8	15	17	8	15	16	16	15	9	14	5	9	14	50
15	42	17	16	8	16	16	8	16	15	17	16	9	15	5	9	15	49
16	41	18	17	8	17	15	8	17	15	18	17	9	16	4	9	16	48
17	40	19	18	8	18	14	8	18	14	19	18	9	17	4	9	17	48
18	39	20	19	8	19	13	8	19	13	20	19	9	18	4	9	18	47
19	39	21	20	8	20	13	8	20	12	21	20	9	19	4	9	19	46
20	38	22	21	8	21	12	8	21	11	22	21	9	20	4	9	20	45
21	37	23	22	8	22	11	8	22	10	23	22	9	21	4	9	21	44
22	36	24	23	8	23	10	8	23	9	24	23	9	22	4	9	22	43
23	35	25	24	8	24	9	8	24	8	25	24	9	23	4	9	23	42
24	34	26	25	8	25	8	8	25	8	26	25	9	24	4	9	24	42
25	33	27	26	8	26	7	8	26	7	27	26	9	25	4	9	25	41
26	33	28	27	8	27	7	8	27	6	28	27	9	26	4	9	26	40
27	32	29	28	8	28	6	8	28	5	29	28	9	27	4	9	27	39
28	31	30	29	8	29	5	8	29	4	30	29	9	28	3	9	28	38
29	30	31	30	9	0	4	9	0	3	31	30	9	29	3	9	29	37

TABLE

TABLE II.

The Sun's Mean Motion in Months and Days.

NOVEMBER.							DECEMBER.								
Longit. ☉			Anom. ☉				Longit. ☉			Anom. ☉					
Co.	Bi.	S.	D.	M.	S.	D.	M.	Co.	Bi.	S.	D.	M.	S.	D.	M.
1	0	10	0	37	10	0	36	1	0	11	0	11	11	0	10
2	1	10	1	36	10	1	35	2	1	11	1	11	11	1	10
3	2	10	2	36	10	2	35	3	2	11	2	10	11	2	9
4	3	10	3	35	10	3	34	4	3	11	3	9	11	3	8
5	4	10	4	34	10	4	33	5	4	11	4	8	11	4	7
6	5	10	5	33	10	5	32	6	5	11	5	7	11	5	6
7	6	10	6	32	10	6	31	7	6	11	6	6	11	6	5
8	7	10	7	31	10	7	30	8	7	11	7	5	11	7	4
9	8	10	8	30	10	8	29	9	8	11	8	5	11	8	4
10	9	10	9	30	10	9	29	10	9	11	9	4	11	9	3
11	10	10	10	29	10	10	28	11	10	11	10	3	11	10	2
12	11	10	11	28	10	11	27	12	11	11	11	2	11	11	1
13	12	10	12	27	10	12	26	13	12	11	12	1	11	12	0
14	13	10	13	26	10	13	25	14	13	11	13	0	11	12	59
15	14	10	14	25	10	14	24	15	14	11	14	0	11	13	58
16	15	10	15	24	10	15	23	16	15	11	14	58	11	14	57
17	16	10	16	23	10	16	23	17	16	11	15	58	11	15	57
18	17	10	17	23	10	17	22	18	17	11	16	57	11	16	56
19	18	10	18	22	10	18	21	19	18	11	17	56	11	17	55
20	19	10	19	21	10	19	20	20	19	11	18	55	11	18	54
21	20	10	20	20	10	20	19	21	20	11	19	54	11	19	53
22	21	10	21	19	10	21	18	22	21	11	20	53	11	20	52
23	22	10	22	18	10	22	17	23	22	11	21	52	11	21	51
24	23	10	23	17	10	23	16	24	23	11	22	52	11	22	51
25	24	10	24	17	10	24	16	25	24	11	23	51	11	23	50
26	25	10	25	16	10	25	15	26	25	11	24	50	11	24	49
27	26	10	26	15	10	26	14	27	26	11	25	49	11	25	48
28	27	10	27	14	10	27	13	28	27	11	26	48	11	26	47
29	28	10	28	13	10	28	12	29	28	11	27	47	11	27	46
30	29	10	29	12	10	29	11	30	29	11	28	46	11	28	45
31	30	11	0	11	11	0	10	31	30	11	29	46	11	29	45

TABLE III.

The Sun's Mean Motion in Hours.

Hours.	Longitude.		Aphe- lion.
	Mi.	Mi.	
1	2	2	
2	5	5	
3	7	7	
4	10	10	
5	12	12	
6	15	15	
7	17	17	
8	20	20	
9	22	22	
10	25	25	
11	27	27	
12	30	30	
13	32	32	
14	34	34	
15	37	37	
16	39	39	
17	42	42	
18	44	44	
19	47	47	
20	49	49	
21	52	52	
22	54	54	
23	57	57	
24	59	59	

TABLE

TABLE IV.

A Table of the Sun's Equation.

Sign 0.	Sign 1.	Sign 2.	Sign 3.	Sign 4.	Sign 5.
Æ. Subj.	Æ. Subj.	Æ. Subj.	Æ. Subj.	Æ. Subj.	Æ. Subj.
D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
00	00	58	41	57	43
10	21	01	42	57	42
20	41	11	43	57	41
30	61	31	44	57	40
40	81	51	45	57	39
50	101	61	46	57	38
60	121	81	47	57	36
70	141	101	47	57	35
80	161	111	48	57	34
90	181	131	49	56	33
100	201	151	50	56	32
110	221	161	50	56	30
120	241	181	51	56	29
130	261	191	52	55	27
140	281	211	52	55	26
150	301	221	53	54	25
160	321	231	54	54	23
170	341	251	54	53	22
180	361	261	54	52	20
190	381	281	55	52	19
200	391	291	55	51	17
210	411	301	56	51	15
220	431	321	56	50	14
230	451	331	56	49	12
240	471	341	57	48	10
250	491	351	57	48	9
260	511	361	57	47	7
270	521	381	57	46	5
280	541	391	57	45	4
290	561	401	57	44	3
300	581	411	57	43	2
Æ. Add.	Æ. Add.	Æ. Add.	Æ. Add.	Æ. Add.	Æ. Add.
11 Sign.	10 Sign.	9 Sign.	8 Sign.	7 Sign.	6 Sign.

The

The Use of the Tables.

TO find the Sun's true Place in the *Ecliptick* at any time, by these *Tables*, this is the

R U L E.

From the several *Tables*, collect the *Years*, *Month*, and *Day* of the *Month*, and also the odd *Hours*, if any be, and set them down, one under another, with the respective *Longitudes* and *Anomalies* answering thereto: Then add the several *Longitudes* and *Anomalies* together, and with the *Sum* of the *Anomalies*, enter (*Table IV.*) of *Equations*, which *Equation*, *Add to*, or *Substrait from*, the *Sum* of the *Longitudes*, and the *Sum* or *Remainder*, shall be the Sun's true Place in the *Ecliptick* for the time proposed.

How Time is to be computed.

1. Any *Day* begins upon its own *Noon*; so that the First *Day* of *January*, at 12 at *Noon*, is the common *Term* of the *Old* and *New Years*.

Example.

Let the true Place of the *Sun* in the *Ecliptick* be required, for the 16th *Day* of *May*, in the *Year* of our Lord 1703. at 5 in the *Afternoon*.

First, Set down the *Year* 1703. then the *Month* and *Day*, *May* 16; and lastly, the *Hours*, 5, as is here due.

Secondly, Look for 1703. (in *Table I.*) against which stands 9 S. 10 D. 14 M. for the ☉ *Longitude*; and 6 S. 12 D. 46 M. for the ☉ *Anomaly*; both which set down as here you see.

Thirdly, Look for *May* 16. (in *Table II.*) against which stands 4 S. 14 D. 3 M. for the *Long.* And

4 S. 12 D. 2 M. for the *Anomaly*; both which set under the former, as you see.

Fourthly, Look for 5 *Hours* (in *Table III.*) against which stands 12 *Minutes*, both for the *Longit.* and *Anomal.* both set down, as you see.

Fifthly,

		Long. ☉			Anom. ☉		
		S.	D.	M.	S.	D.	M.
Year	1703.	9	20	14	06	12	46
May, Day	19.	4	14	03	04	12	02
Hour	5			12			12
☉ Mean Lon.		1	04	29	10	25	00
Equat. Add.			01	06			
☉ true Place.		2	05	35	II Gemini.		

Fifthly, Add all the *Longitudes* together, and they make 14 Sig. 04 Deg. 29 Min. (from which abate 12 Signs, and there remains only Two Signs.) —Also, Add the *Anomalies* together, and they make 10 Sig. 25 Deg. 00 Min.

Sixthly, (in Table IV.) Look for 10 Signs at the Bottom of the Table, and 25 Deg. in the last Column towards the *Right Hand*, so against it, over 10 Signs, you shall find 1 Deg. 6 Min. to be Added. Set them under the *Mean Longitude*, and add them to it, so will the *Sum* be 2 Sig. 05 Deg. 35 Min. And that is the true Place of the Sun in the *Ecliptick*, which is 35 Deg. distant from the *Æquinoctial Point* γ *Aries*. Note, that

Signs	0	1	2	3	4	5	6	7	8	9	10	11
Is	γ	δ	ϵ	ζ	η	θ	ι	κ	λ	μ	ν	ξ

Other Examples.

	Longit. $\odot$	Anom. $\odot$	In this Example, it
	S. D. M.	S. D. M.	being Leap-Year, I take
Year 1720.	9 20 07	6 12 21	the Day of the Month
Febr. Day 29	1 28 09	1 28 09	out of the Column that
Hours 6	15	15	hath Bissex. at the
Mean Motion	11 18 31	8 10 45	Head of it.
Æquat. Add.	1 52		
$\odot$ true Place.	11 20 23	κ Pisces.	

	Longit. $\odot$	Anom. $\odot$	Note also, That if
	S. D. M.	S. D. M.	the Sum of the Sun's
Year 1713	9 20 48	6 13 10	Anomalies be less than
April 23	3 21 23	3 21 22	Six Signs, they will
At Noon	0 00 00	0 00 00	be found at the Head
Mean Motion	1 12 11	10 04 32	of the Equation Ta-
Æqua. Add.	0 1 35		ble, and the Degrees
$\odot$ true Place.	1 13 46	8 Taurus.	in the First Column to-

	Longit. $\odot$	Anom. $\odot$	(always) be Substra-
	S. D. M.	S. D. M.	ed from the Mean Lon-
Year 1761	9 21 10	6 12 42	gitude; whereas in all
Jan. Day 12	11 50	11 50	these Examples it hath
Hours 16	39	39	been Added.
Mean Motion	10 3 39	6 25 11	
Æquat. Add.	51		
$\odot$ true Place.	10 4 30	$\equiv$	

H h

In

In all the Problems in this Section, it is to be understood, That
(in all the Spherical Schemes, or Figures following.

P			Pole of the World.
Æ Q			Æquinoctial Circle, or Æquator.
P P			Parallels (or smaller Circles) of Declination of the Sun, or of a Star.
F			Pole of the Ecliptick.
E C			Ecliptick.
F F			Parallels (or lesser Circles) of Altitude of the Sun, or of a Star, or the Latitude of a Place or Country.
γ			Vernal
♌			Autumnal
A			Ascension
D			Descension
Z			Zenith.
N			Nadir.
H O			Horizon.
Z N			Prime Vertical Circle, or Azimuth of East and West.
O r			East
O c			West
R			Right Angle.
☉			Sun.
*			Star.
S			Side, Sun (and sometimes) Star.
b			North
m			South
R. A. P. T.	Right-angled	} Plain Triangles.	
O. A. P. T.	Oblique-angled		
R. A. S. T.	Right-angled	} Plain Triangles.	
O. A. S. T.	Oblique-angled		

Astronomical Problems.

PROBLEM I.

The Longitude, or Place of the Sun, in the Ecliptick, being given; To find,

1. The Sun's Right Ascension.
2. The Sun's Declination.
3. The Angle of Position made by the Intersection of the Meridian and the Ecliptick.

IN this Problem there is given the Sun's or a Star's Place, in Respect of the Ecliptick; and his Place, in respect of the *Æquinoctial*, is required,

I. By the Cœlestial Globe.

Example. Let the Place of the Sun be in 29 Deg. of *Taurus* δ , (that is, 59 Deg. from the beginning of *Aries* γ , which is the nearest *Æquinoctial* Point.)

The Globe being in any Position, (for in this Problem there is no regard to be had to the Latitude) count the Sun's Place in the Ecliptick upon the Ecliptick Circle, from γ ; and bring that Point to the Graduated Side of the Brass Meridian. Then, (1.) Will the Brass Meridian cut the *Æquinoctial* Circle in 56 d. 46 m. counted also from γ ; and that is the Right Ascension. And, (2.) The Number of Degrees of the Brass Meridian, comprehended between the *Æquinoctial* and Ecliptick Circles, will be 20 d. which is the Sun's Declination. —And, (3.) The Angle made by the Intersection of the Brass Meridian, and the Ecliptick Circle in the Point of the Sun's Place, will be 77 d. 23 m. which is the Angle of Position, in respect of the Meridian and Ecliptick.

II. By Trigonometrical Calculation.

The Globe being in this Position, there is represented upon the Superficies thereof, Two Right-angled Spherical Triangles, such as are expressed in the Diagrams; and are there noted with $\odot R \gamma$ and $\odot R \delta$; in which, the Sides $\odot \gamma$ and $\odot \delta$ are Arches of the Ecliptick Circle, and is the Sun's Longitude: XXVII.

H h 2

The

Fig.

XXVII.

XXVIII.

Fig. XXVII. The Sides γ R and $\simeq$ R are Arches of the *Æquinoctial*, and is the Sun's *Right Ascension*; and the Sides $\odot$ R are Arches of the *Brass Meridian*, and is the Sun's *Declination*. — Also, the Angle at γ or $\simeq$, is the Angle of the greatest *Obliquity* of the *Ecliptick*, (and is equal to the Sun's greatest *Declination* 23 d. 31 m.) The Angle R is a *Right Angle*, and the Angle $\odot$ is the Angle at the Sun's *Position*, in respect of the *Ecliptick* and *Meridian Circles*.

To resolve this *Problem Trigonometrically*, you are to consider the *Quadrant* of the *Ecliptick*, in which the Sun is, which in the *Figures* are signified by one of these Numbers, 1. 2. 3. 4. Of which, the first is of the *Spring*, from the beginning of *Aries* γ , to the beginning of *Cancer* $\simeq$, &c. Then in the Triangle R γ $\odot$ or R $\simeq$ $\odot$.

There is given, $\left\{ \begin{array}{l} 1. \text{ The Angle at } \gamma \text{ or } \simeq, 23 \text{ d. } 31 \text{ m.} \\ 2. \text{ R } \gamma \text{ or R } \simeq: \text{ The Sun's Distance from the} \\ \text{besides the Right Angle at R, next } \textit{Æquinoctial Point}; \text{ to be numbred} \\ \text{from } \gamma \text{ or } \simeq, \text{ unto the Degrees of the Sun's} \\ \text{Longitude or Place in the } \textit{Ecliptick} \text{ given.} \end{array} \right.$

And,

$\left\{ \begin{array}{l} 1. \text{ R } \odot, \text{ The Sun's } \textit{Declination} \text{ for that Sign which he is} \\ \text{in; whether } \textit{North} \text{ or } \textit{South}. \\ 2. \text{ The Arch } \gamma \text{ R, or } \simeq \text{ R, which if it be in} \\ \text{Quadrant, } \left\{ \begin{array}{l} 1. \text{ The Degrees found by the} \\ \text{Canon, are the Degrees of} \\ 2. \text{ The Degrees found, must be} \\ \text{subtracted from } 180, \text{ and the} \\ \text{remained are the Degrees of} \\ 3. \text{ The Degrees found must be} \\ \text{added to } 180, \text{ and the Sum} \\ \text{are the Degrees of} \\ 4. \text{ The Degrees found must be} \\ \text{subtracted from } 360 \text{ d. and the} \\ \text{Remainder are the Degrees of} \end{array} \right. \text{ } \textit{Right Ascension}. \\ 3. \text{ The Angle } \odot, \text{ or the Angle of } \textit{Position}, \text{ in respect of} \\ \text{the } \textit{Meridian} \text{ and } \textit{Ecliptick Circles}. \end{array} \right.$

The Canons for Calculation.

The Sun being in 29 d. of *Taurus* δ , which is 59 d. from *Aries* γ .

For the *Right Ascension* γ or α R. By *Case III.* of R. A. S. T. the *Sine* of 90 d.

Is to the *Co-sine* of the greatest *Obliquity* of the *Ecliptick* 66 d. 29 m.

Is the *Tangent* of the *Sun's Distance* from γ or α , 59 d.

To the *Tangent* of 56 d. 46 m. Which is the *Sun's Right Ascension*; because his *Place* given was in the *First Quadrant*: —But if the *Sun* had been in 1 d. of *Leo* Ω , or 29 d. of *Scorpio* ϖ , or 1 d. of *Aquarius* ϱ . All which *Points* are 59 d. distant from γ or α ; the *Right Ascension* would be found the same, as before, viz. 56 d. 46 m. But by the *Rule* before given in this *Problem*.

D. M.

the <i>Sun's</i> { 1 d. of <i>Leo</i> , in Quad. 2. }	The <i>R. Ascension</i> { 123 14
<i>Place</i> given { 29 d. of <i>Scorpio</i> , in Qu. 3. }	<i>sion</i> would { 236 46
had been in { 1 d. of <i>Aquarius</i> , in Qu. 4. }	have been { 303 15

2. For the *Sun's Declination* R $\odot$. By *Case II.* of R. A. S. T.

Is the *Sine* of 90 d.

Is to the *Sine* of the greatest *Obliquity* of the *Ecliptick* 23 d. 31 m.

Is the *Sine* of the *Sun's Distance* from γ or α 59 d.

To the *Sine* of 20 d. the *Sun's present Declination*.

Which is *North*, because he is in a *Northern Sign*.

3. For the *Angle* of the *Sun's Position* $\odot$. By *Case III.* of R. A. S. T.

Is the *Sine* of the *Sun's present Declination* 20 d.

Is to the *Sine* of greatest *Declination* 23 d. 31 m.

Is the *Sine* of the *Sun's R. Ascension* 56 d. 46 m.

To the *Sine* of 77 d. 23 m. The *Sun's Angle of Position*, made by the *Meridian* and *Ecliptick Circles*.

P R O B. II.

The *Right Ascension*, or *Declination* of the *Sun* given; To find his *Longitude* (or *Place*) in the *Ecliptick*.

THIS is the *Converse* of the foregoing *Problem*; for in this, the *Sun's Place*, in respect of the *Equinoctial Circle*, is given: And his *Place*, in respect of the *Ecliptick Circle*, is required: And may be resolved as followeth.

I. By

I. By the Cœlestial Globe.

Fig.
XXVII.
XXVIII.

Example. Let the Sun's *Right Ascension* be 303 d. 14 m. and his *Declination* 20 d. Southward: And let his *Longitude* (or *Place* in the *Ecliptick*) be required.

Count 303 d. 14 m. the *Right Ascension* given, upon the *Equinoctial Circle*, from the *Vernal Equinoctial Point Aries* and bring that Point to the graduated Side of the *Brass Meridian*: Then will the *Brass Meridian* cut the *Ecliptick Circle* in 1 d. of *Aquarius* ♒, and in that Sign and Degree the Sun is at Noon, when his *Right Ascension* is 303 d. 14 m.

But to find his *Place* by his *Declination*, Count 20 d. (the *Declination* given) upon the *Brass Meridian*, downwards towards the South Pole (because the *Declination* given was *Southerly*) and turn the *Body* of the *Globe* about till 20 d. of the *Meridian* do cut the *Ecliptick Circle*; which it will do in Two Points, one in 29 d. of *Scorpio* ♏, in the third *Quadrant*; and the other in 1 d. of *Aquarius* ♒, in the fourth *Quadrant*; in both which Points the Sun being, he hath 20 d. of *South Declination* and which of those Points you seek is determined by the Degrees of *Right Ascension* given, as here 303 d. 14 m. and the *Globe* being in this Position, will sign out the same Triangle mentioned in the foregoing *Problem*.

II. By Trigonometrical Calculation.

Consider the *Quadrant* of the *Ecliptick* in which the Sun is (which the Degrees of *Right Ascension* given, will determine) and in this Example will be the fourth *Quadrant*: But the given *Declination* is indifferent in all the Four *Quadrants*.

Wherefore, in the Triangle $R \gamma \odot$.

There is given,
besides the
Right Angle
at R .

1. The Angle at γ or $\sphericalangle$, 23 d. 31 m. the Sun's greatest *Declination*.
2. The Side $R \odot$, the Sun's present *Declination* 20 d. And also either of the other Sides $R \gamma$ (or $R \sphericalangle$) an Ark of the *Equator*, to be numbered from the nearest *Equinoctial Point*, to the Degrees of *Right Ascension* given.

The Side $\odot \gamma$ (or $\odot \equiv$) the Sun's Distance in the *Ecliptick Circle*; from the nearest *Æquinoctial Point*, from which Point the *Longitude* required is to be numbred upon the *Ecliptick Circle*. Fig. XXVII. XXVIII.

So the *Right Ascension* 303 d. 14 m. given, it being in the *Fourth Quadrant*, will give in the *Ecliptick* 1 d. of *Aquarius* ♊ .

The Canons for Calculation.

1. By the *Right Ascension* given.

The *Right Ascension* given being 303 d. 14 m. are found in the *Fourth Quadrant*, and therefore must be subtracted from 360 d. and the Remainder will be 56 d. 46 m. which must be made use of in the Calculation, instead of 303 d. 14 m. and then in the Triangle $\triangle \odot R$,

1. By the *Right Ascension* given.

Is the Tangent of the Side $\triangle R$, 56 d. 46 m. the *Right Ascension*,

Is to the Sine of 90 d. or Radius:

As is the Co-sine of the Angle at $\triangle$ 66 d. 29 m.

To the Co-tangent of $\triangle \odot$, 59 d. 00 m. Sun's *Longitude*.

Now, because the *Right Ascension* given was in the *Fourth Quadrant*, the *Longitude* 56 d. 46 m. thus found, must be in the *Fourth Quadrant* also, which will be in 1 d. of *Aquarius* ♊ .

2. By the *Declination* given.

Is the Sine of the Sun's greatest *Declination*, 23 d. 31 m. the Angle at $\triangle$.

Is to the Sine of the Side $\odot R$, 20 d. the Sun's present *Declination* given:

As is the Radius, Sine 90 d.

To the Sine of the Side $\triangle \odot$, 59 d. from *Libra* ♎ .

And being in the *Fourth Quadrant*, it gives the *Longitude* of the Sun to be in 1 d. of *Aquarius* ♊ , as before.

P R O B.

P R O B. III.

Fig.
XXIX.

The Latitude of the Place, 51 d. 30 m. and the Sun's Place the Ecliptick, 29 d. of Taurus, given, to find,

1. The Sun's Declination.
2. The Sun's Amplitude.
3. The Hour from Midnight or Ascensional Difference.
4. The Time of the Sun's Rising and Setting.
5. The Length of the Day and Night.
6. The Angle of the Sun's Position at the time of his Rising or Setting.

I. By the Cœlestial Globe.

BRing the 29 d. of Taurus to the Brass Meridian, and you will find 29 of Taurus to lye under 20 d. of the Meridian, and that is the Sun's Declination North, the Sun being a Northern Sign. Then, set the Index of the Hour-Circle to South 12, and turn the Body of the Globe Eastward, till 29 of Taurus do just touch the Horizon, and there fix it. Then shall you find 29 d. of Taurus to cut the Horizon in 33 18 m. counted from the East, which is the Amplitude of the Sun's Rising from the true East Point of the Horizon Northward: And also, the Index of the Hour-Wheel, will point 11 m. after 4 in the Morning, at which time the Sun Rises. — And if you turn the Body of the Globe about Westward till 29 d. of Taurus doth touch the West Side of the Horizon, then shall the Index point at 49 m. after 7 at Night, at which Time the Sun Setteth.

Again, Turn the Globe about, till 29 d. of Taurus touch the East Side of the Horizon (as before) and set the Index of the Hour-Circle to the North (or undermost) 12: And then turning it Westward till 29 d. of Taurus touch the Horizon the West Side; and then shall the Index point at 3 ho. 38 m. more than 12 Hours, from the South 12: Which shews the Day is then 15 ho. and 38 m. Long. — And if you count the Hours between the North 12 and the Index, you shall find them to be 8 ho. and 22 m. and that is the Length of the Night.

And then, for the Angle of the Sun's Position, that is to be found, as is directed in the 24th Element, and in the Problem to this Second Part, and will be found to be 56 d. 23 m.

The Globe being in this Position, there is represented upon the Superficies of it a Spherical Triangle $P \odot O$, as in Fig. XXIX. compounded of the Arches of such *Great Circles* of the *Sphere*, as are ingredient in the *Proposition*, viz. Of $P O$, an Arch of the *Brass Meridian*; of $\odot O$, an Arch of the *Horizon*; and of $P \odot$, an Arch of that *Meridian*, which cutteth the *Ecliptick* in that Point in which the Sun is. — And in this Triangle, the Angle $P O \odot$ is a *Right Angle*: $\odot P O$, is the *Hour* from *Midnight*: And the Angle $P \odot O$, is the Angle of *Position* made by the *Meridian* and *Horizon*, at the time of the *Sun's Rising*.

II. By Trigonometrical Calculation.

In the Triangle $P O \odot$, you have given, (besides the *Right Angle* at O). (1.) The Hypotenuse $P \odot$, the Complement of the *Sun's Declination*, (70 d.) (2.) The Perpendicular $P O$, the *Latitude* 51 d. 30 m. — To find, (1.) The Base $\odot O$, the *Sun's Amplitude* at his *Rising*. (2.) The Angle $\odot P O$, the *Hour* from *Midnight*: And, (3.) The Angle $P \odot O$; the Angle of the *Sun's Position* at his *Rising*.

The Canons for Calculation.

1. For the *Amplitude*, $\odot O$. By *Case VI.* of R. A. S. T.
Is the Co-sine of $P O$, (the *Latitude*) 38 d. 30 m.
Is to the Radius, 90 d.
As the Co-sine of $P \odot$, (the *Sun's Declination*) 20 d.
To the Co-sine of $\odot O$, (the *Amplitude*) 33 d. 20 m.
Which is equal to the Arch of the *Horizon*. Or $\odot$, which is the *Amplitude* of his *Rising* from the *East* Northward, because the Sun was in a *Northern Sign*: For if he had been in a *Southern Sign*, as in 59 d. of *Scorpio*, the *Amplitude* would have been the same, but from the *East Southward*.
2. For the *Hour* from *Midnight* $\odot P O$. By *Case V.* of R. A. S. T.
Is the Radius,
Is to the Tangent of $P O$ (the *Latitude*) 51 d. 30 m.
As the Co-tangent of $P \odot$ (the *Sun's Distance* from the Pole) 20 d.
To the Co-sine of $\odot P O$ (the *Hour from Six*) 27 d. 14 m.
Whose Complement, 62 d. 46 m. is the *Hour from Midnight*; which turned into *Time*, (by allowing 15 d. for 1 h. and 4 d. for 1 m. of *Time*) makes 4 h. and 11 m. Which is the *Hour from Midnight*.

Fig.
XXIX.

3. For the *Ascensional Difference*.

The Hours from *Midnight*, 4 h. 11 min. subtracted from 6 h. there remains 1 h. 49 m. which is the *Ascensional Difference*. Or, The Time that the *Sun Rises* before *Six* in the *Summer*, or after *Six* in the *Winter*.

4. and 5. For the Time of the *Sun's Rising* and *Setting*, and (consequently) the *Length* of the *Day* and *Night*.

The *Ascensional Difference*, 1 h. 49 m. being added to 6 h. gives 7 h. 49 m. for the *Semidiurnal Ark* in *Summer*; or subtracted from 6 h. gives 4 h. 11 m. for the *Semidiurnal Ark* in *Winter*.

The *Semidiurnal Ark*, 7 h. 49 m. subtracted from 12 h. there remains 4 h. 11 m. for the time of the *Sun's Rising*: And then subtracted from 12 h. leaves 7 h. 49 m. for the time of the *Sun's Setting*. And,

The *Semidiurnal Ark*, 7 h. 49 m. doubled, gives 15 h. 38 m. for the *Length* of the *Day*: —And 15 h. 38 m. subtracted from 24 h. there will remain 8 h. 22 m. for the *Length* of the *Night*.

6. For the *Angle* $P \odot O$, of the *Sun's Position*, at the time of his *Rising*. By *Case IV.* of *R. A. S. T.*

As the *Sine* of $P \odot$ (the *Co-Declination*) 70 d.

Is to the *Radius*:

So is the *Sine* of PO (the *Latitude*) 51 d. 30 m.

To the *Sine* of $P \odot O$ (the *Angle* of *Position* at the *Sun's Rising*) 56 d. 23 m.

P R O B. IV.

To find the *Sun's Meridian Altitude*, and his *Depression* at *Midnight*, he being in any *Point* of the *Ecliptick*.

LET the Place of the *Sun* be the same, as in the former *Problem*, viz. in 29 d. of *Taurus*. Then,

I. By the *Cœlestial Globe*.

Turn the *Globe* about, till the 29 d. of *Taurus* be just under the *Meridian*; then shall you find the Number of Degrees of the *Meridian*, which are comprehended between that *Point* and the *Horizon* to be 58 d. 30 m. which is the *Meridian Altitude*. And if you bring the 29 d. of *Scorpio*, which is the opposite *Point*

Astronomical Problems.

205

Fig.
XXIX.

the *Ecliptick* in which the *Sun* is, to the *Meridian*, the Number of the Degrees of the *Meridian* between that Point and the *Horizon* will be found to be 18 d. 30 m. which is the *Sun's* Depression at *Midnight*.

In like manner,

D. M.		D. M.		D. M.
16 00	Ω	54 36		22 24
25 00	m	19 25		57 25
29 00	≡	26 38		50 22
13 00	Υ	43 39		33 21

P R O B. V.

To know when *Twilight* Begins and Ends.

TWILIGHT begins, when the *Sun* is 18 d. below the *Horizon* before its *Rising*: And endeth, when the *Sun* cometh to be 18 d. below the *Horizon*, after its *Setting*.

The *Globe Rectified*, and the *Sun* in 29 d. of *Taurus*, find the opposite Point thereunto, which is the 29 d. of *Scorpio*; and bring that Point, as also the *Quadrant of Altitude*, both of them on the West-side of the *Meridian*; and then move both the Body of the *Globe*, and the *Quadrant of Altitude* also, till the 29 d. of *Scorpio* lye directly under 18 d. of the *Quadrant of Altitude*: Which done, keep them both together, and then see how many Hours the *Index* is removed from 12, which you shall find to be 1 h. and 8 m. So that *Twilight* begins at 8 m. after 1 in the Morning. And this being taken from 4 h. 11 m. the time of the *Sun's Rising* that Day, there will remain 3 h. 3 m. which is the Length or Continuance of the *Twilight*. Also if you double the time of the beginning of *Twilight* 1 h. 8 m. you shall have the length of dark Night, which will be but 2 h. 16 m.

In like manner, if you would know when the *Twilight* endeth after *Sun-setting*, you must bring the 29 d. of *Scorpio* (the Point opposite to the *Sun*) on the East-side of the *Meridian*, making it, and 18 d. of the *Quadrant of Altitude*, to meet, then the *Index* will shew 10 h. 52 m. and till that time of Night doth *Twilight* continue.

Fig.
XXIX.

And so,

	D.		H. M.		H. M.
The	29 35	Day-break will be at	0 10	Midn. aft.	11 50
Sun in	8 4		5 52		6 8
	2 24		4 6		7 54
	0 8		2 41		9 19
					Night at

And if you go about to find the time of the beginning and end of *Twilight*, all the Time that *Sun* is passing from 2 d. of *Gemini* to 30 d. of *Cancer*, which is from about the 12th of *May* to the 12th of *July*, you shall find that there will be no *Twilight* at all, but all that Time continual Day: For all that Space of Time the *Sun* never descendeth so much as 18 d. under the *Horizon*, in the Latitude of 51 d. 30 m.

The former of these Two last *Problems* needeth no *Trigonometrical Calculation*: But, if to the *Sun's Declination* for the Day (or *Place of the Sun*) given, 20 d. you add the Complement of the *Latitude* given, 38 d. 30 m. the *Sum* of them 58 d. 30 m. is the *Meridian Altitude* for that Day that the *Sun* is in 58 d. of *Taurus*.

But if the *Sun* had been in 29 d. of *Scorpio*, a Southern Sign, then the *Declination* 20 d. subtracted from 38 d. 30 m. the Complement of the *Latitude*, the Remainder 18 d. 30 m. will be the *Meridian Altitude*.

For the *Depression* of the *Sun*, he being upon the *Meridian* at *Midnight*, it is the same with the *Meridian Altitude*, when the *Sun* is in the opposite Sign.

So the *Depression* at *Midnight* when the *Sun* is in
 29 d. of *Taurus*, } is { 18 d. 30 m.
 29 d. of *Scorpio*, } is { 58 d. 30 m.

P R O B.

P R O B. VI.

The Latitude of Place, and the Sun's Place in the Ecliptick (and consequently his Declination) being given; To find

1. What Altitude the Sun shall have at Six a Clock.
2. What Azimuth he will be upon, at Six a Clock, Morning or Evening.
3. The Angle of the Sun's Position, made by the Intersection of the Vertical Circle, and the Meridian which passes by the Sun at the time of the Question.

ALL these Problems are only in use, when Sun is in a Northern Sign, and so hath North Declination: For the Sun is never above the Horizon (in either of the Temperate Zones) when he is in a Southern Sign, or hath South Declination. .

Fig. XXIX.

I. By the Cœlestial Globe.

1. For the Altitude at Six. Bring the 29 d. of *Taurus* to the Meridian, and set the Index of the Hour-Circle to 12; then turn the Globe Eastward, till the Index of the Hour-Circle come just to Six a Clock: Then holding the Globe there, lay the Quadrant of Altitude just over the 29 d. of *Taurus*, and there you shall find it to cut 15 d. 30 m. of the Quadrant. And such Altitude shall the Sun have at Six a Clock in the Morning, and the same at Six at Night.

And so,

	D. M.		D. M.
When the Sun is in	{ 16 00 Ω }	The Sun's Altitude	{ 12 32
	{ 13 00 γ }	at Six will be	{ 4 2
		found to be	

2. For the Azimuth at Six. Bring the 29 of *Taurus* to the Meridian, and set the Index of the Hour-Wheel to 12; then move the Globe till the Index lye upon Six; and holding the Globe there, lay the Quadrant of Altitude just over 29 d. *Taurus*: Then shall you find, that there are 77 d. 14 m. of the Horizon contained between the Intersection of the North Part of the Meridian, and the Quadrant of Altitude, which is the Azimuth from the North; or 12 d. 46 m. from the East, which is the Azimuth from the East, or 102 d. 46 m. from the South, which is the Azimuth therefrom.

In

Ancilla Mathematica.

In like manner,

	D. M.		D. M.	D. M.	D. M.
Sun in	16 0 0	The Sun's Azimuth at 6 will be found to be from the	North	East or West	South
			79 49	10 11	100 11
	13 0 0		86 47	3 13	93 13

II. By Trigonometrical Calculation.

Fig.
XXX.

The *Globe* being in this Position, you may discover upon the Superficies thereof a *Spherical Triangle*, such as in the *Figure*, is noted with $Z P \odot$; in which, $Z P$ is an Arch of the *Meridian*, $P \odot$ an Arch of the *Hour-Circle* of Six, and $Z \odot$ an Arch of an *Azimuth*, or *Vertical Circle*: In which *Triangle* there is given, (besides the *Right Angle* at P). (1.) The Perpendicular $Z P$, 38 d. 30 m. equal to the Complement of the *Latitude*. (2.) The Base $\odot P$, 70 d. equal to the Complement of the Sun's *Declination*: To find, (1.) The Hypotenuse $Z \odot$, the Complement of the Sun's *Altitude* at Six. (2.) The Angle $\odot Z P$, the Sun's *Azimuth* at Six a Clock: (3.) $Z \odot P$, the *Angle* of the Sun's Position.

The Canons for Calculation.

1. For the Hypotenuse $Z \odot$, (the Complement of the Sun's *Altitude* at Six) by *Case XIV.* of R. A. S. T.

As the Radius,

Is to the Co-sine of $Z P$, 51 d. 30 m.So is the Co-sine of $\odot P$, 20 d.To the Sine of 15 d. 32 m. (the Complement of $Z \odot$.)And such *Altitude* the Sun will have, at Six a Clock, (Morning or Evening) when he is in 29 d. of *Taurus*.

2. For the Angle $P Z \odot$, the Sun's *Azimuth* at Six a Clock, by *Case XIII.* of R. A. S. T.

As the Tangent of $\odot P$ (the Co-*Declination*) 70 d.

Is to the Radius:

So is the Sine of $Z P$ (the Co-*Latitude*) 38 d. 30 m.

To the Tangent of 12 d. 46 m.

Whose Complement 77 d. 14 m. is the Angle $\odot Z P$, the Sun's *Azimuth* from the North Part of the *Meridian*.

3. For the *Angle* of the Sun's Position, $Z \odot P$: By Case XI. Fig. XXVII.
of R. A. S. T.

As the Sine of $\odot P$, 70 d.

Is to the Sine of $\odot Z P$, 77 d. 14 m.

So is the Sine of $Z P$, 31 d. 30 m.

To the Sine of $Z \odot P$, 40 d. 15 m.

Which is the *Angle* of the Sun's Position.

P R O B. VII.

Having the Latitude, 51 d. 30 m. the Sun's Place, 59 of Taurus, (or Declination 20 d.) Given, as in the foregoing Problem; To find,

1. At what Hour the Sun shall be upon the East or West Azimuth.
2. What Altitude the Sun shall have when he is upon the East or West Azimuth.
3. The Angle of the Sun's Position.

I. By the Coelestial Globe.

Bring the 29 d. of Taurus to the Meridian, and the Index to 12 of the Clock. Also bring the beginning of the Degrees of the Quadrant of Altitude to the East Point of the Horizon, and turn the Globe about till the 29 d. of Taurus do touch the Degrees of the Quadrant of Altitude; then shall the Index point at 7 m. past Seven; at which time, in the Morning, will the Sun be exactly upon the East Azimuth, or Point of the Compass. And if you carry the Quadrant of Altitude to the West Point of the Horizon, and turn the Globe about till 29 d. of Taurus touch the edge of Degrees thereof, the Hour-Index will point at Four of the Clock, and 53 m. at which time, in the Afternoon, will the Sun be upon the West Azimuth or Point of the Compass.

In the same manner,

	D. M.		D. M.		D. M.
When the	16 00 Ω	It will	6 57	West at	5 03
Sun is in	13 00 γ		6 17		5 43
		be due			
		East at			

Then for the Sun's Altitude when he will be upon the East or West Azimuth.

Bring 29 d. of Taurus to the Meridian, and the Quadrant of Altitude to the East or West Points of the Horizon: Then turn the Globe

Fig. XXX. *Globe* about, till the 29 d. of *Taurus* touch the *Quadrant of Altitude*, and you shall find it to touch at 25 d. 55 m. of the *Quadrant of Altitude*; and such *Altitude* hath the Sun, when he is upon the *East* or *West Azimuth*.

In like manner,

	D.	M.			D.	M.
When the	16	00	Ω	its <i>Altitude</i> , when	20	19
Sun is in	13	00	γ	<i>East</i> or <i>West</i> , will		
				be found,	6	36

II. By Trigonometrical Calculation.

Fig. XXXI.

The *Globe* being in any of the former *Positions*, you will find represented upon the Superficies of it a *Spherical Triangle*, Z P ⊙, (such as in the *Figure*) *Right-angled* at Z, which is composed of Z P, an Arch of the *Brass Meridian*, equal to the *Complement* of the *Latitude* 38 d. 30 m. Z ⊙, an Arch of the *Quadrant of Altitude*: And of P S, an Arch of that *Meridian* (or *Hour-Circle*) which cutteth the *Ecliptick* in the Place the Sun is in. — And in this *Triangle* there is given (besides the *Right Angle* at Z, (1.) The *Perpendicular* P Z, the *Complement* of the *Latitude* 38 d. 30 m. (2.) The *Hypotenuse* P ⊙, the *Complement* of the Sun's *Declination* 70 d. To find, (1.) The *Angle* Z P ⊙, the *Hour* at which the Sun will be upon the *East* or *West Azimuth*, (2.) The *Side* Z ⊙, the *Complement* of the *Altitude* he shall then have: And, (3.) The *Angle* Z ⊙ P of his *Position*.

The Canons for Calculation.

1. For the *Angle* Z P ⊙, (the *Hour* from *Noon*, that the Sun will be due *East* or *West*; by *Case* V. of R. A. S. T.

As the *Radius*,

Is to the *Tangent* of Z P, 31 d. 30 m.

So is the *Co tangent* of P ⊙, 20 d.

To the *Co-sine* of Z P ⊙, 73 d. 10 m.

Which converted into *Time*, is 4 h. 53 m. Whose *Complement* is 7 h. 7 m. at which *Time* the Sun will be due *East* or *West*.

2. For

Astronomical Problems.

211

Fig.
XXXI.

2. For the Base $Z \odot$, the Complement of the Sun's *Altitude*, when due *East* or *West*: By *Case VI.* of R. A. S. T.

As the Co-sine of $Z P$, (the *Co-Latitude*) 51 d. 30 m.

To the Radius:

So is the Co-sine of $P \odot$ (the *Declination*) 20 d.

To the Co-sine of $Z \odot$ (the *Altitude*) 25 d. 55 m.

And that is the Sun's *Altitude*.

3. For the Angle $Z \odot P$, of Position: By *Case IV.*

As the Sine of $P \odot$, 70 d.

To the Radius:

So Sine $Z P$, 38 d. 30 m.

To the Sine of $Z \odot P$, 41 d. 29 m.

Which is the Angle of Position.

P R O B. VIII.

The *Latitude of the Place*, the *Sun's Place in the Ecliptick* (or his *Declination*) being given, to find,

1. *What Altitude the Sun shall have at any time of the Day.*

2. *What Azimuth he shall then have. And,*

3. *The Sun's Angle of Position.*

Let the *Latitude* given be 51 d. 30 m. North, the *Sun's Place in the Ecliptick* 29 d. of *Taurus*, (and so his *Declination* 20 d.)

And let the *Hour* be 9 in the Morning, or 3 in the Afternoon.

I. By the *Cœlestial Globe*.

Bring the 29 d. of *Taurus* to the *Meridian*, and set the *Hour-Index* to 12 a Clock. Then turn about the *Globe*, till the *Hour-Index* point to the given *Hour*, (suppose 9 in the Morning, or 3 in the Afternoon:). There keep the *Globe*; and laying the *Quadrant of Altitude* over the 29 d. of *Taurus*, you shall find 43 d. cut thereby; and such *Altitude* shall the Sun have at 9 in the Morning, or 3 in the Afternoon. And at the same time you will find the *Quadrant of Altitude* to cut the *Horizon* in 24 d. 37 m. counted from the *East* or *West* Points thereof; and such *Azimuth* will the Sun have at 9 a Clock in the Morning, or 3 in the Afternoon, when he is in 29 d. of *Taurus*. And by this *Problem* the Sun's *Altitudes* in any Sign or degree of the *Ecliptick* at all Hours may be found, as in these following *Synopsis* or *Tables*.

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Fig.
XXXI.

A Table, shewing what Altitude the Sun shall have at every Hour of the Day, in the beginning of every of the Twelve Signs of the Zodiack.

So the Sun being in the beginning of		Cancer or Leo.		Gemi. or Virgo.		Aries or Libra.		Scorp. or Pisces.		Aqua. or Sagit.		Capric. or Sagit.	
At the H.		D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
XII.		62	00	58	42	50	00	38	30	27	11	18	15
XI.	I.	59	43	56	34	48	12	36	58	25	40	17	6
X.	II.	53	45	50	55	43	12	32	37	21	51	13	38
IX.	III.	44	42	43	3	36	0	26	7	15	58	8	12
VIII.	VI.	35	41	34	13	27	31	18	8	8	33	1	15
VII.	V.	27	17	24	56	18	18	9	17	0	6		
VI.		18	11	15	40	9	0						
V.	VII.	9	32	6	50								
IV.	VIII.	1	32										

His Altitude will be

Its Altitude on these Deg. of Azi.

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A Table, shewing what Altitude the Sun shall have, he being upon every Tenth Azimuth from the South, in the beginning of every of the Twelve Signs. Fig. XXXI.

The Sun in the beginning of		Cancer.	Leo or Gemi.	Virgo or Taur.	Libra or Aries.	Scorp. or Pisces.	Srgitt. or Aqua.	Capric.
d. Az.		D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
Its Altitude on these Deg. of Azimuth from the South, viz.	0	62 0	58 42	50 0	38 30	27 0	28 18	15 0
	10	61 43	58 24	49 38	38 4	26 30	17 45	14 25
	20	60 51	57 28	48 33	36 46	25 10	16 5	12 41
	30	59 52	55 52	46 40	34 34	22 27	13 15	9 45
	40	57 20	53 29	43 51	31 21	18 48	9 14	5 34
	50	54 3	50 12	40 11	27 5	13 58	3 57	0 6
	60	49 56	45 53	35 23	21 41	8 0		
	70	44 40	40 25	29 27	15 13	1 0		
	80	28 11	33 46	21 29	7 52			
	90	30 38	26 10	14 25				
	100	22 27	18 2	6 45				
	110	14 14	9 58					
	120	6 34	2 30					

These Tables are of good use for the making of *Cylinders*, *Quadrants*, and other Instruments, that give the *Hour* and *Azimuth* by the height of the Sun, and also to insert the *Tropicks* and other Signs of the *Zodiacks* and *Azimuths* into Sun-Dials, &c.

II. By Trigonometrical Calculation.

The *Globe* continuing in the same Position you left it, you may discover upon it an *Oblique Spherical Triangle* Z P ☉, (as in the Figure) constituted by the Intersections of Z P, an Arch of the *Brass Meridian*, Z ☉, a Part of the *Quadrant* of *Altitude*: And of P ☉, an Arch of a *Meridian*, passing through the 29 d. of *Taurus* in the *Ecliptick*, or *Hour-Circle* of 9 or 3 a Clock. Fig. XXXII.

In which *Triangle* there is given,

1. The Side Z P, (the Complement of the *Latitude*) 38 d. 30 m.

K k 2

2. The

Fig.
XXXII.

2. The Side $P \odot$ (the Complement of the Sun's *Declination*, or his Distance from the Pole) 70 d.
3. The Angle $Z P \odot$, (the Hour-Circle of 9 or 3 a Clock) 45 d.

To find,

1. The Side $Z \odot$ (the Complement of the Sun's *Altitude*.)
2. The Angle $\odot Z P$, (the Sun's *Azimuth* from the North Part of the *Meridian*.)
3. The Angle of Position, $Z \odot P$.

The Canons for Calculation.

By Case III. and Case IX. of O. A. S. T.

(1.) As the Sine of half the *Sum* of the Two given Sides, $Z P$, 38 d. 30 m. and $P \odot$ 70, viz. 54 d. 15 m. Is to the Sine of half the *Difference* of those Sides, viz. 15 d. 54 m.

So is the Co-tangent of half the given Angle, $Z P \odot$, viz. 22 d. 30 m.

To the Tangent of half the *Difference* of the unknown Angles, at Z and $\odot$, viz. 38 d. 48 m.

Then,

(2.) As the Co-sine of half the *Sum* of the given Sides $Z P$ and $P \odot$, viz. 35 d. 45 m.

Is to the Co-sine of half the *Difference* of those Sides, viz. 74 d. 15 m.

So is the Co-tangent of half the given Angle $Z P \odot$, viz. 22 d. 30 m.

To the Tangent of half the *Sum* of the Two unknown Angles $P Z \odot$ and $Z \odot P$, viz. 75 d. 49 m.

Then,

The half Sum before found is

The half Difference before found is

The Sum of them is

The Difference of them is

The one equal to the greater Angle $\odot Z P$ 114.37, which is the Sun's *Azimuth* from the North Part of the *Meridian*: From the South Part 65 d. 23 m. and from the East or West 24 d. 37 m.

The other, equal to the lesser Angle $Z \odot P$ 37 d. 1 m. which is the Angle of the Sun's Position.

PROB

P R O B. XI.

The Latitude of the Place (51 d. 30 m.) the Sun's Place in the Ecliptick (29 d. of Taurus; or 29 d. of North Declination) and the Sun's Altitude (12 d.) being given; To find,

1. *The Sun's Azimuth, from the East, West, North or South Points of the Horizon:*
2. *The Hour of the Day, from Noon or Midnight.*
3. *The Angle of the Sun's Position.*

I. *By the Cœlestial Globe.*

1. *For the Sun's Azimuth.*

THE *Globe* being Rectified, &c. and the *Quadrant of Altitude* fixed, and brought to the *Horizon*, turn 29 d. of *Taurus* toward the *East*, if in the Morning; or towards the *West*, if in the Evening, till it come to lye just under 12 d. of the *Quadrant of Altitude*; and then note at what Degree in the *Horizon* the *Quadrant of Altitude* resteth; which will be at 17 d. 8 m. from the *East*, if in the Morning, or 17 d. 8 m. from the *West*, if in the Afternoon, which is the *Azimuth* from the *East* or *West* towards the *North*. And this *Azimuth*, if reckoned by the Points of the Compass upon the *Horizon*, will be E. by N. 5 d. 35 m. Northward, if in the Morning; or W. by N. 5 d. 53 m. Northward, if in the Evening, when the *Sun* is in 29 d. of *Taurus*, and hath 12 d. of *Altitude*. Now if you count the Degrees of the *Horizon* between the *Quadrant of Altitude*, and the *North Part* of the *Meridian*, you shall find them to be 72 d. 52 m. which is the *Azimuth* from the *North*: And if you count them from the *South Part* of the *Meridian*, you shall find them to be 107 d. 8 m. which is the *Azimuth* from the *South*.

In like manner,

	D.	M.
The Latitude being	51	30
The Sun's Place	1	00 <i>Aquarius.</i>
The Sun's Altitude	12	00

Then will the *Azimuth* be found to be 56 d. from the *East* or *West*, towards the *South*; which (by the Points of the Compass upon the *Horizon*) will appear to be S. E. by S. if in the Morning, or S. W. by S. if in the Evening.

2. *For*

Fig.
XXXII.

2. For the Hour of the Day.

Bring 29 d. of *Taurus* to the *Meridian*, and set the *Index* to 12 a Clock: Then, if it be in the Forenoon, set the *Quadrant of Altitude* on the *East Side* of the *Meridian*; but on the *West Side*, if it be in the Afternoon. And turn the *Globe* about, till the 29 d. of *Taurus* meet with 12 d. of the *Quadrant of Altitude*; and then shall the *Index* of the *Hour-Circle* point at a Clock, and 36 m. if it be in the Morning; or at 24 m. after 6 of the Clock, if it be at Night. And that is the true Hour of the Day.

In like manner,

	D.	M.
The <i>Latitude</i> being	51	30
The <i>Sun's Place</i>	1	00 <i>Taurus</i> .
The <i>Altitude</i>	36	00

Then will the Hour of the Day be found to be either 9 in the Morning, or 3 in the Afternoon. And which of these Hours is, may best be known by a second Observation of the *Altitude*. For if the *Altitude* do increase, it is the Forenoon; but if decrease, it is the Afternoon.

Again,

	D.	M.
The <i>Latitude</i> being	52	30
The <i>Sun's Place</i> in the beginning of <i>Taurus</i> .		
The <i>Sun's Altitude</i>	25	56

The Hour of the Day would be found to be either 8 m. past 4 in the Afternoon. Or if it were in the Forenoon, 52 m. after 8 in the Morning.

3. For the Angle of the Sun's Position.

This may be found by the 26th Element, and as is taught in the *Præme* before this *Second Part*: And in this Example will be found to be 39 d. 16 m.

II. By Trigonometrical Calculation.

Fig.
XXXIII.

The *Globe* resting in the same *Position* you left it, when you wrought these *Problems* upon it, you will discover upon it an *Oblique-angled Spherical Triangle*, (such as in the *Figure*, is now with *Z* $\odot$ *P*: Which *Triangle* consists of, (1.) *Z P*, an Arch of the *Brass Meridian*, equal to the Complement of the given *Latitude*, 38 d. 30 m. (2.) Of *Z* $\odot$, an Arch of the *Quadrant of Altitude*.

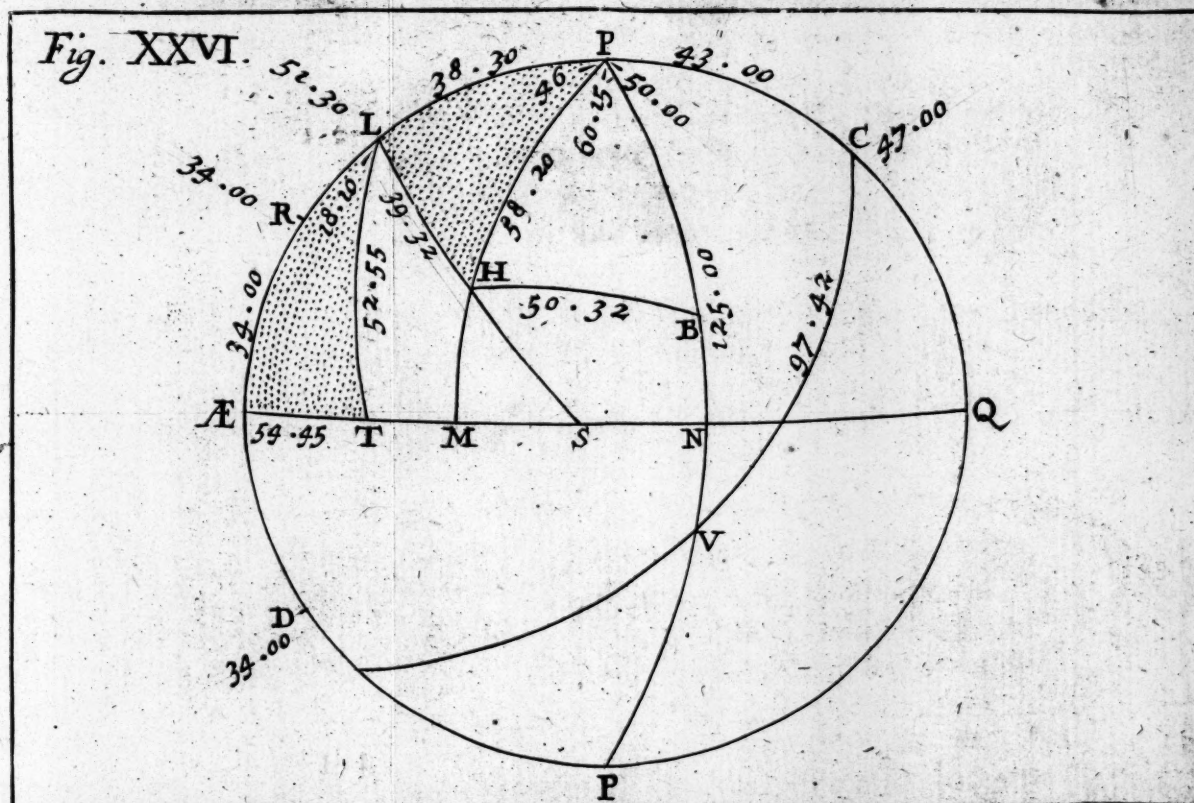


Fig. XXVII.

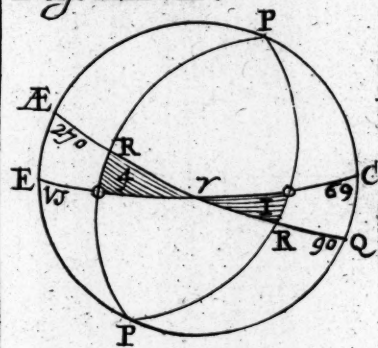


Fig. XXVIII.

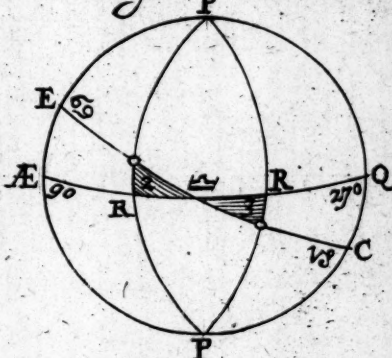


Fig. XXIX.

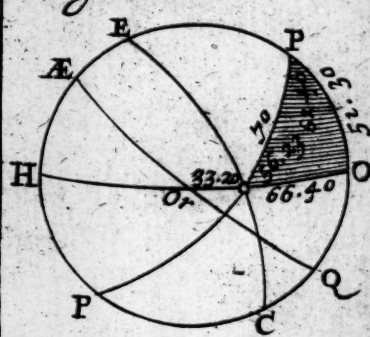


Fig. XXX.

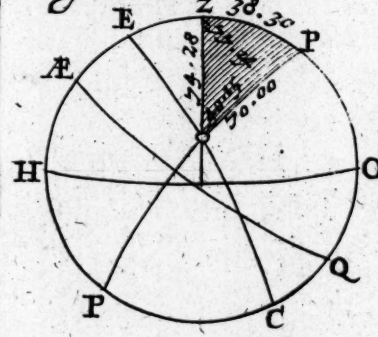


Fig. XXXI.

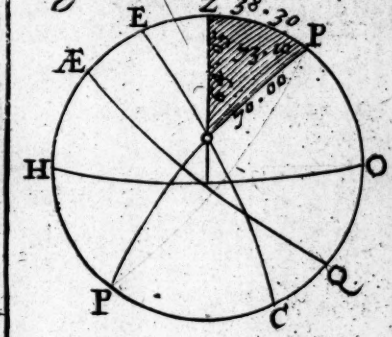
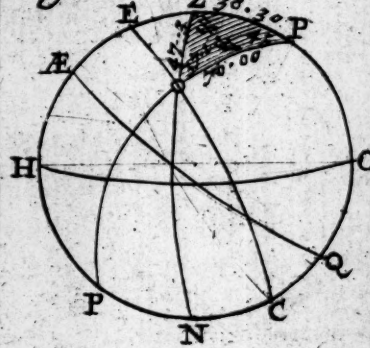


Fig. XXXII.



Altitude, crossing the *Ecliptick* in 29 d. of *Taurus*, where the Sun is 12 d. high, and is equal to the *Complement* of the Sun's *Altitude* 78 d. (3.) An Arch of a *Meridian* (or *Hour-Circle*) P $\odot$, passing through 29 d. of *Taurus*, and crossing the *Quadrant* of *Altitude* in 12 d. thereof, and is equal to the *Complement* of the Sun's *Declination* (or his Distance from the Pole) 70 d. So that the *Three Sides* of this *Triangle* are *Given*, and the *Three Angles* are *Required*.

The Canons for Calculation.

1. For the Sun's *Azimuth*, the Angle $\odot$ Z P. By Case IX. of O. A. S. T.

	D.	M.
The Side $\odot$ P, the <i>Co-declination</i> , is	70	00
The Side $\odot$ Z, the <i>Co-altitude</i> , is	78	00
The Side Z P, the <i>Co-latitude</i> , is	38	30
Their <i>Sum</i> is	186	30
Their <i>half Sum</i> is	93	15
The <i>Difference</i> between the <i>half Sum</i> , and 70 d. $\odot$ P	23	15

Being thus prepared, the Proportions are,

- (1.) As the *Radius*, Sine 90 d.
Is to the Sine of Z P (one of the *Sides* containing the enquired Angle Z) 38 d. 30 m.
So is the Sine of Z $\odot$ (the other Side containing the enquired Angle Z) 78 d.
To the Sine of 37 d. 31 m.
- (2.) As this Sine of 37 d. 31 m.
Is to the Sine of the *half Sum*, 93 d. 15 m. (or 86 d. 45 m.)
So is the Sine of the *Difference*, 23 d. 15 m.
To the Sine of 40 d. 20 m.
- (3.) To this Sine of 40 d. 20 m. add the *Radius*, and it will be 19.811063; the half whereof 9.905531 is the Sine of 53 d. 34 m. Whose *Complement* 36 d. 26 m. doubled, makes 72 d. 52 m. And that is the Quantity of the Angle $\odot$ Z P: Or the Sun's *Azimuth* from the North Part of the *Meridian*. And that taken from 90 d. leaves 17 d. 8 m. for the Sun's *Azimuth* from the East or West. And that added to 90 d. gives 107 d. 8 m. for the Sun's *Azimuth* from the South.

2. For

Fig.
XXXIII.2. For the *Hour*, the *Angle* $Z P \odot$. By *Case* I. of O. A. S. T.As the Sine of the Side $P \odot$ (70 d.)Is to the Sine of the *Angle* $\odot Z P$, (72 d. 52 m.)So is the Sine of the Side $Z \odot$ (78 d.)To the Sine of the *Angle* $Z P \odot$, (84 d. 7 m.)And that is the *Hour*, counted from the *North* Part of the *Meridian* (which in Time is 5 h. 36 m. that is, 36 m. after 5 in the Morning, or 95 d. 53 m. (or 6 h. 24 m.) from the *South*.3. For the *Angle* $Z \odot P$, the *Angle* of *Position*. By *Case* I. of O. A. S. T.As the Sine of the Side $P \odot$, (70 d.)Is to the Sine of the *Angle* $\odot Z P$, (72 d. 52 m.)So is the Sine of $Z P$, (38 d. 30 m.)To the Sine of the *Angle* $Z \odot P$, 39 d. 16 m.Which is the *Angle* of the *Sun's Position*.

P R O B. X.

To find the Longitude and Latitude of any Star.

THE Longitude of any *Star* is an *Arch* of the *Ecliptick*, contained between the beginning of *Aries*, and the Intersection of an *Arch* of a great Circle, which passeth through both the Poles of the *Ecliptick*, and also through the Body of that *Star*.

The Latitude of a *Star* is that Part of an *Arch* of a great Circle, which passeth through both the Poles of the *Ecliptick*, and through the Body of the *Star*, and is contained between the *Ecliptick* Line and that *Star*.

I. For the Longitude $\odot$, Skrew the *Quadrant of Altitude* over that Pole of the *Ecliptick* which is nearest to the *Star*, whose Longitude you seek. Then laying the *Quadrant* just over the Centre of the *Star*, look what Degrees of the *Ecliptick*, are cut by the (counting them from the beginning of *Aries*) *Quadrant of Altitude*, and those Degrees are the Degrees of the *Star's Longitude*. So the *Quadrant of Altitude* skrewed over the *North Pole* of the *Ecliptick*, and laid upon the bright *Star Capella*, the *Quadrant* shall cut 77 d. 16 m. of the

the *Ecliptick* Circle, counted from the beginning of *Aries*; and that is that *Star's* Longitude.

Fig.
• XXXIII.

Note, That the Poles of the *Ecliptick* are distant from the Poles of the World $23\frac{1}{2}$ d. on either side.

For the *Latitude*, the *Quadrant* fitted as before, and laid over the Centre of *Capella*, the *Star* shall cut 22 d. 50 m. of the *Quadrant of Altitude*; and such is the *Latitude* of that *Star*, North, for that it lyes on the North Side of the *Ecliptick* Line.

This needs no Trigonometrical Calculation.

P R O B. XI.

To find the Right Ascension and Declination of a Star.

THE *Right Ascension* of a *Star* is that Arch of the *Æquinoctial*, which is contained between the beginning of *Aries*, and that Point which comes to the *Meridian* with that *Star*.

The *Declination* of a *Star* is an Arch of the *Meridian* contained between the *Æquinoctial* and any *Star*.

For the *Right Ascension*, (the *Globe* being rectified) bring *Capella* to the *Meridian*, and then shall you find 73 d. 7 m. of the *Æquinoctial* contained between the beginning of *Aries* and the *Meridian*; and that is the *Right Ascension* of *Capella*.

For the *Declination*, bring *Capella* to the *Meridian*, so shall you find 45 d. 37 m. of the *Meridian* contained between the *Æquinoctial* and *Capella*; and that is the *Declination* of that *Star*. And in this manner you may find the *Longitude*, *Latitude*, *Right Ascension*, and *Declination*, of any other *Star* upon the *Cælestial* *Globe*, as in this following Table of the principal Fixed Stars of the first Magnitude you shall find.

This needs no Trigonometrical Calculation.

L I

Stars

Fig.
XXXIII.

Stars Names.	Longit.		Latit.			R. Ascen.		Declin.		
	D.	M.	D.	M.		D.	M.	D.	M.	
<i>Arcturus</i>	119	39	31	2	B	210	13	20	58	B
<i>Lucida Lyra</i>	280	43	61	47	B	276	27	38	30	B
<i>Algol</i>	51	37	22	22	B	41	46	39	39	B
<i>Capella</i>	77	16	22	50	B	73	7	45	37	B
<i>Aldebaran</i>	65	12	35	31	A	64	17	15	48	B
<i>Regulus</i>	145	17	0	26	B	147	43	13	33	B
<i>Cauda Leonis</i>	167	3	12	18	B	173	4	16	25	B
<i>Spica Virgin.</i>	199	16	1	59	A	196	56	2	31	A
<i>Antares</i>	245	13	4	27	A	242	23	25	37	A
<i>Fomahant</i>	329	11	21	00	A	339	46	31	17	A
<i>Regel</i>	72	17	1	11	A	74	44	8	37	A
<i>Syrius</i>	99	33	39	30	A	97	42	16	14	A
<i>Procyon</i>	111	18	15	57	A	110	34	6	3	B

P R O B. XII.

To find the Distance of Two Stars.

1. **I**F the Two Stars be both of them under the same Meridian, Bring them under the General (or Brass) Meridian, and see what Degrees of the Meridian are contained between them, for that is their Distance.

2. If they lye not under the same Meridian, but have the same Declination, or lye in the same Parallel, Bring one of them to the Meridian, and see what Degrees of the *Æquinoctial* are cut thereby: Then bring the other Star to the Meridian, and count what Degrees of the *Æquinoctial* are contained between the Meridian and the Degrees before found; for that is the Distance of those Two Stars.

3. If the Two Stars do neither lye under the same Meridian, nor in the same Parallel, Then lay the Quadrant of Altitude (it being loose) to both the Stars, and the Degrees of the Quadrant contained between the Two Stars is their Distance. And if the Quadrant be too short, you may use the Circle of Position, or take their Distance with a pair of Calope-Compasses, and measure their Distance upon the *Æquinoctial*, or any other great Circle.

Thus

Astronomical Problems.

221

Thus,

The Right Shoulder of *Auriga*, and the Right Shoulder of *Fig. XXXIII.*
Orion, being under the same *Meridian*, their Distance will be
found to be 37 d. 38 m.

Also,

Arcturus and the Lion's Neck, being near in the same Parallel,
their Distance will be found to be 57 d.

Likewise,

Lyra the Harp, and *Marchad* in the Wing of *Pegasus*, will be
found to be distant 63 d.

The finding of the Distance of *Stars* upon the *Cælestial Globe*
is the same as the finding of Distance of Places upon the *Ter-*
restrial, and consequently the *Triangles* made upon the *Globe* are the
same also: So that the Canons for Calculation for finding them
will be the same also, and needs not be here again repeated.

P R O B. XIII.

To know what *Stars* will be upon the *Meridian* at any Hour
of the Night.

THE *Sun* being in 29 d. of *Taurus*, what *Stars* will be up-
on the *Meridian* at 10 a Clock, and 12 m. at Night. Bring
10 d. of *Scorpio* (which is the opposite Sign to *Taurus*) to the
Meridian, and set the *Index* of the Hour-Circle to 12. Then
turn the *Globe* about Westward till the *Index* point at 12 m.
after 10 a Clock, and there hold the *Globe*; and all those *Stars*
which lye under the *Brass Meridian* are then upon the *Meri-*
an, of which *Arcturus* is the Chief.

This needs no Trigonometrical Calculation.

P R O B. XIV.

To know what Day in the Year any *Star* shall be upon the *Meri-*
dian at 12 a Clock at Night.

Bring the *Star* to the *Meridian*, and mark what Degree of
the *Ecliptick* is just under the *Meridian* at the same time:
then find that Degree of the *Ecliptick* in the *Horizon*, and note
that Day of the Year standeth against it, for that Day of the
Year will that *Star* be upon the *South Part* of the *Meridian*
at 12 at Night: And when the *Sun* is in the opposite Point of
the

L 1 2

Fig. the *Ecliptick*, the same *Star* will be upon the *North Part* of the
XXXIII. *Meridian* at 12 at Noon.

P R O B. XV.

The Sun's Place, and the Altitude of a known Star given; To find the Hour of the Night.

THE *Sun* being in 21 d. of *Capricorn*, the *Altitude* of the *Great Dog* 14 d. I demand the *Hour of the Night*.

The *Globe Rectified*, &c. Bring 21 d. of *Capricorn* to the *Meridian*, and the *Index* to 12 a Clock. Then move the *Globe* and *Quadrant of Altitude* so together, that the *Great Dog* meet with 14 d. of the *Quadrant*; and then shall the *Index* point at 8 of the Clock, and 22 m. which is the true *Hour of the Night*.

And thus,

	D.		D.		D. M.
When	20 4	and the	The Bull's Eye 39	The ho.	7 12
the Sun	20 m	Alti-	The Bull's Eye 30	will be	9 2
is in	5 II	tude of	Arcturus 50		11 3

P R O B. XVI.

The Altitude of Aldebaran (or any other Star) being given, in a known Latitude; To find the Star's Azimuth.

THE *Quadrant of Altitude* being fixed in the *Zenith*, move it and the *Globe* till the *Degrees of Altitude* given do meet with the *Centre of the Star*; then shall the end of the *Quadrant of Altitude* shew you upon the *Horizon* the *Azimuth* in which the *Star* then is. And thus, if you bring the *Quadrant of Altitude* on the *East Side* of the *Globe*, moving it and the *Globe* both, till the *Centre of Aldebaran* do meet with 42 d. of the *Quadrant*, you shall then find the *Quadrant of Altitude* to rest at 33 d. of the *Horizon*, counted from the *East*; or at 57 d. if you count them from the *South*: And that is the *Azimuth* of *Aldebaran* when he hath 42 d. of *Altitude* and that is near the S. E. by E. Point of the *Compass*.

The Latitude of the Place, (51 d. 30 m.) and the Declination of a Star, (suppose the Bull's Eye, Aldebaran) given: To find

P R O B. XVII.

Its Right Ascension.

THE Globe Rectified to the Latitude, &c. Bring *Aldebaran* to the Meridian: Then count how many Degrees of the *Equinoctial* are contained between the Meridian and the beginning of *Aries*; which will be 64 d. 17 m. and that is the *Right Ascension* of that Star; which in time (by allowing 15 d. for an Hour, and 1 d. for 4 m. of time) in 4 h. 16 m. its *Right Ascension* in time.

And in the same manner may you find

	D.	M.		H.	M.
The Right { <i>Arcturus</i> }	210	13	} to be { in time {	14	I
Ascensi- { <i>Spirus</i> }	97	42		6	30
on of { <i>Algol</i> }	39	39		2	38

The same Trigonometrical Calculation, as for the *Sun's Right Ascension*, as in *Prob. III.* serves for this also.

P R O B. XVIII.

Its Ascensional Difference.

Bring the Star to the Meridian, and the Hour-Index to 12: Then bring the Star either to the East or West Side of the Horizon, and there you shall find 1 h. and 27 m. contained between the Index and 6 a Clock: And such is the *Ascensional Difference* of *Aldebaran*.

In like manner you may find

	D.	M.		H.	M.
The Ascen- { <i>Arcturus</i> }	28	40	} to be { or in time {	1	55
sional Dif- { <i>Syrus</i> }	21	28		1	26
ference of { <i>Algol</i> }	—	—		—	—

Algol his Declination being more than the Complement of the Latitude, never rises nor sets, but is always above the Horizon.

The same Trigonometrical Calculation, as for the *Sun*.

P R O B.

P R O B. XIX.

Its Amplitude.

BRing *Aldebaran* to the *Horizon* on either Side of the *Globe*, and you shall find it to touch the *Horizon* at 25 d. 56 m. from the *East* or *West* Northward; which is the *Amplitude* of the *Bull's Eye* rising or setting. And according to the Points of the *Compass* it riseth E. N. E. 2 d. 26 m. Northerly, and sets W. N. W. 2 d. 26 m. Northerly.

And thus may you find

			D.	M.
That	{ <i>Arcturus</i> <i>Syrius</i> <i>Algol</i> }	Riseth from	Northward	35 6
		the <i>East</i> or	Southward	26 41
		<i>West</i>	Never rises or sets.	

The same Trigonometrical Calculation as for the *Sun*.

P R O B. XX.

The Semidiurnal Arch, and the time that *Aldebaran* (or any other Star) continues above the *Horizon*.

BRing the *Aldebaran* to the *Meridian*, and set the *Hour-Circle* to 12. Then turn the *Globe* Westward, till *Aldebaran* touch the *Horizon*; then shall the *Hour-Index* point at 7 h. 27 m. And so long time is *Aldebaran* above the *Horizon*, before he comes to the *Meridian*, and continues so many Hours and Minutes above the *Horizon*, after he hath past the *Meridian*, and sets in the *West*. And those 7 h. and 27 m. is the *Semidiurnal Arch* of that *Star*; which doubled, is 14 h. 54 m. And so long doth that *Star* continue above the *Horizon* after the time of his rising.

And in this manner you find

			D.	M.		H.	M.
The Semi-diurnal Arch of	{ <i>Arcturus</i> <i>Syrius</i> <i>Algol</i> }	to be	7	55	And his Continuance above the <i>Horizon</i>	15	50
			4	34		9	8
			12	00		24	00

The same Trigonometrical Calculation as for the *Sun*.

Astronomical Problems.

215

Fig.
XXXIII.

P R O B. XXI.

At what Hour (any time of the Year) Aldebaran comes to the Meridian.

LET the time be the First of January, at which time the Sun is in 22 d. of Capricorn. Bring 22 d. of Capricorn to the Meridian, and set the Hour-Index to 12. Then turn the Globe about till Aldebaran be under the Meridian, and then you shall find the Index to point at 42 m. after 8 of the Clock, at which time Aldebaran will be upon the Meridian that Night.

In like manner you may find that

Upon	October 28	Arcturus	{ will be upon the Meridi- an at	11	10
	January 21	Syrus		9	33
	January 1	Algol		7	12

The same Trigonometrical Calculation as for the Sun.

P R O B. XXII.

At what Hour (at any time of the Year) Aldebaran (or any other Star) riseth or setteth.

LET the time be January 1. By the last before-going, you found that Aldebaran came to the Meridian at 8 h. 42 m. and by the last but one you found his Semidiurnal Arch to be 7 h. 27 m. This being taken from 8 h. 42 m. the time of his rising South, leaveth 1 h. 15 m. the time of its rising; so that upon the First of January Aldebaran did rise at 15 m. after 1 of the Afternoon. Again, if you add his Semidiurnal 7 h. 27 m. to the time of its being South 8 h. 42 m. the Sum will be 16 h. 9 m. from which take 12 h. and the Remainder will be 4 h. 9 m. So that Aldebaran did set at 9 m. after 4 of the Clock the next Morning.

And in like manner you may find that

Upon	October 28	Arcturus	{ Rises at	3	8	{ Sets at	7	12
	January 21	Syrus		5	3		2	3
	January 1	Algol		—	—		—	—

Algol never Rises nor Sets.

The same Trigonometrical Calculation as for the Sun.

P R O B.

P R O B. XXIII.

At what Horary Distance from the Meridian, Aldebaran will be due East or West : And what Altitude he shall then have.

BRing *Aldebaran* to the *Meridian*, and the *Hour-Index* to 12, and the *Quadrant of Altitude* to the *West Point* of the *Horizon*: Then turn the *Globe Eastward*, till the *Centre* of the *Star* be just under the *Edge* of the *Quadrant*; then shall the *Index* point at 5 h. and 40 m. So that when *Aldebaran* is due *East or West*, he will be 5 h. 40 m. of time short of, or gone beyond, the *Meridian*.

And when the *Centre* of *Aldebaran* is just under the *Edge* of the *Quadrant of Altitude*, you shall find it to touch 20 d. 21 m. And such is the *Altitude* of *Aldebaran* when he is upon the *East or West Azimuth*.

In like manner may you find that

	H. M.		D. M.
<i>Arcturus</i>	} will be upon the <i>East or West Azimuth</i> , when he is distant from the <i>Meridian</i> .	} and his <i>Altitude</i> will be	
<i>Syrius</i>			
<i>Algol</i>			
	4 49	27 13	
	5 04	20 56	
	3 15	54 37	

The same Trigonometrical Calculation as for the *Sun*.

P R O B. XXIV.

What Altitude and Azimuth Aldebaran (or any other Star) shall have, when Six Hours distant from the Meridian.

BRing *Aldebaran* to the *Meridian*, and the *Index* to 12. Then turn the *Globe* about till the *Index* point at 6; then lay the *Quadrant of Altitude* over the *Centre* of the *Star*, and you shall find it to lye under 12 d. 18 m. of the *Quadrant*: And such is the *Altitude* of *Aldebaran*. At the same time look what *Degrees* of the *Horizon* are cut by the *Quadrant of Altitude*, and you shall find 8 d. between it and the *East or West Points Northwards*. And such is the *Azimuth* of *Aldebaran*. And according to this Rule you shall find that when

	Altitude	Azimuth	
	D. M.	D. M.	
<i>Arcturus</i>	} is 6 h. distant from the <i>Meridian</i> his	16 15	76 36
<i>Algol</i>		12 38	79 43
			} from the North.

Syrius

Syrus is never 6 h. distant from the *Meridian*, nor any other
 ar that hath *South Declination*.

Fig.
 XXXIII.

The Trigonometrical Calculation as for the *Sun*.

P R O B. XXV.

to find what *Altitude and Azimuth any Star bath when he is at
 any horary Distance from the Meridian.*

THIS is no other than the last. For having brought the *Star*
 to the *Meridian*, and the *Index* to 12, move the *Globe* till it
 come to the designed *Hour*. Then the *Quadrant of Altitude* being
 id over the *Star*, shall at the same time shew you both the
Altitude and Azimuth thereof as before. This needeth no
 example.

The same Trigonometrical Calculation as for the *Sun*.

P R O B. XXVI.

to find the *Azimuth of a Star, to find at what Horary Distance
 that Star is from the Meridian, and what Altitude that Star
 then bath.*

Bring the *Star* to the *Meridian*, the *Index* to 12, and the *Qua-
 drant of Altitude* to the *Given Azimuth*; then turn the
Globe about till the Centre of the *Star* lye just under the *Qua-
 rant of Altitude*; the *Index* at that time shall give the *Horary
 Distance*, and *Quadrant* the *Altitude* of the *Star*.

Example. *Aldebaran* being seen upon 80 d. of *Azimuth* from
 the North Westward, that is, near upon the W. by N. Point of
 the *Compass*; the *Star* brought to the *Meridian*, and the *Quadrant
 of Altitude* to 80 d. and the *Hour-Index* to 12. If you bring
 the *Star* to the *Quadrant of Altitude*, you shall find the *Index* to
 point at 6 h. which is the *Star's Horary Distance* from the *Meri-
 dian*. And the *Quadrant of Altitude* will shew 12 d. 18 m. the
Altitude of Aldebaran at that time.

These Twelve last *Problems* may be resolved by Trigonometri-
 Calculation in all Respects as those of the *Sun*, if (in all
 them) instead of the Word [*Sun*] there, read [*Star*] here.
 And the *Canons* for Calculation will be the same in both.

M m

P R O B.

Fig.
XXXIII.

P R O B. XXVII.

The Longitude and Latitude of a Star, which is situate out of the Ecliptick, being given; To find the Right Ascension and Declination of that Star.

HERE is given the Situation of the Star, in respect to the Ecliptick, altho' its Situation be out of the Ecliptick; and his Place, in respect of the Æquinoctial, is required

I. By the Cœlestial Globe.

Screw the Quadrant of Altitude over the Pole of the Ecliptick and lay it to the Degree of the Star's Longitude in the Ecliptick. —Then count the Star's Latitude given upon the Quadrant of Altitude: And observe what Meridian passeth through that Point: For the Degrees of that Meridian, comprehended between that Point, and the Pole of the World, are the Degrees of the Star's Declination: And the same Meridian continued will cross the Æquinoctial Circle in the Point of the Star's Right Ascension.

II. By Trigonometrical Calculation.

Fig. XXXIV. When the Longitude of the Sun falls to be in any Sign, Ascending, as in Fig. XXXIV. }
XXXV. Descending, as in Fig. XXXV. } make choice of the Triangle

P F S, for the Latitude of the given Star if it be { North,
South, }
the which the Letters *b* and *m* written within them will determine. In which Triangle there is

- Given, {
1. The Side P F, the Distance of the Pole of the Æquinoctial from the Pole of the Ecliptick, 23 d. 30. m.
 2. The Angle F, whose Measure is the Arch of the Ecliptick, intercepted between the Sides F P and F S, from whom the Longitude of the Star is produced, and the Solstitial Point found out.
 3. The Side F S, the Complement of the given Star's Latitude.

Requiritur

- required, {
1. The Side P S, the Complement of the Star's Declination. Fig. XXXIV.
 2. The Angle P, which the Ark of the *Æquator* intercepted between the Sides P F and P S; of which, this *Right Ascension* of the Star is made, whither from the Degree of the *Ecliptick* 270, or 90 d.

Example. Let there be given the *Head of Andromeda*, which is a Star of the Second Magnitude; whose *Longitude* is in 9 d. 8 m. γ , and *Latitude* 25 d. 42 m. b . The *Longitude* of the Star falls out to be in an Ascending Sign *Aries*; therefore, make choice of the Triangle P F S, Fig. XXXV. the *Latitude* being North, in which there is given, (1.) P F, the Distance of the Pole of the *Æquator* from the Pole of the *Ecliptick*, 23 d. 30 m. (2.) The Angle F, which the Ark of the *Ecliptick* intercepted between the Sides F P and F S, being produced, measureth, and gives the *Longitude* of the Star to be 9 d. 38 m. γ , so that the Solstitial Point $\ominus$ is nearest; subtract therefore 9 d. 38 m. γ from 90 d. the beginning of $\ominus$, the Remainder will be 80 d. 22 m. which is the Angle at F. (3.) F S, the Complement of the *Latitude* of the given Star, 64 d. 18 m. And let there be sought, (1.) P S, the Complement of the Declination 62 d. 45 m. therefore the Declination of the Star is 27 d. 15 m. (2.) The Angle P, 87 d. 47 m. Which added to 270 d. makes the *Right Ascension* of the Star to be 357 d. 7 m.

The Canons for Calculation.

I. For the Angles S and P.

- 1.) As the Sine of half the Sum of the Sides S F and F P, 43 d. 54 m.
Is to the Sine of half the Difference of those Sides, 20 d. 24 m.
So is the Co-tangent of half the Angle at F, 40 d. 11 m.
To the Tangent of half the Difference of the Angles at S and P, 30 d. 46 m.
- 2.) As the Co-sine of half the Sum of the Sides F S and F P, 46 d. 6 m.
Is to the Co-sine of half the Difference of those Two Sides, 39 d. 36 m.

M m 2

So

Fig.
XXXIV.
XXXV.

So is the Co-tangent of half the Angle at F, 40 d. 11 m.
To the Tangent of half the Sum of the Angles at S and P,
57 d. 1 m.
Which 57 d. 1 m. added to the aforefound Difference, gives
87 d. 47 m. for the greater Angle at P; and 30 d. 46 m.
the Difference aforefound subtracted from 57 d. 1 m.
leaves 26 d. 15 m. for the lesser Angle at S.

Then,

(3.) As the Sine of the Angle at S, 26 d. 15 m.
Is to the Sine of the Side F P, 23 d. 30 m.
So is the Sine of the Angle at F, 80 d. 22 m.
To the Sine of the Side S P, 62 d. 45 m.
Whose Complement, 27 d. 15 m. is the *Star's Declination*.

P R O B. XXVIII.

The Declination and Right Ascension of a Star being given: To find the Longitude and Latitude of that Star.

THIS is the Converse of the former Problem, and may be resolved as followeth.

I. By the Cœlestial Globe.

Screw the *Quadrant of Altitude* over the Pole of the *Ecliptick*: Then count upon the *Æquinoctial Circle* the Degrees of *Right Ascension*; and upon that *Meridian* which passes through that Point, count the *Star's Declination* from the *Æquinoctial* towards the Pole, and to that Point bring the *Quadrant of Altitude*; then will the *Quadrant* cut the *Ecliptick* in the Degrees of the *Star's Longitude*; and the Degrees of the *Quadrant*, comprehended between the *Ecliptick Circle* and its Pole will give the Degrees of the *Star's Latitude*.

II. By Trigonometrical Calculation.

When the *Right Ascension* of the *Star* falls out to be in the *First* or *Fourth Quadrant* of the *Æquinoctial*; as in Fig. XXXIV. or in the *Second* or *Third*: As in Fig. XXXV. make use of the Triangle P F S for the *Star's Declination* North or South. In which there is

Given

- Given, {
1. P F, the Distance of the Pole of the *Ecliptick* from the Pole of the *World*, 23 d. 30 m. Fig. XXXIV.
 2. P S, the Complement of the *Star's Declination*, 64 d. 13 m. XXXV.
 3. The Angle P, whose Measure is the Arch of the *Æquator* intercepted between the Sides P F and P S, 87 d. 47 m. from whence the *Right Ascension* of the *Star* is produced, and will be found to be 357 d. 47 m.

- Required, {
1. The Angle F, whose Measure is the Arch of the *Ecliptick* intercepted between the Sides F P and F S, from whence the *Longitude* of the *Star* from the next *Solstitial Point* is found.
 2. F S, the Complement of the *Star's Latitude*.

The Canons for Calculation.

(1.) As the Sine of half the Sum of the given Sides P F and P S, 43 d. 8 m.
 Is to the Sine of half the Difference of those Sides, 19 d. 37 m.
 So is the Co-tangent of half the given Angle F, 43 d. 54 m.
 To the Tangent of half the Difference of the Angles at F and S,
 viz. 27 d. 3 m.

(2.) As the Co-sine of half the Sum of the Sides F S and F P, 43 d. 8 m.
 Is to the Co-sine of half the Difference of those Sides, 19 d. 37 m.

So is the Co-tangent of half the given Angle at F 43 d. 54 m.
 To the Tangent of half the Sum of the Angles at F and S,
 viz. 53 d. 18 m.

The half Difference before found 27 d. 3 m. added to the half Sum now found, gives 80 d. 21 m. for the Angle at F, the Complement of the *Star's Longitude*. And subtracted from the half Sum 53 d. 18 m. leaves 26 d. 15 m. for the Angle at S.

Then say,

(3.) As the Sine of the Angle at F, 80 d. 21 m.
 Is to the Sine of the Side S P, 62 d. 45 m.
 So is the Sine of the Angle at P, 87 d. 47 m.
 To the Sine of the Side S F, 64 d. 18 m.

Whose Complement 25 d. 42 m. is the *Star's Latitude*.

A Table of the Right Ascension and Declination of 100 Eminent Fixed Stars, Calculated for the Year of Christ 1700, completed according to Ricciolus. And their Difference of Right Ascension and Declination in 100 Years.

Names of the STARS.	Right Ascension for the Year 1700.		Declination for the Year 1700.		Diff. of Ascen. in 100 Years.		Decl. in 100 Years.
	De.	Mi. Se.	De.	Mi. Se.	D. M.	Mi.	
Head of Andromeda.	358	14	08	27	06	B	A
Girdle of Andromeda.	13	11	20	34	2	B	A
South Foot of Andromeda.	26	21	51	40	52	B	A
Fomalhaut	340	11	00	31	08	A	S
Right Shoulder	327	56	55	1	43	A	S
Left Shoulder	318	55	54	6	48	A	S
Left Hand	307	45	54	10	33	A	S
Bright Star, in the Eagle.	294	2	47	8	6	B	A
First in Horn of γ .	24	17	2	17	48	B	A
Second in γ Horn.	24	30	3	16	19	B	A
Bright one in Aries.	27	35	58	22	1	B	A
Goat of Auriga.	73	35	56	45	40	B	A
Right Shoulder of Auriga.	84	29	42	44	51	B	A
Arcturus in Bootes.	210	33	2	20	48	B	S
Left Shoulder of Bootes.	215	2	33	39	35	A	S

Name

Right Ascension

Names of the STARS.	Right Ascension for the Year 1700.		Declination for the Year 1700.		Diff. of Ascen. in 100 Years.		Decl. in 100 Years.
	De.	Mi. Se.	De.	Mi. Se.	D.	M.	Mi.
<i>Prosepe</i> in Cancer.	125	46	2	20	43	B	19
Northern <i>Asinego</i> in ☌.	126	26	0	22	3	B	20
Southern <i>Asinego</i> in ☌.	126	54	3	19	15	B	20
Great Dog <i>Cyrus</i> .	97	57	6	16	18	A	A
Little Dog <i>Porcyon</i> .	110	54	33	5	59	B	12
Upper Horn of ☿.	300	24	34	13	22	A	16
Lower Horn of ☿.	301	7	29	15	38	A	17
First in the Tail of ☿.	320	56	29	17	54	A	26
Second in the Tail of ☿.	322	43	40	17	22	A	27
Bright one in <i>Cassiopea's</i> Chair.	358	14	33	57	32	B	34
<i>Scheder</i> . or the Breast of <i>Cassiopea</i> .	5	56	0	54	55	B	34
In the Flexure of <i>Cassiopea</i> .	9	45	58	59	7	B	34
In <i>Cassiopea's</i> Knee.	16	36	0	58	40	B	33
<i>Cephus</i> his Girdle.	321	6	20	69	17	B	26
Bright one in the <i>Whale's</i> Jaw.	41	38	7	2	50	B	25
Northern in the <i>Whale's</i> Belly.	24	12	0	11	44	A	31
Southern in the <i>Whale's</i> Belly.	7	5	8	19	35	A	34
Northern in the <i>Whale's</i> Tail.	1	4	12	10	24	A	34

Names

Names of the STARS.	Right Ascension for the Year 1700.		Declination for the Year 1700.		Diff. of Ascension in 100 Years.		Declination in 100 Years.
	De.	Mi. Se.	De.	Mi. Se.	D.	M.	M.
Bright one in the North. Crown.	230	39	0	27	45	20	S 21
In the Beak of the Swan.	289	39	48	27	22	40	A 11
In the Swan's Breast.	302	55	52	39	20	5	A 18
In the Swan's Tail.	307	47	17	44	14	51	A 20 $\frac{1}{2}$
Upper Wing of the Swan.	293	56	2	44	26	21	A 14
Lower Wing of the Swan.	308	29	10	32	51	24	A 21
Bright one of the Dragon.	267	25	20	51	35	2	S 2
In the Head of Castor.	108	50	46	32	30	26	S 11
In the Head of Pollux.	111	43	36	28	43	2	S 12
Bright one in the Twins Foot.	95	3	32	16	37	32	S 2
<i>Hercules</i> Head.	255	21	37	14	46	48	S 8
<i>Hercules</i> Right Shoulder.	244	19	35	22	11	40	S 15
<i>Hercules</i> Left Shoulder.	255	31	33	25	15	48	S 8
Heart of <i>Hydra</i> .	138	12	22	7	21	30	A 25
Lion's Heart <i>Regulus</i> .	148	4	15	13	25	16	S 28 $\frac{1}{2}$
Lion's Tail.	175	25	34	16	14	4	S 24
Bright one in <i>Juba Leonis</i> .	150	48	47	21	21	0	S 29
Bright one in <i>Lumbis Leonis</i> .	164	32	20	22	7	44	S 34
Uppermost in the Neck.	149	58	52	24	53	54	S 29
Lowest in the Neck.	147	47	52	18	13	33	S 28

Uppermost in the Neck.
Lowest in the Neck.

Names

Names of the STARS.	Right Ascension for the Year 1700.		Declination for the Year 1700.		Diff. of Ascen. in 100 Years.		Decl. in 100 Years.
	De.	M. Se.	De.	Mi. Se.	D.	M.	
Thigh of the Hare.	78	51	30	0	1	5	S
Northern Scale of <i>Libra</i> .	225	15	26	14	1	21 $\frac{1}{2}$	A
Southern Scale of <i>Libra</i> .	218	38	12	45	1	25	A
Bright one in <i>Lyra</i> .	276	39	32	32	0	50	A
Head of	260	15	38	12	1	11	S
Left Hand of	239	47	37	2	1	23	A
Right Knee of	252	39	40	53	0	50	A
Left Knee of	245	11	37	15	1	23	A
Right Shoulder of	162	8	38	44	1	13	S
} <i>Ophiucus</i> .							
Uppermost in the Head of <i>Orion</i> .	79	41	10	9	1	22	A
Right Shoulder of <i>Orion</i> .	84	43	4	7	1	22	A
Left Shoulder of <i>Orion</i> .	97	16	40	6	1	19	A
Foot of <i>Orion</i> Rigel.	75	2	50	8	1	15 $\frac{1}{2}$	S
1 } In the Belt of <i>Orion</i> . 2 } 3 }	79	9	48	0	1	17 $\frac{1}{2}$	S
	80	12	54	1	1	17	S
	81	18	25	2	1	16	S

N =

Names

Names of the STARS.	Right Ascension for the Year 1700.		Declination for the Year 1700.		Diff. of Ascension in 100 Years.		Declination in 100 Years.
	De.	Mi. Se.	De.	Mi. Se.	D.	M.	M.
Mouth of Pegasus.	322	27	36	8	1	18	26 A
Sa'd Albarus in the Leg.	342	20	36	26	1	12	32 A
Marcab in the joining of the Wing.	342	28	10	13	1	15	32 A
End of Pegasus Wing.	359	27	25	13	1	16	34 A
Bright one in the Side of Perseus.	45	32	18	48	1	28	12 A
Ras Al Gol of Perseus.	42	12	42	39	1	37	25 A
The hindmost in the Head of } the Southern Fish.	345	24	5	1	1	17	33 A
In the Knot in the Line of κ .	26	38	5	1	1	18	30 A
Bright one in the Head of $\uparrow$.	283	1	5	21	1	31	8 S
Antares, Heart of Scorpius.	242	47	28	25	1	32	16 A
Northern Front of } Middlemost } Southern Front of } Brightest in the Neck of Scorpius.	236	58	15	18	1	28	16 A
	236	14	34	21	1	30	20 A
	235	18	0	25	1	37 $\frac{1}{2}$	21 A
	232	24	0	7	1	15	21 S
Aldebaran, or Southern Eye of γ .	64	41	35	15	1	26 $\frac{1}{2}$	15 A
In the Northern Horn of γ .	76	51	18	28	1	37	8 A
Southern Horn of γ .	79	55	20	20	1	31	7 A
Northern Eye of γ .	62	43	36	18	1	24	17 A
Lowest of the Hyades γ .	60	39	35	14	1	25	17 A
Bright one of the Pleiades γ .	52	27	35	23	1	29	21 A

Lowest of the Hyades &
Bright one of the Pleyades &.

Names

Names of the STARS.	Right Ascension for the Year 1700.		Declination the Year 1700.		Diff. of Ascen. in 100 Years.		Decl. in 100 Years.
	De.	Mi. Se.	De.	Mi. Se.	D.	M.	
<i>Spica Virginis.</i>	197	22 55	9	33 30	B	I 19½	32½ A
<i>Girdle of Virgo.</i>	190	10 22	5	2 54	B	I 18	34 S
<i>Vindemiatrix in Virgo.</i>	191	52 20	12	34 58	B	I 17	33 S
The Bright one in the Shoulder of the Greater Bear.	161	17 5	63	22 2	B	I 41	32 S
The Bright one in its Side.	160	52 20	57	59 2	B	I 37	32 S
The Bright one in the hindermost Thigh.	174	23 34	55	23 42	B	I 23	34 S
On the Back near the Tail.	180	8 2	58	41 42	B	I 20	34 S
{ First Second Laft	190	7 56	57	36 58	B	I 9	33 S
	197	55 2	56	30 52	B	I 3	32 S
	203	53 50	56	50 56	B	I 2	31 S
The laft in the Tail of the lesser Bear, now the Pole Star.	9	52 10	87	42 51	B	3 10	34½ A
The Bright one in the Shoulder, heretofore called <i>Cyndura</i> .	222	39 20	75	37 30	B	I 15	2½ A

P R O B. XXIX.

The Latitude of a Place, (or Elevation of the Pole) and Declination of the Sun, or of a Star, being given: To find the Ascensional Difference of that Star: And from thence, (the Right Ascension of the same Star being known) to find its Oblique Ascension: The Semidiurnal Ark, the Continuance of it above and below the Horizon: Its Ortive and Occasive Amplitude: And the Oriental Angle, viz. that of the Point of the Ecliptick.

I. By the Cœlestial Globe.

Fig.
XXXVI.

THE *Globe* being rectified to the *Latitude*, and the *Place of the Sun* brought to the *Meridian*; and the *Quadrant of Altitude* screw'd to the *Zenith*, and brought to the *East or West Points of the Horizon*; the several *Arches and Angles* made by the *Positions and Intersections of these Circles* being measured or counted upon the *Globe*, will resolve the several *Demands*: But with some additional *Works*; which I here omit. Wherefore,

II. By Trigonometrical Calculation.

Here the *Situation of a Star* is given, in respect of the *Æquinoctial*; and Enquiry is made where its *Place* should be, in respect of an *Oblique Horizon* given.

The *Globe* continuing in the same *Position* as when you wrought the *Problem* upon it, you will find constituted upon it *Two Spherical Triangles*; such as in the *Figure* are noted with *R Or S*, for the *Declination of the Star Northern or Southern*. In either of which *Triangles*, there is (besides the *Right Angle at R*)

Given, { 1. The *Angle Or*, which is the *Angle* made by the *Æquator* and the *Horizon of London*, 38 d. 30 m.
2. The *Perpendicular R S*, the *Star's Declination North*.

And

EX
(
o

1. Or R, the *Ascensional Difference*; which, when a *Star* declineth towards

An Elevated Pole. { Subtract the Decl. from the R. Ascension, it gives the Obl. Ascension.
Add the Decl. to the R. Ascension, it gives the Obl. Descension.
Add 90 d. it gives the Semidiurnal Ark.

A Depressed Pole, { Add the Decl. to the R. Ascension, it gives the Obl. Ascension.
Subtract the Decl. from the R. Ascension, it gives the Obl. Ascension.
Subtract 90 d. it gives the Semidiurnal Ark.

The Semidiurnal Ark being turned into Hours, gives half the Time of the Stars Continuance above the Horizon: Whose Complement to 12 Hours is its Seminocturnal Ark, or half his Continuance below the Horizon.

2. Or S, the *Amplitude* of the *Star*, at his Rising or Setting, having either North or South Declination.

3. The Angle S; which Subtracted in Signs, { Ascending, from the Angle of the Ecliptick and Meridian,
Descending, from the Complement of the Angle of the Ecliptick; adding 180 d. } Gives the Oriental Angle

EXAMPLE I. Let there be given an Elevation of the Pole (as at London) 51 d. 30 m. And the Sun's Declination, 0 d. 50 m. South.

In

Fig. XXXVI. In the Triangle R Or S, (the Declination being Southward) there is given, (besides the Right Angle at R,) (1.) The Angle R Or S, the Complement of the given Latitude, 38 d. 30 m. (2.) The Side R S, the Declination of the Sun, 1 d. 2 m. by which may be found,

(1.) The Side Or R, the Ascensional Difference, 1 d. 2 m. Which, when the Sun declines (as here) towards the Depressed Pole, being added to the Right Ascension before found to be 358 d. 3 m. gives the Oblique Descension, to be 359 d. 7 m. But being taken from the said Right Ascension, 358 d. 3 m. gives the Oblique Descension to be 356 d. 59 m. And being taken likewise from 90 d. gives 88 d. 56 m. for the Semidiurnal Ark: Which being turned into Time, gives 5 h. 55 m. for the half continuance of the Sun above the Horizon, whose Complement to 12 h. is 6 h. 5 m. the half continuance of the Sun below the Horizon: But double the continuance of the Sun above the Horizon, the Length of the Day will be 11 h. 50 m. And double the continuance below, and the Length of the Night will be 12 h. 10 m.

(2.) The Side Or S, the Ortive Amplitude of the Sun, 1 d. 20 m.

(3.) The Angle R S Or, 51 d. 30 m. the given Latitude.

Which being subtracted (because, in this Example, the Sign is Ascending, viz. ♈) from the Angle of the Ecliptick and Meridian, before found, to be 66 d. 31 m. gives 15 d. 1 m. for the Oriental Angle of the Point of the Ecliptick.

This Example was of the Sun, I will here give another Example of a Fixed Star, viz. The Head of Andromeda.

EXAMPLE II. The Declination of the Head of Andromeda is 27 d. 15 m. represented by the Side R S.

The Complement of the given Latitude is 38 d. 30 m. represented by the Angle R Or S.

These being given, we are to find,

1. The Side R Or, the Star's Ascensional Difference.
2. The Side Or S, the Amplitude of the Star.
3. The Angle at S.

III. The Canons for Calculation.

1. For the Side R Or, by Case VII. of R. A. S. T.

As the Radius,

Is to the Tangent of R S, the Star's Declination, 27 d. 15 m.

Astronomical Problems.

241

Fig.

XXXVI.

o is the Co-tangent of the *Angle Or*, 38 d. 30 m.

To the Sine of the Side R Or. The Star's *Ascensional Difference*, 40 d. 21 m.

2. For the *Side Or S*. By *Case IX.* of R. A. S. T.

s the Sine of the Angle R Or S, 38 d. 30 m.

Is to the Radius:

o is the Sine of the Side R S, 27 d. 15 m.

To the Sine of the Side S R, (the Star's *Amplitude* at his *Rising*) 47 d. 22 m.

3. For the *Angle R S Or*. By *Case VIII.* of R. A. S. T.

s the Co-sine of the Side R S, 27 d. 15 m.

To the Radius:

o is the Sine of the Angle R Or S, 38 d. 30 m.

To the Sine of the Angle R S Or, 61 d. 41 m.

By which the *Oriental Angle* may be found.

D. M.

the *Right Ascension* of *Andromeda's Head* was found } 357 47

by (Pr. 27.) to be

and his *Declination* given

North 27 15

his *Ascensional Difference* (by the (1.) hereof) is } 40 21

found to be

Which subtracted from the Star's *Right Ascension*, } 317 26

gives the *Oblique Ascension* to be

and the *Declination* added to the *Right Ascension*, } 398 08

gives

From which subtract the whole Circle

360 00

there will remain for the *Oblique Descension*

38 08

To which add

90 00

and it gives the *Semidiurnal Arch* to be

128 08

H. M.

Which turned into Time, is

8 32

Which doubled, is

17 04

and so long the *Star* continues above the *Horizon* of *London*.

and that subtracted from 24 h. leaves 6 h. 56 m. for the Star's

continuance below the *Horizon* of *London*.

by the (2.) hereof, the *Amplitude* of the Star's *Rising* } 47 22

from the *East Northward*, is found to be

47 22

and by the (3.) hereof, the *Angle* at S,

61 41

Which } Subtracted in Signs Ascending, from }

{ Added in Signs Descending, to }

the *Angle* of

the *Ecliptick* and *Meridian*, gives the *Oriental Angle*.

P R O B.

The Declination of the Sun, and the Length of a Day being given, to find the Latitude or Height of the Pole.

I. *By the Cœlestial Globe.*

BRing the *Solstitial Colure* to the *Meridian*, and count the *Sun's Declination* upon it, either *North* or *South*, (suppose 23 d. 30 m. *North*) then let the *Sun's Declination* be 23 d. 30 m. *North*, and the *Length of the Day* 16 h. 26 m.

Bring the *Æquinoctial Colure* to the *Meridian*, and upon it count 23 d. 30 m. *North*, and there fix the *Body of the Globe*. Then, upon the *Parallel of 23 d. 30 m. of Declination*, count 8 h. 13 m. the half *Length of the Day*, and there make a *Mark*. Then move the *Brass Meridian* up and down in the *Frame*, till the *Mark* you made do justly touch the *Horizon*; for then the *Degrees of the Brass Meridian*, comprehended between the *Horizon* and the *Pole*, will be the *Latitude* sought, viz. 31 d. 30 m.

II. *By Trigonometrical Calculation.*

This being (in effect) but the *Converse* of the foregoing *Problem*, the *Spherical Triangle* will be the same upon the *Globe* as in that; namely, The *Triangle R Or S*: In which there is given, (besides the *Right Angle* at *R*) (1.) The *Side R S*, the *Sun's Declination*, 23 d. 30 m. *North*. (2.) The *Side R Or*, the *Ascensional Difference*, which the *Sun* hath when the *Length of a Day* given is either *Greater* or *Less* than 12 *Hours*.

1. Let the *Sun's Declination* be 23 d. 30 m.
2. The longest *Day at London*, 16 h. 26 m.
3. The *Ascensional Difference*, 2 h. 13 m. which is half the *Time* by which the longest *Day* exceeds 12 h. viz. 4 h. 26 m. the half whereof, 2 h. 13 m. turned into *Degrees* is 33 d. 52 m.

III. *The Canons for Calculation.* By *Case XIII.* of *R. A. S. T.* As the *Tangent of the Side R S*, (the *Sun's Declination* given) 23 d. 30 m.

Is to the *Radius*:

So is the *Sine of the Side R Or*, (the *Ascensional Difference*) 33 d. 52 m.

To the *Tangent of 51 d. 30 m.* the *Latitude* sought. Whose *Complement*, 38 d. 30 m. is the *Angle R, Or S*. This

This Problem is of good use in Geography, where the Length of the Longest Day, of every Clime is given, to find what Elevation of the Pole answers thereunto. Fig. XXXVI.

P R O B. XXXI.

Latitude of a Place, or Elevation of the Pole, and the Oblique Ascension of a Star being given: To find the Cosmical, Achronical and Heliacal Rising of that Star.

THE Cosmical Rising or Setting of a Star, is that which happens in the Morning, but the Achronical in the Evening; one commencing at the Sun's Rising, the other at his Setting. The Heliacal Rising is, when a Star begins to get from under the Sun Beams; the Setting, when it begins to go under them.

But for the better seeing of the Stars, there must be a certain depression of the Sun below the Horizon, which is called The Arch of Vision; for the Light of a Star is Greater or Less. In Ptolomy's Opinion, Venus should have an Arch of 5 Degrees, tho' visible even in the Day-time, at a very great Distance.

	D.	M.
Jupiter and Mercury	10	00
	11	00
	11	30
	12	00
	13	00
	14	00
	15	00
	16	00
	17	00
	18	00

The Fixed Stars of the

First
Second
Third
Fourth
Fifth
Sixth
Nubilus

Magnitude

in this and the following Problem, the Situation of a Star is given in respect of the Horizon and Aequator together; and his situation is sought for in respect of the Sun's being in, or near, the Horizon, and in the Ecliptick.

then therefore the Oblique Ascension of a Star falls in the

First
Second
Third
Fourth

Quadrant of the Ecliptick, as in

Fig. XXXVII.
Fig. XXXVIII.
Fig. XXXIX.
Fig. XL.

Fig.
XXXVII.
XXXVIII.
XXXIX.
XL.

I. By the Coelestial Globe.

1. For the Cosmical Rising and Setting.

Rectifie the Globe to your Latitude, and bring the Place of the Sun (suppose 11 d. of ♊) to the East Part of the Horizon: Then look what Stars are about the Verge of the Eastern Semicircle of the Horizon, for all those Stars do at that time Rise Cosmically. And those Stars which touch, or are near the Rim of the Western Semicircle of the Horizon, do Set at that time Cosmically. Shall you find

May 27. { Aldebaran or the Bull's Eye, with divers other smaller Stars, } Rising, and { The Right Leg of Serpentarius, and several other smaller Stars, } Setting Cosmically.

2. For the Acronical Rising and Setting.

Rectifie the Globe to the Latitude, bringing the Place of the Sun, suppose 5 d. of *Scorpio* to the West Part of the Horizon: then shall all those Stars which you see on the Verge of the Eastern Side of the Horizon, be Rising Acronically. And all those which are about the Verge of the Western Part of the Horizon, are then Setting Acronically. And so upon the forementioned Time, you shall find

May 27. { A Star in the Whale's Tail, and several other smaller Stars } Rising, and { The Tail of the Lion, the South Ballance, and several other smaller Stars } Setting Acronically.

3. For the Heliacal Rising and Setting.

Rectifie the Globe to the Latitude, and the Quadrant of Altitude in the Zenith; then bring the given Star (suppose *Regulus*, or the *Lion's Heart*) to the East Side of the Horizon, and the Quadrant of Altitude to the West Side; then *Regulus* being a Star of the First Magnitude (by the Rule of the Ancients) may be seen when the Sun is but 12 d. below the Horizon; wherefore see what Degree of the *Ecliptick* doth cut the Quadrant of Altitude in 12 d. which you will find to be 9 d. of *Pisces*; the opposite Degree to which is 9 d. of *Virgo*. To which Sign and Degree, when the Sun cometh (which will be about the 23th of August) then will *Regulus*, or the *Lion's Heart*, Rise Heliacally.

en, for his *Heliacal Setting*, bring the *Star* to the *West Side* of the *Horizon*, and turn the *Quadrant of Altitude* to the *East*, and see what *Degree* of the *Ecliptick* is elevated upon the *Quadrant*, as the *Magnitude* of the *Star* you are to deal with require. For when the *Sun* comes to the opposite *Degree* of the *Ecliptick*, that *Star* shall *Set Heliacally*. So,

Fig.
XXXVII.
XXXVIII.
XXXIX.
XL.

{ *Pleiades* } shall Rise { *June 4.* } And Sets { *April 20.*
{ *Aldebaran* } Heliacal- { *June 26.* } Heliacal- { *April 22.*
{ *Arcturus* } ly upon { *Sept. 26.* } ly upon { *Novem. 19.*

And so for others, as in the following *Table*.

Table shewing the Time of the Year when 50 Eminent Fixed Stars do Rise, both Cosmically and Achronically.

<i>Names of the Stars.</i>	<i>Cosmical Rising.</i>	<i>Achronical Rising.</i>
<i>Marchab. Pegass</i>	<i>January 1</i>	<i>March 9</i>
<i>Right Shoulder of Aquarius</i>	<i>6</i>	<i>Febr. 14</i>
<i>Stream Star in the Wing of Pegasus</i>	<i>28</i>	<i>19</i>
<i>Following Tail of the Goat</i>	<i>February 5</i>	<i>Janu. 26</i>
<i>Right Star in the Ram's Head</i>	<i>March 2</i>	<i>April 20</i>
<i>The following Horn of the Ram</i>	<i>5</i>	<i>15</i>
<i>The former Horn of the Ram</i>	<i>April 10</i>	<i>30</i>
<i>North Tail of the Whale</i>	<i>12</i>	<i>March 19</i>
<i>Brightest of the Pleiades</i>	<i>22</i>	<i>May 10</i>
<i>The Knot in the Net of Pisces</i>	<i>May 1</i>	<i>April 25</i>
<i>North Horn of the Bull</i>	<i>15</i>	<i>June 7</i>
<i>North Eye of the Bull</i>	<i>20</i>	<i>May 12</i>
<i>The Belly of the Whale</i>	<i>23</i>	<i>March 16</i>
<i>The Bull's Eye, Aldebaran</i>	<i>27</i>	<i>May 11</i>
<i>North Horn of the Bull</i>	<i>June 5</i>	<i>June 17</i>
<i>Lower Head of Gemini</i>	<i>21</i>	<i>July 20</i>
<i>Right Foot of Gemini</i>	<i>25</i>	<i>June 5</i>
<i>Middle Star in Orion's Girdle</i>	<i>July 2</i>	<i>May 4</i>
<i>North Asellus</i>	<i>12</i>	<i>July 25</i>
<i>Resep</i>	<i>14</i>	<i>19</i>
<i>South Asellus</i>	<i>17</i>	<i>17</i>
<i>Star Dog, Porcyon</i>	<i>18</i>	<i>June 6</i>
<i>Great Dog, Palilicium</i>	<i>30</i>	<i>May 3</i>
<i>Orion's Heart</i>	<i>August 8</i>	<i>August 9</i>

Fig.
XXXVII.
XXXVIII.
XXXIX.
XL.

Names of the Stars.	Cosmical Rising.	Achronical Rising.
Lion's Back	August 10	Octob. 19
Hydra's Heart	20	June 16
Lion's Tail	22	Octob. 16
Hare's Thigh	Septem. 9	April 14
Vindemiatrix	13	Nov. 8
Arcturus	15	Decem. 13
Virgin's Girdle	19	Octob. 18
Bright Star of the Crown	28	Jan. 7
Virgin's Spike	Octob. 3	Sept. 24
Right Shoulder of Hercules	6	Jan. 9
Left Shoulder of Hercules	10	21
Head of Hercules	21	8
North Ballance	21	Nov. 17
South Ballance	23	Octob. 24
Swan's Bill	31	Febr. 16
Right Shoulder of Ophiucus	Novem. 5	January 4
Left Knee of Ophiucus	6	Decem. 6
Lower Wing of the Swan	4	March 11
Vulture's Tail	11	Jan. 27
Right Knee of Ophiucus	16	Decem. 6
Scorpion's Heart	22	Novem. 4
The Eagle	25	Decem. 31
Pegasus's Scheat	Decem. 11	March 24
Andromeda's Girdle	22	April 29
Andromeda's Head	25	6
Upper Horn of the Goat	25	Jan. 14

II. By Trigonometrical Calculation.

The Latitude of London is 51 d. 30 m.

The Oblique Ascension of the Head of Andromeda is 317 d. 26 m.
and therefore in the Fourth Quadrant of the Ecliptick. Wherefore in the Oblique-angled Spherical Triangles γ Or. A, α Or. A.

Astronomical Problems.

247

- In which there is Given
1. The Angle at γ or $\sphericalangle$, (the greatest Declination of the Sun, or Obliquity of the Ecliptick, 23 d. 30 m. Fig. XXXVII.
 2. The Angle at Or. (which is at the Interfection of the *Æquator*, and the *Horizon* of London) 38 d. 30 m. XXXVIII. XXXIX. XL.
 3. The Side Or. γ , or Or. $\sphericalangle$, (the Distance of the Oblique Ascension from the nearest *Æquinotial* Point, 42 d. 34 m.

To find { 1. The Side γ A, or $\sphericalangle$ A; for, A is that Point of the Ecliptick, which Ascends with a Star.

When therefore the Sun is in that Point, a *Star Riseth Cosmically*: But when the Sun is in that Point of the Ecliptick that is opposite to A, a *Star Riseth Achronically*.

To find { 2. The Oriental Angle at the Point A, viz. the Angle γ A Or. or $\sphericalangle$ A Or.

Moreover, in the Triangle R A $\odot$, (besides the Right Angle at R)

- There is Given { 1. The Side R $\odot$, the *Ark of Vision*, agreeing with the Star, viz. 13 d. for the second Magnitude.
2. The Angle R A $\odot$ (the same as γ A Or. or $\sphericalangle$ A Or. before found, 26 d. 50 m.

To find the Side A $\odot$, the Arch which must be added to the Point A, to find out the Point $\odot$; which, when the Sun is in, a *Star Rises Heliacally*.

The Canons for Calculation.

1. For the Sides A Or. and Or. γ . By Case III. of O. A. S. T.
The Angles Given are A Or. γ , 141 d. 30 m. and Or. γ A, 23 d. 30 m. And the Given Side is Or. γ comprehended by them, 42 d. 34 m.

The Sum of the Given Angles is 164 d. Their Difference 118 d.
The half Sum 82 d. The half Difference 59 d. And half the Given Side is 21 d. 17 m.

Then,

(1.) As the Sine of half the Sum of the Angles A Or. γ , and Or. γ A, 82 d.

Is to the Sine of half the Difference of those Angles:

So is the Tangent of half the Given Side Or. γ , 21 d. 17 m.

To the Tangent of 18 d. 38 m. Which is half the Difference of the Two unknown Sides Or. γ , and Or. A.

(2.) As

Fig.
XXXVII.
XXXVIII.
XXXIX.
XL.

(2.) As the Co-sine of half the Sum of the Angles A Or. γ , and Or. γ A. 82 d.

Is to the Co-sine of half the Difference of those Angles:

So is the Tangent of half the Given Side Or. γ , 21 d. 17 m.

To the Tangent of 55 d. 19 m. Which is half the Sum of the Two unknown Sides Or. A, and Or. γ . — The half Difference 18 d. 38 m. subtracted from 55 d. 19 m. (the half Sum) leaves 36 d. 41 m. for the Lesser Side A Or. — And the half Sum 55 d. 19 m. added to the half Difference, gives 73 d. 57 m. for the Greater Side A γ , but (because the Angle Or. opposite thereto, is above a Quadrant) add 90 d. to 73 d. 57 m. and the Sum 163 d. 57 m. is the Side A γ .

2. For the Angle Or. A γ . By Case II. of R. A. S. T.

(3.) As the Sine of the Side A Or. 36 d. 41 m.

Is to the Sine of Or. γ A, 23 d. 30 m.

So is the Sine of the Side Or. γ , 42 d. 34 m.

To the Sine of the Angle Or. A γ , 26 d. 50 m.

Then in the Triangle A R $\odot$, Right-angled at R, to find the Side A $\odot$; by Case IX. of R. A. S. T.

(4.) As the Sine of the Angle at A, 26 d. 50 m.

Is to the Sine of the Side R $\odot$ (the Angle of Vision, 13 d. it being a Star of the Second Magnitude).

So is the Radius,

To the Sine of the Side A $\odot$, 29 d. 53 m. Which added to the Side A γ , 163 d. 57 m. the Sum is 193 d. 50 m. from which subtract 180 d. there will remain 13 d. 50 m.

Wherefore, the Head of *Andromeda* Riset *Cosmically* at London, with 13 d. 50 m. of ϖ Capricorn, in which Sign and Degree the Sun is about the 24th of December: And *Achronically* with 13 d. 50 m. of $\♋$ Cancer, in which Sign and Degree the Sun is about the 25th Day of June. — The Oriental Point of Riset is at A, or the Angle γ A Or. 25 d. 50 m. And the Head of *Andromeda* Riset *Heliacally*, when the Sun is in 13 d. 50 m. of $\♈$ Aquarius, in which Sign the Sun is about the 22th of February, and Sets *Heliacally* when in the opposite Point, viz. 13 d. 50 m. of $\♌$ Leo, that is about the 26th of August.

P R O B. XXXII.

Fig.
XLI.
XLII.
XLIII.
XLIV.

The Elevation of a Pole, and the Oblique Descension of a Star given; To find the Cosmical, Acronical and Heliacal Setting of that Star.

I. By the Cœlestial Globe.

THE Practical Performance of this and the following Problems is shewed before in Prob. XXXI. and needs not be here again recited. Wherefore,

II. By Trigonometrical Calculation.

When the Oblique Descension of a Star falls

In the $\left\{ \begin{array}{l} \text{First} \\ \text{Second} \\ \text{Third} \\ \text{Fourth} \end{array} \right\}$ Quadrant of the $\left\{ \begin{array}{l} \text{Figure XLI.} \\ \text{Figure XLII.} \\ \text{Figure XLIII.} \\ \text{Figure XLIV.} \end{array} \right\}$ Then,

By Trigonometrical Calculation.

In the Oblique-angled Triangle γ Oc. D, or $\simeq$ Oc. D, there is given,

- (1.) The Angle γ Oc. $\simeq$, 23 d. 30 m.
- (2.) The Angle Oc. the Angle of the *Æquator* and *Horizon* of London, 38 d. 28 m. its Comp. to 180.
- (3.) γ Oc. or $\simeq$ Oc. the Distance of the Oblique Descension, from the nearest Part of the *Æquinotial*.

To find, 1. The Side γ D or $\simeq$ D

For, D is the Point of the Ecliptick, which descends with the Star: When therefore the Sun is in that Point, where D is, a Star Sets *Achronically*: But when the Sun is in the Point of the Ecliptick, opposite thereto, the Star then Sets *Cosmically*.

2. The Angle γ D Oc. or $\simeq$ D Oc.

Moreover, in the Triangle R D $\odot$ (besides the Right Angle at R) there is given,

- (1.) The Side R $\odot$, the Arch of Vision.
- (2.) The Angle R D $\odot$, the same with the Angle γ D Oc. or $\simeq$ D Oc. before found.

To find the Side D $\odot$, the Arch which must be taken from the Point D in the Ecliptick, to get the Point $\odot$ in the Ecliptick; to which Point, when the Sun comes, the Star Sets *Heliacally*.

E X A M.

Fig.
XLI.
XLII.
XLIII.
XLIV.

- E X A M P L E.

The Head of *Andromeda* (at London) sets *Achronically* with 26 d. 46 m. of γ : — *Cosmically*, with 26 d. 46 m. of α . — But, the Angle γ D Oc. is found to be 121 d. 32 m. and its Complement to 180 d. the Angle R D $\odot$, 58 d. 28 m. Wherefore, the Head of *Andromeda* sets *Heliacally*, when the Sun is in 11 d. 31 m. γ .

P R O B. XXXIII.

The Latitude of a Place, the Oblique Ascension or Descension of a Star, and the Longitude of the Sun being given: To know whether a Star may be seen Heliacally.

I. By the Cœlestial Globe.

TO perform this by the Globe, see before *Prob.* XXXI.

II. By Trigonometrical Calculation.

This is but the *Converse* of the Two foregoing *Problems*: And the *Spherical Triangles* upon the Globe will be the same, and therefore, in the Triangle R A $\odot$ or R D $\odot$

There is given,
besides the
Right Angle
at R,

1. The Oriental Angle at the Point A, or the Occidental at the Point D: The one being found in the Triangle γ Or. A, or α Or. A, as in the *Problem*: The other in the Triangle γ Oc. D, or α Oc. D, as in the *Problem*.
2. The Side A $\odot$, or D $\odot$; the Arch which is intercepted between the *Longitude* of the Sun, and the Ascending or Descending Point of the *Ecliptick*.

To find out the Arch R $\odot$.

Which Arch, if it be greater, or equal, to the visible Arch of the Given Star, that Star doth appear: But, if it be less, it lyes hid under the Sun's Beams.

[*Note.*] Sometimes the Oriental Angle is less than the Arch of Vision; and then (even at Midnight) such a Star cannot be seen: But, if you subtract the Arch of Vision from the greatest Depression of the *Æquator* under the *Horizon* (which is the Complement of the Latitude (or Elevation of the Pole at London 38 d. 30 m.) there remains the Northern Declination of a Degree of the *Ecliptick*, unto which the Sun must first come, before that Star will begin to appear at Midnight.

Fig.
XLI.
XLII.
XLIII.
XLIV.

As for Example. All Stars of the Fifth Magnitude, have an Arch of Vision of 16 d. therefore subtract 16 from 38 d. 30 m. the Depressi^on of the *Æquator*, and there will remain 22 d. 14 m. the Northern Declination of the Sun, viz. as much as is required that it be depressed 16 d. below the *Horizon*; that the Stars of the Fifth Magnitude may appear. But the Sun hath as great Declination in 13 d. 26 min. of *Gemini* II, and in 16 d. 3 m. of *Cancer* ☉: Therefore, whilst the Sun goes through that Arch of the *Ecliptick* (which will be between the 24th of *May*, and the 28th of *June*) the Stars of the Fifth Magnitude (are not visible at *London* (even at Midnight) because the Twilight hinders. Here needs no Canons for Calculation.

P R O B. XXXIV.

The Elevation of the Pole, and the Longitude of the Sun: To find the Beginning of the Morning, and End of the Evening, Twilight.

I. By the *Cœlestial Globe*.

IN this and the following *Problem*, the Situation of the Sun is given, in respect of the *Æquator*, and its Situation is sought in respect of the *Meridian*.

But, because, at every Cessation of the Twilight, there is commonly required a Depression of the Sun, of 18 d. below the *Horizon*.

Now, in the Latitude of 48 d. 30 m. the Sun being in the beginning of *Cancer* ☉, at Midnight, is depressed below the *Horizon*, precisely 18 d. Hence it follows, that in Places, having a greater Elevation of the Pole than 43 m. the Sun being in the *Tropick*, must (at Midnight) be depressed less than 18 d. from whence it follows, that Twilight will neither Begin nor End, but will last all Night. Therefore, to know when this *Problem* is in Use, subtract 18 d. from the greatest Depression of the *Æquator*, (that is, at *London*, 38 d. 30 m.) and there remains the Northern Declination 20 d. 30 m. (as by the *Problem* conversed): So the Sun, when he hath 20 d. 30 m. of North Declination, which he will have when he is in 1 d. 20 m. of *Gemini* II, and in 28 d. 40 m. of *Cancer* ☉: Therefore, whilst the Sun goes through that Arch of the *Ecliptick* (which is between the 12th Day of *May*, and

P p

the

the 11th Day of July) the Twilight lasts all Night at London all which Time there is no Use of this Problem there. At other Times of the Year it will be in Use, and may be solved as followeth.

II. By Trigonometrical Calculation.

Fig.
XLV.

Upon the Globe, the Triangle resolving this Problem, is the Oblique-angled Spherical Triangle $P N \odot$ in Fig. XLV. for the Declination of the Sun, Northern or Southern. In which there

- Given, {
1. The Side $P N$, the Distance of the Pole of the Equator, and the Horizon of London, 38 d. 30 m.
 2. The Side $N \odot$, the Complement of the Depreffion of the Sun under the Horizon, which the Sun must have when the Twilight Begins or Ends, viz. 72 d.
 3. The Side $P \odot$, the Complement of the Declination of the Sun, which will be

Greater	Than 90	When in Northern	Signs
Lesser	Degrees,	When in Southern	

Required, The Angle at P . Which must be turned into Time, which, being

Counted from Midnight { Shews the Beginning of the Morning Twilight.
And subtracted from 12 Hours, shews the End of the Evening Twilight.

The Canon for Calculation.

By Case XI. of O. A. S. T.

The Sum of the Three Sides of the Triangle is

The Half Sum is 99 d. 24 m. Or.

The Difference between the Half Sum 99 d. 24 m. }

And the Side opposite to P is

Then say,

(1.) As the Radius,

To the Sine of $P N$, (one of the Sides containing the enquired Angle P) 38 d. 30 m.

So is to the Sine of $P \odot$ (the other Side containing the enquired Angle P) 88 d. 48 m.

To the Sine of 38 d. 29 m.

D.

199

89

27

(2.)

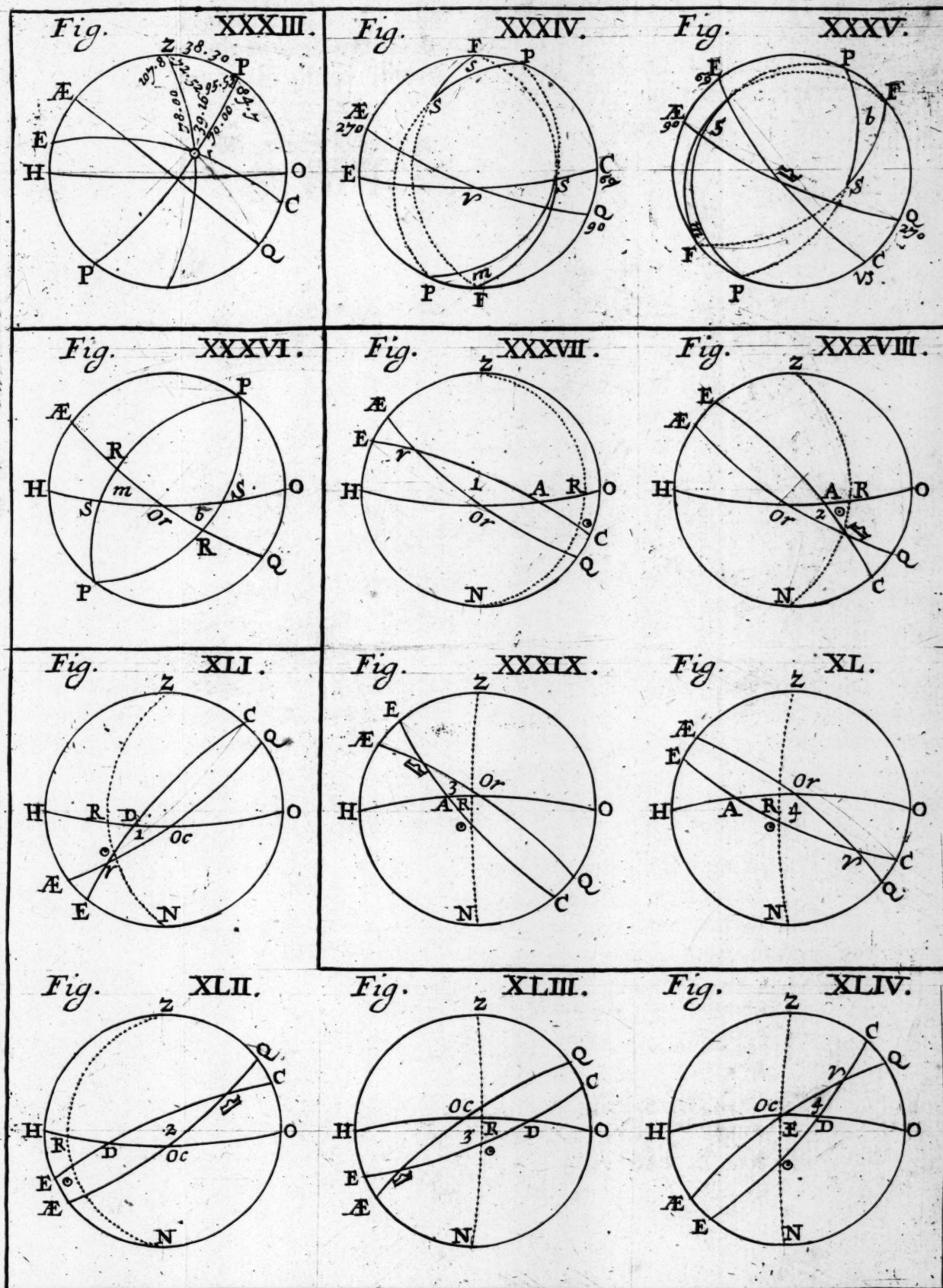


Plate XVII. From Page 216. to P. 228.

Astronomical Problems.

233

As the Sine of 38 d. 29 m. last found.
 Is to the Sine of the Half Sum of the Sides, 99 d. 14 m. (or XLV.
 89 d. 36 m.)
 Is the Sine of the Difference, 27 d. 24 m.
 To the Sine of 47 d. 41 m.
 To which Sine add the Radius, and Half thereof will be the
 Sine of 59 d. 19 m. Whose Complement 30 d. 41 m. is
 Half the Angle, at P.
 Thus the Angle at P, being 61 d. 22 m. that reduced into Time,
 makes 4 h. 6 m. *scilicet*.
 which Time the Twilight Begins in the Morning. And that
 subtracted from 12 h. leaves 7 h. 54 m. for the Time that
 Twilight will End in the Evening.

P R O B. XXXV.

The Elevation of the Pole, and the Altitude of a Star; whose Right
 Ascension and Declination are both known: To find out the Mo-
 ment of Time, and the Azimuth of the Star at that Time.

I. By the Celestial Globe.

Rectifie the Globe to the Latitude, (suppose London) 51 d.
 30 m. Bring the Place of the Sun (suppose 27 d. 54 m.
 Pisces) to the Meridian, and the Hour Index to 12, and (the
 quadrant of Altitude to the Zenith. Then turn the Body of the
 Globe about, till the Star (suppose Caput Andromeda) meet with
 24 m. (the given Altitude thereof.) Then will the Hour-
 Index point at the Time required, *viz.* 3 h. 47 m. *Morn.*

II. By Trigonometrical Calculation.

The Sun at Noon is at the highest. Take away therefore the Fig.
 Sun's Right Ascension, from the Right Ascension of the Star; and XLVI.
 Remainder being turned into Time, shews how many Hours XLVII.
 Afternoon the Star is at the highest. Therefore,

When a Star is in the { Eastern } Fig. XLVI.
 { Western } Parts, as in { Fig. XLVII.

Ancilla Mathematica.

In the Triangle P Z S there is

- Given, {
1. The Side P Z, the Distance of the Pole of the Equator, and the Horizon of London, 38 d. 30 m.
 2. The Side Z S, the Complement of the Star's Altitude, 84 d. 36 m.
 3. The Side P S, the Complement of the Star's Declination, 62 d. 42 m.

- Required, {
1. The Angle P, and must be turned into Time:
And { In Fig. XLVI. } must be { Taken from } the Time of Calculation,
 { In Fig. XLVII. } must be { Added to }
 that the Time of Observation may appear.
 2. The Angle at Z, the Azimuth of the Star, at the Time of Observation.

The Canon for Calculation. By Case XI. of O. A. S. T.

1. For the Angle at P.

(1.) As the Radius,

Is to the Sine of P Z, 38 d. 30 m. (the Complement of the Latitude.)

So is the Sine of Z S (the Complement of the Star's Altitude) 84 d. 36 m.

To the Sine of 38 d. 18 m.

(2.) As the Sine of 38 d. 18 m.

Is to the Sine of the half Sum of the Three Sides of the Triangle 92 d. 54 m. (or 87 d. 6 m.)

So is the Sine of the Difference (between the Half Sum, and the Side Z S, opposite to the Angle at P) 8 d. 18 m.

To the Sine of 61 d. 10 m. The Double whereof 122 d. 20 m. is the Quantity of the Angle at P. Which, turned into Time gives 8 h. 10 m.

2. For the Angle at Z.

As the Co-sine of the Star's Altitude Z S, 84 d. 36 m.

Is the Sine of the Angle at P, 122 d. 20 m. Or. (comp. to 180 d. 57 d. 40 m.)

So is the Sine of the Side S P, (the Comp. of the Star's Declination) 62 d. 42 m.

To the Sine of the Angle at Z, 48 d. 57 m. And that is the Star's Azimuth from the North-Eastward.

Examp.

Astronomical Problems.

255

Example in Caput Andromeda.

Fig.

XLVI.
XLVII.

In the Latitude of 51 d. 30 m. the Altitude of the Head of Andromeda, was Observed in the Eastern Part to have 5 d. 24 m. of Altitude above the Horizon: The Sun's Longitude at that time being in 27 d. 54 m. of Pisces ♊. At which D. M.
Time also the Right Ascension of the Sun is 358 04
The Right Ascension of Cap. Andromeda, found as before 357 45
To which add 360 d. it makes 717 45

From which subtract the Sun's Right Ascension 358 d. } 359 41
4 m. there remains

Which turned into Time, gives 23 h. and 57 m. which is as many Hours as the Star is coming to the Meridian after the Sun.

Thus the Angle at P, being found to be 122 d. 20 m. which (the Star being in the Eastern Part) must be subtracted from the Time of Culmination, 23 h. 57 m. and then there will remain 15 h. 47 m. for the Time of Observation Afternoon, which is 3 h. 47 m. in the Morning.

P R O B. XXXVI.

The Time of Observation, and Latitude of the Place, being given :
To find out the Altitude and Azimuth of a Star given.

I. By the Cœlestial Globe.

Rectifie the Globe to the Latitude, the Quadrant of Altitude to the Zenith, the Place of the Sun to the Meridian, and the Index to 12. Then turn the Body of the Globe about, till the Index points at the Time given, and there keep it: Then the Quadrant of Altitude laid over the Star will give you upon it the Star's Altitude; and when it cuts the Horizon, the Azimuth thereof also.

II. By Trigonometrical Calculation.

This is but the Converse of the former Problem: And the Triangle made upon the Globe the same, viz. the Triangle Z P S, in Fig. XLVI, and XLVII. In which Triangle there is

Given,

Fig.
XLVI.
XLVII.

Given, {
1. The Side Z P (the Distance of the Poles of the Equator, and Horizon of London) 38 d. 30 m.
2. The Side P S, the Complement of the Star's Declination (Cap. Andromeda) 62 d. 42 m.
3. The Angle S P Z (the Time between the Time of Observation, and the Time of the Stars next Culmination, viz. 8 h. 10 m) which is 122 d. 20 m.

Required, {
1. The Side Z S, (the Complement of the Star's Observed Altitude).
2. The Angle at Z (the Azimuth of the Star, from the North Part of the Meridian.)

The Canons for Calculation. By Case IX. and Case I. of O. A. S. T.

1. For the Angle at Z.

(1.) As the Sine of half the Sum of the Sides Z P and Z S, 50 d. 36 m.

Is to the Sine of their half Difference, 12 d. 6 m.

So is the Co-tangent of half the Given Angle, Z P S, 61 d. 10 m.

To the Tangent of 8 d. 30 m. the half Difference of the Two Angles, Z and S.

(2.) As the Co-sine of half the Sum of the Given Sides, 50 d. 36 m.

Is to the Co-sine of their half Difference, 12 d. 6 m.

So is the Co-tangent of half the given Angle Z, 61 d. 10 m.

To the Tangent of 40 d. 22 m. Which is the half Sum of the Two Angles.

Then 8 d. 30 m. { Added to } D. M. { D. M. } For the { Z.
30 m. { Subtracted from } 40 22 { 48 52 } Angle { S.
31 52

So that the Azimuth of the Star is 48 d. 52 m. from the North Part of the Meridian.

2. For the Side Z S.

(3.) As the Sine of the Angle at Z, (last found) 48 d. 52 m.

Is to the Sine of the Side SP (the Complement of the Star's Declination) 62 d. 42 m.

So is the Sine of the Angle at P, 122 d. 20 m. (or 57 d. 40 m.)

To the Sine of 84 d. 36 m. for the Side Z S. Whose Complement, 5 d. 24 m. is the Altitude of the Star at the Time of Observation.

Astrono-

Astronomical Problems

Relating to

ASTROLOGY.

INTRODUCTION.

Astrology consisteth principally of Two Parts, viz. the one *Mathematical*, as is the *Astronomical Part*; the other *Judiciary*, as is the *Astrological Part*.

The *Mathematical Part* teacheth how in a *Scheme* or *Figure* (as they call it) to represent the *Face* of the Heavens in *Plano*, for any Hour of the Day or Night, at all Times of the Year, and in all Parts of the World.

The *Astrological Part* teacheth how (from the Sight of the said Position of the *Scheme* or *Figure* of the Heavens at the Time of its Erection) to give a determinate Judgment of what was demanded upon that Erection of the *Scheme* or *Figure*; as of *Annual Revolutions*, *Elections*, the *Nativity* of a Person, &c.

The principal Authors that have given their Opinions concerning the dividing of the Heavens into 12 *Mansions* or *Houses*, are, 1. Ptolomy, 2. Alcabitius, 3. Campanus, and 4. Regiomontanus: Which last Way is now generally received and practiced among the *Astrologers* of these Times, and by them termed the *Rational Way* of Regiomontanus.

Now, because (as I said before) that the Erection of a *Figure* of the Heavens is the *Mathematical Part* of *Astrology*, I shall therefore shew how by the *Globes* to erect a *Figure* of the Heavens according to the *Rational Way* of Regiomontanus.

P R O B.

P R O B. I.

How to Erect a Figure of the Heavens in the Latitude of London, 51 d. 30 m. N. for the 10th Day of March, at 49 m. after 9 in the Forenoon; at which Time the Sun entred into the first Scruple of Aries, in the Year 1675.

THE Heavens are divided in XII Houses or Mansions, by 12 Semicircles of Position; for which Purpose, to some Globes, there is made one of Brass, which is fixed in the Intersections of the Meridian and Horizon, by the Elevation or Depression whereof the Heavens may be divided into Parts or Houses through each Degree of the Ecliptick.

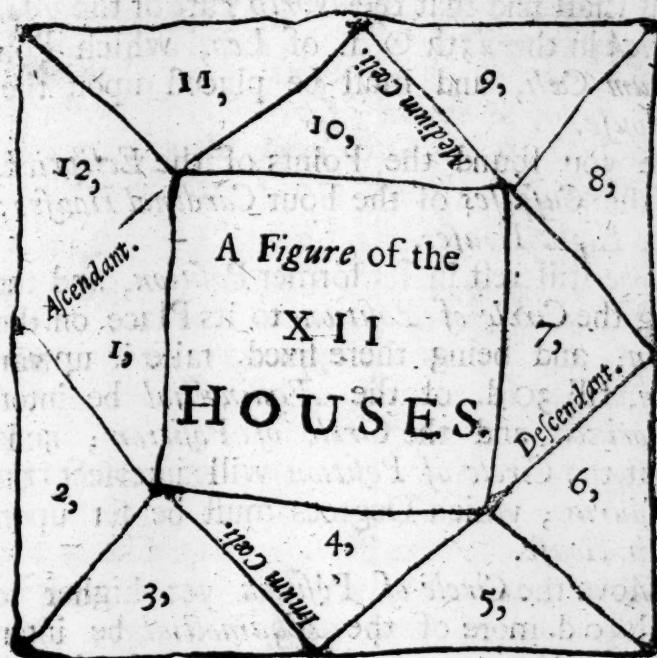
Of these XII Houses, or Mansions of Heaven, Four are called Cardinal, as, (1.) The Horoscope, or Ascendent, or Cuspis of the First House. (2.) The Medium Cæli, or Angle of the South, or Cuspis of the Tenth House. (3.) The Descendent, or Angle of the West, or the Cuspis of the Seventh House. (4.) The Imum Cæli, or the Angle of the North, or the Cuspis of the Fourth House.

Regiomontanus divides the Heavens into XII Houses according to his Way, by the Circle of Positions passing through every 30th Degree of the Æquinoctial, and cutting the Ecliptick at several Points, which are the Cuspises of the several Houses:—So that when the Globe is set to the Latitude, and the Hour-Wheel Rectified and brought to the Given Hour, you have the Cuspises of the Four Cardinal Houses: For,

The Degree of the Ecliptick cut by the	{	East Side of the Horizon,	{	Gives the Cuspis of the	{	First	{	Houses
		South Part of the Meridian,				Tenth		
		West Side of the Horizon,				Seventh		
		North Part of the Meridian,				Fourth		

The Cuspises of the other 8 Houses are found by the Motion of the Circle of Position, as shall be shewed by and by.

The Houses are denominated by 1, 2, 3, 4, &c. to 12, from the Ascendent downwards to the Imum Cæli, up again to the Descendent, and again by Medium Cæli, down to the Ascendent. As in the following Scheme.



Let this suffice for *Definition*, and now we will come to the *Practise* by the *Globes*.

First, To the Day proposed, the 10th of *March*, find (by the *Astronomical Problem*) the Sun's Place in the *Ecliptick* at Noon, which you shall find to be in 0 d. 5 m. of *Aries*.

Secondly, Set the *Globe* to the *Latitude* 51 d. 30 m.

Thirdly, Bring the Sun's Place at Noon (0 d. 5 m. of *Aries*) to the *Meridian*.

Fourthly, Turn the *Globe* about till the *Hour-Index* point to the *Hour* given, viz. to 49 m. after 9 in the *Morning*.

Lastly, The *Globe* being fixed in this *Position*, you shall find that the *East-Semicircle* of the *Horizon* doth cut the *Ecliptick* in 0 d. 29 m. of *Cancer*, which is the *Sign* then *Ascending*, and must be placed upon the *Cuspis* of the *First House*.

Then cast your Eye upon the Intersection of the *South Part* of the *Meridian* and the *Ecliptick*, and there you shall find the *Ecliptick* cut by the *Meridian* in 25 d. of *Aquarius*, and that Point of the *Ecliptick* is then in the *Medium Caeli*, and must be set upon the *Cuspis* of the *Tenth House*.

Also, you find that the *West Semicircle* of the *Horizon*, cuts the *Ecliptick* in 00 d. 29 m. of *Capricorn*; which Point is then *Descending*, and must be placed upon the *Cuspis* of the *Seventh House*.

Q q

Lastly,

Lastly, You shall find that the *North Part* of the *Meridian* doth cut the *Ecliptick* in the 25th $\odot$ d. of *Leo*, which Point is then upon the *Imum Cæli*, and must be placed upon the *Cuspis* of the *Fourth House*.

Thus have you found the Points of the *Ecliptick*, which do occupy the *Cuspises* of the *Four Cardinal Houses*: Now for the other *Eight Houses*.

Let the *Globe* still rest in its former *Position*, and then,

First, Bring the *Circle of Position* to its Place on the *East Side* of the *Horizon*; and being there fixed, raise it upwards towards the *Meridian*, till 30 d. of the *Æquinoctial* be intercepted between the *Horizon* and the *Circle of Position*; and then you shall find that the *Circle of Position* will intersect the *Ecliptick* in 20 d. of *Taurus*; which Degrees must be set upon the *Cuspis* of the *Twelfth House*.

Secondly, Move the *Circle of Position* yet higher towards the *Meridian*, till 30 d. more of the *Æquinoctial* be intercepted between it and the *Horizon*, (in all 60 d.) and when in doth so, you shall find the *Circle of Position* will cut the *Ecliptick* in 36 d. of *Pisces*; which Point must be set upon the *Cuspis* of the *Eleventh House*.

The *Meridian* gives the *Cuspis* of the *Tenth House* in 25 d. of *Aquarius*, as before.

Thirdly, Move the *Semicircle of Position* from the *East Side* of the *Horizon* to the *West Side*, and move it downwards from the *Meridian*, till 30 d. of the *Æquinoctial* be intercepted between the *Meridian* and *Circle of Position*; and then you shall find the *Circle of Position* will intersect the *Ecliptick* in 7 d. of *Aquarius*; which Point must be set upon the *Cuspis* of the *Ninth House*.

Fourthly, Move the *Circle of Position* yet lower by 30 d. i. e. 60 d. from the *Meridian* downwards, and then you shall find the *Position-Circle* to cut the *Ecliptick* in 21 d. of *Capricorn*, which Point must be set upon the *Cuspis* of the *Eighth House*.

The *Descendant* or *Cuspis* of the *Seventh House*, is the Intersection of the *West Side* of the *Horizon* and *Ecliptick*, which is in 00 d. 29 min. of *Capricorn*, as before.

And thus have you found the *Cuspises* of the *Four Houses* above the *Horizon*, beside the *Ascendant* and the *Medium Cæli*, viz. of the 12, 11, 9 and 8 Houses. Now the *Cuspises* of the

the Four other Houses under the Earth have the same Degrees of the opposite Signs upon them. For,

		House.		House.	
20 d. $\varnothing$	$\left. \begin{array}{l} \text{Being upon the} \\ \text{Cuspis of the} \end{array} \right\}$	$\left. \begin{array}{l} 12 \\ 11 \\ 9 \\ 8 \end{array} \right\}$	20 d. m	$\left. \begin{array}{l} 6 \\ 5 \\ 3 \\ 2 \end{array} \right\}$	
26 d. $\times$			26 d. μ		will be on the
7 d. $\equiv$			7 d. Ω		Cuspis of the
21 d. $\wp$			21 d. Ω		

For the Six Signs

Aries, Taurus, Gemini, Cancer, Leo, Virgo,
are opposite to

Libra, Scorpio, Sagitt. Capric. Aquar. Pisces.

And this is the manner how (by the Globe) to erect a Figure according to the (reputed) Rational Way of Regiomontanus.

Now if you would insert the Places of the Planets into your Figure, (for it is them that the *Astrologer* principally giveth Judgment by) your best Way will be to have Recourse to some good Ephemeris, or Calculate them from Astronomical Tables).

Having in the foregoing Precepts shewed how to erect a Scheme (or Figure of the Heavens) by the Globe, I shall now (to retain the same Method throughout this whole Treatise) shew how to effect the same Trigonometrically; the which I shall do in the Three following PROBLEMS.

P R O B. I.

The Time of the Day, and Latitude of the Place given: How to Erect a Scheme or Figure of the Heavens for that Time and Place.

By Trigonometrical Calculation.

YOU must first find (either by Tables already Calculated, Fig. 1. or by the foregoing *Astronomical Problems*) the true Place XLVIII. of the Sun in Longitude; and also, his Right Ascension, both for the Moment of Time given.

Then turn the Time given, (counted from the preceding Noon) to Degrees and Minutes of the *Aequator*; and add them (being turned into Degrees and Minutes) to the Right Ascension of the Sun before-found; the Agregate (casting away 360 d. if

Q q a

need

Fig. XLVIII. need be) is the *Right Ascension* of the *Mid-Heaven*: To which you must find what Degree of the *Ecliptick* doth answer; for that will be the *Sign* of the *Xth House*.

Then, To the *Right Ascension* of the *Mid-Heaven*, add 30 d. then 60 d. again 90 d. and 120 d. and 150 d. And thus you will find the *Oblique Ascension* of the *Xlth*, *XIth*, and *Ist House*; or of the *IId* and *IIId Horoscope*.

By the *Erection* of the *Circle of Position* above the *Horizon*. at every 30 Degrees, there are made several *Spherical Triangles*, as $H \text{ } \text{Æ} \text{ } 11$, &c.

In which Triangle (besides the *Right Angle* at Æ .) there is

Given, { 1. The Side $H \text{ } \text{Æ}$, (the *Elevation* of the *Æquinoctial* above the *Horizon* of *London*) 38 d. 30 m.
2. The Side $\text{Æ} \text{ } 11$, 30 d. (and $\text{Æ} \text{ } 12$, 60 d. &c.

To find, The Angle $\text{Æ} \text{ } 11 \text{ } H$, — $\text{Æ} \text{ } 12 \text{ } H$, &c. By *Case XIII* of *R. A. S. T.*

As the Tangent of $\text{Æ} \text{ } H$, 38 d. 30 m.

Is to the Radius:

So is the Sine of $\text{Æ} \text{ } 11$, 30 d.

To the Tangent of 32 d. 9 m.

Whose Complement 57 d. 51 m. is the Quantity of the Angle $H \text{ } 11 \text{ } \text{Æ}$.

			D. M.		
So will	$\left\{ \begin{array}{l} H \text{ } 11 \text{ } \text{Æ} \\ H \text{ } 12 \text{ } \text{Æ} \\ H \text{ } 1 \text{ } \text{Æ} \\ H \text{ } 2 \text{ } \text{Æ} \\ H \text{ } 3 \text{ } \text{Æ} \end{array} \right\}$	Be found	$\left\{ \begin{array}{l} 57 \text{ } 51 \\ 42 \text{ } 32 \\ 7 \text{ } 10 \\ 42 \text{ } 32 \\ 57 \text{ } 51 \end{array} \right\}$	to which	$\left\{ \begin{array}{l} Q \text{ } 3 \text{ } 0 \\ Q \text{ } 2 \text{ } 0 \\ Q \text{ } 1 \text{ } 0 \\ Q \text{ } 12 \text{ } 0 \\ Q \text{ } 11 \text{ } 0 \end{array} \right\}$
the An-					
gle					
		to be		the Angle	

These Angles being once found will be always in Use in the *Latitude* of 51 d. 30 m. For that they shew the Quantity of the Angle, that the *Ecliptick* makes with every *Circle of Position*.

Moreover,

In the Triangle $11 \text{ } \gamma \text{ } X$, — $12 \text{ } \gamma \text{ } Y$, &c. there is

Given, { 1. The Angle $X \text{ } 11 \text{ } \gamma$, which is a contiguous Angle to the before-found Angle $H \text{ } 11 \text{ } \text{Æ}$.
2. The Side $\gamma \text{ } 11$, the Complement of the *Oblique Ascension* of the *Xlth House*, to a whole Circle, or 360
3. The Angle $11 \text{ } \gamma \text{ } X$, 23 d. 30 m.

To find the Side γ X, or the Distance of the Sign of the XIth House from the Vernal Equinox. By Case IV. of R. A. S. T. XLVIII.

But it will be sufficient to count the Signs of the Oriental Houses: For those which are opposite to them, (as the IVth to the Xth, — the Vth to the IXth, &c.) are counted as many Degrees and Minutes as the others; but of an Opposite Sign.

Example, Let it be required to Erect a Scheme of the Heavens, (in the Latitude of London, 51 d. 30 m.) for the 6th Day of March, the 16 h. 5 m. before Noon.

The Place of the Sun then was in 27 d. 54 m. κ .

The Sun's Right Ascension, 358 d. 4 m.

The Time 16 h. 5 m. turned into Degrees, gives 241 d. 15 m.

Which added to the Right Ascension of the Sun, make 599 d. 19 m. from which subtract the Circle (360 d.) there remains 239 d. 19 m. for the Right Ascension of the Mid-Heaven. To which answer 1 d. 27 m. of π , which is the Sign of the Xth House. And from thence I collect the Oblique Ascension of the rest of the Houses as followeth, viz.

		D. M.	
The Oblique Ascension of the	XI	House to be	The Sign thereof is
	XII		
	I		
	II		
	III		
		269 19	15 15 π
		299 19	1 09 ν
		329 19	29 27 ν
		359 19	28 46 κ
		29 29	11 45 δ

But because the IVth House is opposite to the Xth House, it will have as many Degrees and Minutes upon the Cuspis thereof, but of the opposite Sign, viz. π , and the Vth the same as the XIth, viz. 15 d. 15 m. of the opposite Sign, π . The VIth 1 d. 00 m. of δ . The VIIth 29 d. 27 m. of δ . The VIIIth 28 d. 46 m. of ν . The IXth 11 d. 45 m. of ν . As may be seen in the Erected Fig. XLIX.

PROB. II.

How to Direct any Point of an Erected Scheme to any other Point.

1. THE Point that is Directed is called, otherwise, the First Place, or Significator: But that to which we direct it, the Second Place, or the Promissor: But the Significators are directed whither they tend: Directed, that is, Going according to the Series of the Signs; but Retrograde in Antecedens, or against the Succession of the Signs.

2. Let

2. Let the *Position* made by any direct *Significator*, or by the *Promissor* to which the *Retrograde* is *Directed*, be called the *Horizon* of a *Star*: And it is either a right one, passing through both the *Poles*, as the *Meridian*; or an *Oblique* one, having One of the *Poles* elevated.

3. The *Oblique* one is, sometimes, the same as the *Horizon* of the *Place*; and consequently, the *Elevation* of the *Pole* is the same: But, when it is *different* (which most commonly happens) you must first enquire how much One of the *Poles* is *Elevated* above the *Horizon* of the *Star*: But if a *Star* be

Fig. L.
I. II.
III. IV.

Oriental, $\left\{ \begin{array}{l} \text{With Decli-} \\ \text{nation,} \end{array} \right. \left\{ \begin{array}{l} \text{North,} \\ \text{South,} \end{array} \right. \left\{ \begin{array}{l} \text{North,} \\ \text{South,} \end{array} \right. \text{As in } \left\{ \begin{array}{l} \text{Fig. I.} \\ \text{Fig. II.} \\ \text{Fig. III.} \\ \text{Fig. IV.} \end{array} \right.$

Occidental, $\left\{ \begin{array}{l} \text{North,} \\ \text{South,} \end{array} \right.$

In the Triangle P O S, or P H S, for the Situation of the *Star* above or below: There is

Given, $\left\{ \begin{array}{l} 1. \text{ The Side P O, or P H, the } \textit{Elevation} \text{ of the } \textit{Pole} \text{ at } \textit{London}, 51 \text{ d. } 30 \text{ m.} \\ 2. \text{ The Side P S, the Complement of the } \textit{Star's Decli-} \\ \text{nation.} \\ 3. \text{ The Angle S P O, comprehended between them} \\ \text{(whose } \textit{Measure} \text{ is the } \textit{Arch} \text{ of the } \textit{Æquator}, \text{ which} \\ \text{is intercepted between the Sides P O (P H) P S:} \\ \text{One of which being produced, shews the } \textit{Right As-} \\ \text{scension of the } \textit{Star}; \text{ the other, the } \textit{Right Ascension} \\ \text{of } \textit{Imum Cæli}. \end{array} \right.$

Required, The Angle P O S, or P H S. By *Case III.* of R. A. S. T.

Again, In the Triangle R P O, or R P H, (which are in different *Hemispheres*, when O or H (before found) are greater than a *Quadrant* or 90 d.)

There is Given, $\left\{ \begin{array}{l} 1. \text{ P O or P H, the } \textit{Elevation} \text{ of the } \textit{Pole}. \\ \text{besides the} \\ \text{Right Angle} \\ \text{at R,} \end{array} \right. \left\{ \begin{array}{l} 2. \text{ The Angle O or H, before found, (which} \\ \text{if it exceeds a } \textit{Quadrant}, \text{ is Complement to} \\ 180 \text{ d.} \end{array} \right.$

To find R P, The *Elevation* of the *Pole* above the *Horizon* of the *Star*. By *Case III.* of R. A. S. T.

4. Having found the *Elevation* (by the Third foregoing) then

Astronomical Problems.

265

when the *Horizon* hath any Star that is { Oriental, the Asc. }
 { Occidental, the } Obl. Fig. L.
 { Descension } L. II.
 III. IV.

both of the *Significator* and *Promissor*.

5. Having { *Significators Direct,*
 found out the { *Promissor, to whom the* } *Ascension* or *Descension*
 { *Retrograde is Directed,* }
 for the *Horizon* of the *Star*: Let the *Right* one, or the *Oblique*
 one, be subtracted from the like *Ascension* or *Descension* of the
 other *Star* (adding 360 d. if Need be) the *Remainder* is the *Ark*
 of *Direction* sought for.

P R O B. III.

The *Ark of Direction* of any *Significator* being given; To find
 how far it will reach.

THIS is but the *Converse* of the foregoing: Therefore look
 for the *Elevation* of the *Pole* above the *Horizon* of a *Direct*
Significator, as before; and at that *Elevation* you will get (by the 3d.)
 the *Oblique* { *Ascension* } of the same *Significator* { *Oriental,* }
 { *Descension* } { *Occidental,* }
 to which { *Ascension* } add the given *Ark of Direction*, so will
 { *Descension* }
 you get the *Oblique* { *Ascension.* } To which find what *Degree* of
 { *Descension.* }
 the *Ecliptick* answers (being counted from the *Elevation* of the
Pole above the *Horizon* of the *Significator*) which is the very
Place to which the *Ark* is *Directed*, and to which the *Significator*
 will come.

But subtract the *Ark of Direction* given, from the *Right Ascen-*
tion of the *Retrograde Significator*, and there will remain the
Degree of the *Æquator*, which is found out by the *Retrograde*
Significator, by *Direction*, tending to the *Promissor*: The *Circle*
of Position passing through this *Degree* (which ought well to be
 observed) will be the *Horizon* of the *Promissor*: Above which,
 at the *Elevation* of the *Pole*, as before: For, if that *Degree* fall
 in the { *North, Fig. I.* } the { *Eastern* } Part of Heaven, the
 { *South, Fig. II.* } { *Western* }
Significator inclining unto the { *North, Fig. III.* } in the Triangle
 { *South, Fig. IV.* } POS, or PHS.
 For

Fig. L.

I. II.

III. IV.

For the Situation of the Degree observed, whether Above or Below.

From the given Angles, as above, (the Angle P being counted in the *Æquator*, not from the *Right Ascension* of the *Significator*, but from that Degree which hath, oftentimes, been taken notice of, as that at the *Medium* or *Imum Cæli*) I seek for the Angle O or H.

And moreover, in the other *Triangle*, R P, the very *Elevation* of the *Pole* above the *Horizon* of the *Promissor*: At this *Elevation* of the *Significator*, if you observe that Degree

Oriental } you will gain the *Oblique* { *Ascension.* }
Occidental } { *Descension.* } From which

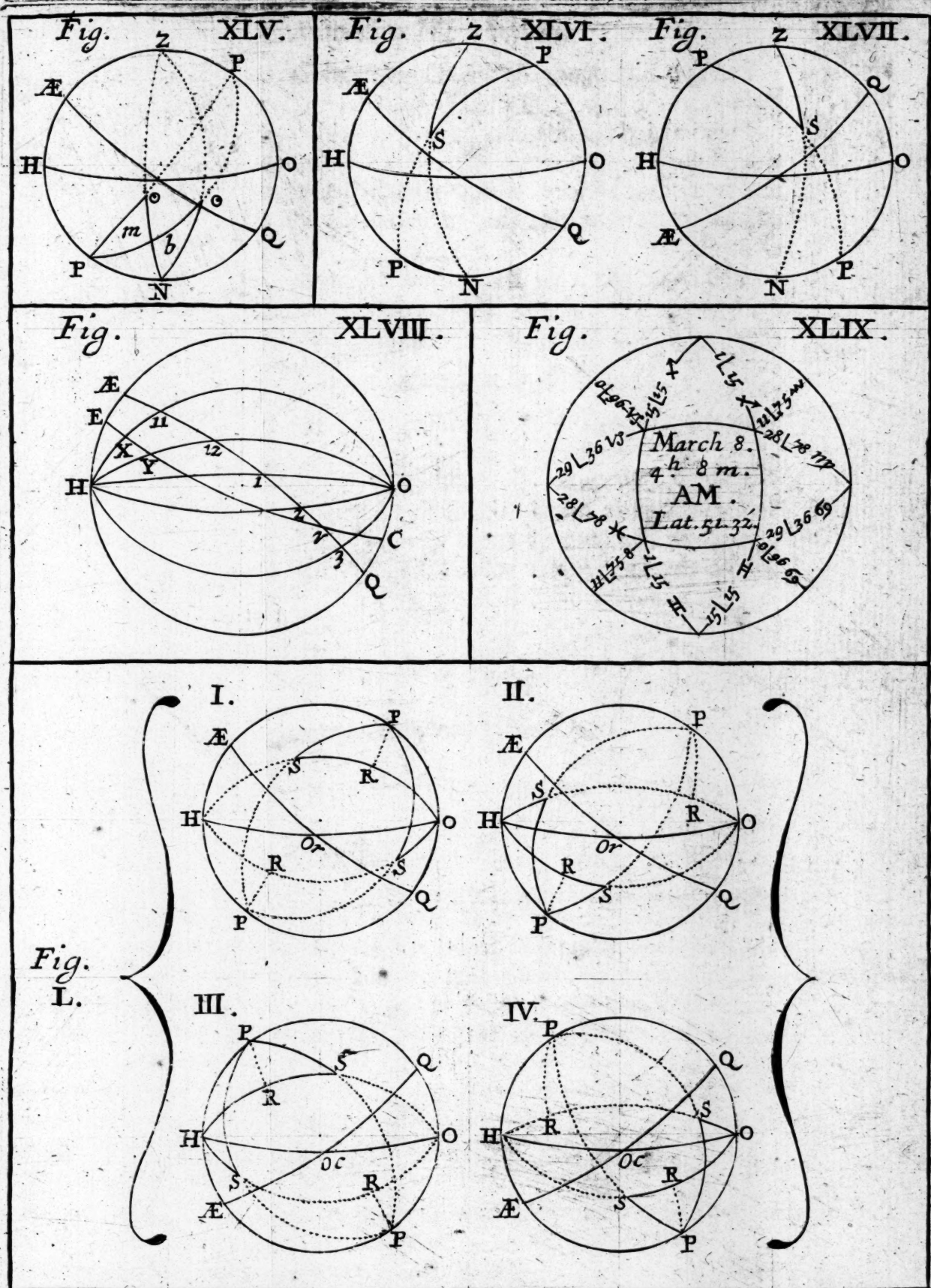
take the given *Ark of Direction*, and the *Remainder* will be the

Oblique { *Ascension,* }
 { *Descension,* } to which the Degree of the *Ecliptick* an-

swers, (but it must be counted from the *Elevation* of the *Pole* above the *Horizon* of the *Promissor*) and is the very *Place*, to which the *Significator* will come, by this *Ark of Direction* given.

The End of the Astronomical Problems.

A N.



ANCILLA MATHEMATICA.

V E L,

Trigonometria Practica.

SECTION V.

O F

SCIOGRAPHIA,

O R

DIALLING.

CHAP. I.

Of Dialling in general.

OF the *Mathematical Sciences*, I know none more Ingenious and Useful then This of the Description of *Sundials* upon all sorts of *Plains* howsoever situate : Neither is there any thing that draws more Admiration from all knowing Men; then to see streight Lines drawn upon a *Plain* at *equal Distances* to measure out exactly the *Equal Divisions* of the *Day* of an *Artificial Day* : For, although the Sun appears in different places of Heaven, according to the different *Seasons* of *Year* ; yet, the same *Streight Lines*, do still determine the same *Hours*, at all those different *Seasons*. And as this *Science* is of such

A a a

Excellency

Excellency and Use, I shall in this *Section*, briefly shew (and that by an *Artifice* not usually practiced) how such *Hour-Lines* may be described upon all *Plains* in any part of the *World*: And that by *Trigonometrical Calculation*, as well as by projecting the *Circles* of the *Globe* upon a *Plain*.

Now, All *Plains* have a particular respect to some *Point*, in the *Latitude* of that *Place* in which they stand: Wherefore, some particular *Place* must be assigned for *Exemplifying* the same, in all the *Varieties*. We will therefore take *London*, the *Metropolis* of *England* (which is situate in 51 deg. 30 m. of *North Latitude*) for our *Examples* following.

CHAP. II.

Of the Diversity of Plains, upon which Hour-lines may be described.

ALL *Plains* upon which *Sum-Dials* may be made, in any *Latitude* or *Place* of the *World*, are situate either

Parallel

Perpendicular

Oblique

Place (or *Country*) in which they are made. And, of these severally.

I. A *Plain* that lies *Parallel* (or *Level*) to the *Horizon*, Is called an *Horizontal Plain*, for the *Latitude* of that *Place* in which it is to stand.

II. Of *Plains* that are *Perpendicular*, or *Erect*, to the *Horizon* there are several *Varieties*. For, of such *Plains*, if the *Face* thereof doth directly behold the

East

West

North

South

} Point of the *Horizon* of the *Place*

wherein they stand, They are then called,

East

West

North

South

} *Erect, Direct Plains.*

But if such *Erect Plains* do not behold the *Direct East, West, North* or *South* Points of the *Horizon*, they then will behold either

The { *South-East*
South-West
North-East
North-West } And then they are called

North { *Erect Plains* ; *Declining* } East
 or { } or
 South { } West.

III. Of *Plains* that lie *Obliquely* to the *Horizon* : (such as the *Roofs* of *Houses*, the *Cooping* of *Walls*, &c.) are called *Reclining Plains*. And if such *Plains* do behold the direct

East {
 West {
 North { Points of the *Horizon* : They are then
 South {

called *Direct East, West, North* or *South Plains* : *Reclining* from the *Zenith* of the Place in which such Plain stands.

But if such *Reclining Plain* doth not respect the true *East, West, North*; or *South* Points, then they will lie open either to the

South-East {
 South-West {
 North-East { And then they are called
 North-West {

South {
 or { *Reclining Plains* : *Declining* } East
 North { } or
 { } West.

And these are all the *Varieties* of *Plains*, upon which *Hour-Lines* may be described.

Only note,] That all *Reclining Plains* whatsoever; whether *Direct* or *Declining*, have *Two Faces*; the *Upper-Face*, which beholds the *Zenith* of the Place, is called the *Reclining Plain* : And the *Under-Face*, which beholds the *Horizon* of the Place, is called the *Incliner* : And one and the same *Dial* serves for both Places.

C H A P. III.

How to find the Situation of any Plain, in respect of Declination and Reclination, in any Latitude.

I. To find the Reclination.

Fig. LI. Definition. THE Quantity of the Reclination of a Plain, is the Arch of that Vertical or Azimuth Circle which is perpendicular to the Reclining Plain, comprehended between the Zenith of the Place and the Plain.

To find which, let A B C D represent such a Reclining Plain : Draw, first, thereon, by help of a Ruler and Quadrant, a Right Line G H, parallel to the Horizon of the place ; which shall be the Horizontal Line of the Plain. Cross this Line G H, with another Right Line K S at Right-Angles to it ; which Line K S shall be the Vertical Line of the Plain.

To this Line K S, apply a straight Ruler K L : And to that end of it which lyeth clear of the Plain, as at L ; apply a Quadrant O E P, having a Thrid and Plummet hanging from the Centre. Then see what number of Degrees of the Quadrant are contained between O and E ; for so much doth that Plain Recline from the Zenith of the Place ; and is the Reclination of the Plain.

II. To find the Declination.

Definition. THE Declination of a Plain, Is an Arch of the Horizon comprehended between the Pole of the Plain, and the Meridian of the Place. — Or, It is the distance of the Plain it self, from the Prime Vertical Circle, or Azimuth of East or West.

To find out the Declination of any Plain, there are required two Observations. to be made by the Sun, both at the same Time, as near as may be. — The First, Of the Horizontal Distance of the Sun, from the Pole of the Plain. — And the Second, Of the Sun's Altitude.

I. To find the Horizontal Distance.

Apply one Edge of the Quadrant to the Horizontal Line of the Plain, so that the other may be Perpendicular to it ; and let the Limb of the Quadrant be towards the Sun. The Quadrant thus applied

applied to the *Plain*, and held *Level* (as near as you can conjecture) hold up a *Thrid* and *Plummet* at full Liberty, near the *Limb* of the *Quadrant*; so that the *Shadow* of the *Thrid* may pass through the *Centre* and *Limb* of the *Quadrant*: And then observe the *Degrees* cut by the *Shadow* of the *Thrid* in the *Limb* of the *Quadrant*; and number them from that side of the *Quadrant*, that standeth *Square* or *Perpendicular* to the *Plain*: For those *Degrees* are the *Horizontal Distance* sought for.

II. To find the Sun's *Altitude*.

Hold up a *Quadrant* with both your Hands, turning the *Left* side of your *Body* to the *Sun*; then raise up, or depress the *Quadrant*, so held, till the *Sun* shining through the *Hole* in that *Sight* which is nearest the *Centre*, do cast its Beam of *Light* upon the *Hole* in the other *Sight* farthest from the *Centre*: And at such time mark exactly what *Degrees* of the *Quadrant* are cut by the *Thrid*, for those *Degrees* are the *Sun's Altitude* at that time.

The *Horizontal Distance*, and the *Sun's Altitude* thus Observed, at the same instant (as near may be) will help you to the *Plain's Declination*; by the *Rules* following. For,

- I. By having the *Sun's Altitude*, you may find the *Sun's Azimuth*, as in *PROBL. IX.* of the *Use of the Cælestial Globe in Astronomy*: And by *CASE XI.* of *O. A. S. T.* Then,
- II. When you make your *Observation* of the *Horizontal Distance*; Mark whither the *Shadow* of the *Thrid* did fall between the *South*, and that *Side* of the *Quadrant* which was *Perpendicular* to the *Plain*:

First. If the *Shadow* fall between them: Then the *Sun's Azimuth* from the *South*, and the *Horizontal Distance* added together, do give the *Declination* of the *Plain*: And (in this Case) the *Declination* is unto the same *Coast* with the *Sun's Azimuth*; that is, *Eastward*, if the *Observation* were made in the *Forenoon*; or *Westward*, if in the *Afternoon*.

Secondly. If the *Shadow* fall Not between them:

Then, The *Difference* between the *Sun's Azimuth* and *Horizontal Distance* is the *Declination* of the *Plain*: And in this Case)----If the *Azimuth* be the *Greater* of the two, then the *Plain Declines* to the same *Coast* whereon the *Sun* is; But if the *H. Distance* be the *Greater*; then the *Plain Declines* the contrary *Coast*.

And

Fig. LI.

And here *Note*] That the *Declination* thus found, is always accounted from the *South*; and that all *Declinations* are counted from either *North* or *South*, towards either *East* or *West*: And must never exceed 90 Degrees.

- I. If therefore, The Degrees of *Declination* do exceed 90 deg. you must take the residue of that Number to 180 deg. and that shall be the *Plain's Declination* from the *North*.
- II. If the Degrees of *Declination* exceed 180; then the *Excess* above 180, gives the *Plain's Declination* from the *North*, towards that *Coast*, which is *Contrary* to the *Coast* whereon the *Sun* was, at the time of *Observation*.

C H A P IV.

How Hour-Lines may be described upon an Horizontal Plain in any Latitude; viz. of London, 51 d. 30 m.

I. By the Globe.

Elevate the *Globe* to the *Latitude* of the place for which you would make your *Dial*, (suppose for *London*, in the *Latitude* of 51 deg. 30 min.) Then bring the *Vernal Equinoctial Colure* (which is the first point of *Aries* also) to the *Meridian*, and (if you will) the *Index* of the *Hour-Circle* to 12. This done,

1. Turn the *Globe* about *Westward*, till the *Hour-Index* points at 1 a Clock, or rather [till 15 degrees of the *Equinoctial* come to be just under the *Meridian*] and there keeping the *Globe*, look upon the *Horizon* how many degrees thereof are cut by the *Equinoctial Colure*; which you shall find to be 11 deg. 30 min. which

		d. m.
Latitude 51. 30		
12		00.00
11	1	11.50
10	2	24.20
9	3	38. 3
8	4	53.35
7	5	71. 6
6		90. 0

set down in a little Table, as you see here is done; for this 11 deg. 30 min. is the distance that the *Hour-lines* of 11 and 1 a clock are distant from the *Meridian* upon the *Dial Plain*.

2. Turn the *Globe* more *Westward*, till 30 degrees of the *Equinoctial* comes to the *Meridian*, and then see what degrees of the *Horizon* are cut by the *Equinoctial Colure*; which you will find to be 24 deg. 20 min. which note down in a Table as before, for that is the *Hour distance* of 10 and 2 a clock from the *Meridian*.
3. Turn

3. Turn the *Globe* still more *Westward*, till 45 degrees of the *Equinoctial* come to the *Meridian*; and then shall the *Equinoctial Colure* cut 38 deg. 3 min. of the *Horizon* counted from the *Meridian*, which is the distance of 9 and 3 a clock.

Do thus with the other hours of 8 and 4, of 7 and 5, and so shall the *Colure* cut 90 degrees at 6 a clock, or when 90 degrees of the *Equinoctial* comes to the *Meridian*. And this being done, your *Dial* is so far made as the *Globe* can assist you.

II. The Geometrical Construction of this Dial, in order to the Trigonometrical Calculation.

1. With 60 Degrees of a *Scale of Chords*, describe a Circle representing your *Dial-plain*, and *Horizon* of the *Place*, (viz. *London*.) Cross it with 2 Diameters *S N*, representing the *Meridian* of the *Globe*, and *Hour-line* of *XII*; and the Line *E W*, for the *Hour-line* of *VI*: then will *Z* represent the *Zenith* of the *Place*, and be the Centre of the *Dial*.

Fig.
LII.

2. The *Latitude* of the *Place* being 51 d. 30 m. Set them from *S* to *a*, and from *W* to *b*: Then a *Ruler* laid from *W* to *a*, will cross the *Meridian* *S N* in *P*, the *Pole* of the *World*: And laid from *E* to *b*, it will cross the *Meridian* *N S* in *Æ*, the intersection of the *Meridian* and *Equinoctial*. And now you have three Points, viz. *W*, *Æ*, *E*; through which you may describe the *Equinoctial* Circle *W Æ E*, whose Centre will be at *c*, always in some part of the Line *N S*, extended, if need be.

3. Divide the Semicircle *W N E*, into 12 equal parts, at the points $\odot \odot \odot$, &c. And laying a *Ruler* to *Z*, and every of those points $\odot \odot \odot$, it will cross the *Equinoctial* Circle *W Æ E*, in the points $***$, &c. dividing that Semicircle into 12 unequal parts.

4. A *Ruler* laid to *P*, the *Pole* of the *World*, and to the several points $***$, &c. it will cut the Circle of your *Plain* in the points 1, 2, 3, &c. on the *East-side* of *N*, and 11, 10, 9, &c. on the *West-side* of *N*.

Lastly. If you lay a *Ruler* to the Centre *Z*, and the respective points 1, 2, 3, &c. and 11, 10, 9, &c. they shall be the true *Hour-lines* belonging to an *Horizontal Dial* for the *Latitude* of 51 de. 30 m. And their respective distances from *N* will be the same as in the *Table* they were found to be by the *Globe*.

III. By

III. By Trigonometrical Calculation.

In these Horizontal Dials, there is nothing to be found by Calculation Trigonometrical, but the *Hour-distances* upon the *Plain* from the *Meridian*; for which, This is

The Canon for Calculation.

As the Sine of 90 de.

Is to the Sine of the *Latitude* P N, 51 de. 30 m.

So is the *Tangent* of 15 d. (the *Æquinoctial Distance* of One hour; of 30 d. for Two hours; of 45 d. for Three hours, &c.)

To the *Tangent* of 11 d. 50 m. for 11 and 1--- of 24 d. 20 m. for 10 and 2--- for 38 d. 3 min. for 9 and 3, &c. as in the Table, before found by the Globe.

So have you all the Hour-lines between 6 in the morning and 6 at night; and for the Hour-lines of 4 and 5 in the morning, and of 7 and 8 at night, draw the same Lines before and after 6, through the Centre, as in the Figure, and they shall be the true Hour-lines: And so is your *Dial* finished.

The *Stile* must stand upright at 12 of the clock, not inclining on either side.

And in this manner may you describe Hour-lines upon an Horizontal Plain in any Latitude.

C H A P. V.

How to describe Hour-Lines upon an Erect direct South Plain, in the Latitude of London, 51 deg. 30 m.

AN *Erect Direct South Dial*, in any Latitude, is no other than an *Horizontal Dial* in that Latitude, which is equal to the Complement of that *Latitude* in which it is an *Erect Direct South Dial*: So that an *Erect Direct South Dial* in the Latitude of 51 d. 30 m. will be the same as an *Horizontal Dial* in the Latitude of 38 de. 30 m. which is the Complement of 51 de. 30 m. — So that the making of such a *Dial*, both by the *Globe* and by *Tri. Calculation* is the same with the other, only instead of 51 de. 30 m. *Latitude* you set your *Globe* to 38 d. 30 m. and so in the Calculation also: But in these *Dials* there needs no Hour-Lines

Lines to be drawn through the Centre; for that the Sun never Shines upon them before 6 in the Morning, nor after 6 at Night.

Fig.
LIII.

The *Stile* of these *Dials* must stand upon the *Hour-Line* of 12, and must point downwards towards the *South Pole*. As in Figure LIII.

CHAP. VI.

To make an *Erect Direct North Dial*, in the *Latitude* of London 51 de. 30 m.

THE *North Erect Direct Dial*, is the same with the *South*, only the *Stile* must point upwards towards the *North-Pole*, and the hours about Midnight, as 9, 10, 11, 12, at Night; and 1, 2 and 3 in the Morning must be left out, and 4 and 5 in the Morning; and 7 and 8 at Night must be drawn through the Centre: So is your *North-Dial* also finished, as in Fig. LIV.

Fig.
LIV.

CHAP. VII.

To make an *Erect Direct East or West Dial* in the *Latitude* of London, 51 de. 30 m.

I. By the Globe.

THE *Globe* rectified to the *Latitude*, the *Index* to 12, the *Quadrant of Altitude* in the *Zenith*: If you turn the *Quadrant of Altitude* so about till the graduated edge thereof do behold the direct *East* or *West*-points of the *Horizon*, you shall find that it will lie in the very *Plain* of the *Meridian-Circle*, and so the *Pole* will have no elevation over it; for turning the *Globe* about, the *Equinoctial Colure* will not cut the *Quadrant of Altitude* in any particular degree, but it will cut all the degrees thereof at the same time; wherefore the *Hour-lines* of these *Plains* will make no *Angles* at the *Pole*, and therefore must be parallel one to the other, which the *Globe* evidently demonstrates, but will not conveniently give the parallel distance of each from other, they being

Fig.
LV.

B b b

nearer

Fig.
L.V.

nearer or farther off each other according as the *Stile* is proportioned to the Plain, which I shall now come to shew.

For the Reasons aforesaid, there is no *Trigonometrical Calculation* required in the making of these *Dials*; and therefore, I shall proceed to

II. The Geometrical Construction of these Dials.

Let the Plain upon which you would make an *East* or *West Dial*, be A B C D.

1. Upon D (or any where towards the lower part of the Line B D, for an *East Dial*, or of A C for a *West*) with 60 degrees of your *Chord*, describe an Arch F G, upon which set the Complement of the *Latitude* of the place, viz. 38 deg. 30 min. from F to G, and draw the Line D G E for the *Equinoctial*.

2. Towards the upper part of this Line, as at P, assume any point, and through it draw the Line 6 P 6 perpendicular to the *Equinoctial*, for the Hour-line of Six. — Also, towards the lower part of the same Line, assume another point as L, and through it draw the Line 11 L 11 perpendicular also to the *Equinoctial* for the Hour-line of Eleven.

3. With 60 degrees of your *Chords*, upon the point L, describe a small Arch of a Circle, as H K, and upon it (always) set 15 degrees (or one hours distance) from H to K, and draw the Line L K M, cutting the Hour-line of Six in M.

4. Upon M as a Centre, with 60 degrees of your *Chord*, describe an Arch of a Circle N O, which divide into five equal parts in the points ○ ○ ○ ○.

5. Lay a Ruler upon M, and each of these points ○ ○ ○ ○ and the Ruler will cut the *Equinoctial*-line E D in the points **** through which points, if you draw Right Lines parallel to the Hour-line of 6, they shall be the Hour-lines of 7, 8, 9, and 10 of the Clock, the Hour-lines of 6 and 11 being drawn before.

6. For the Hour-lines of 4 and 5 in the Morning, before 6 they retain the same distance from 6, as do the hours of 7 and 8, and thus is your *Dial* finished.

The *Stile* must stand upon the Hour-line of 6, and be elevated so high as is the length of the Line M P, and may either be a Pin of Wyre, or a Plate of Brass or Iron.

The *West Dial* is the same, with the *East*, only changing the names of the Hours.

Fig. LI.

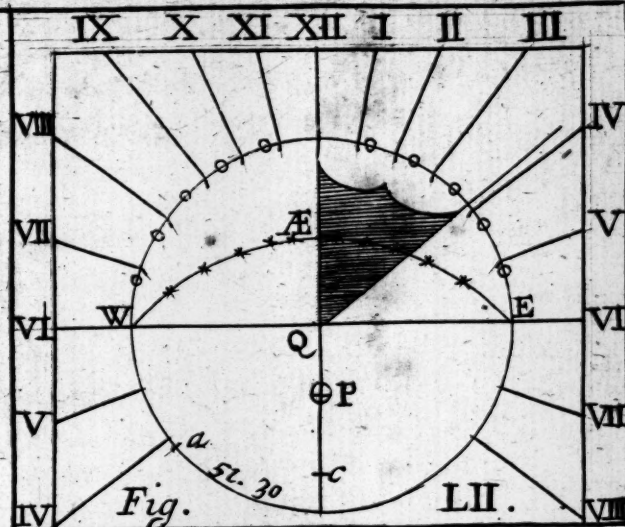
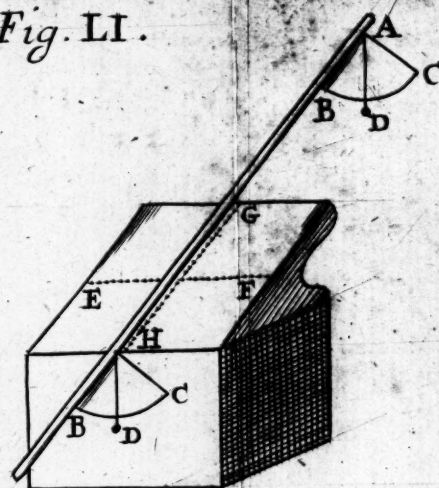


Fig. LIII

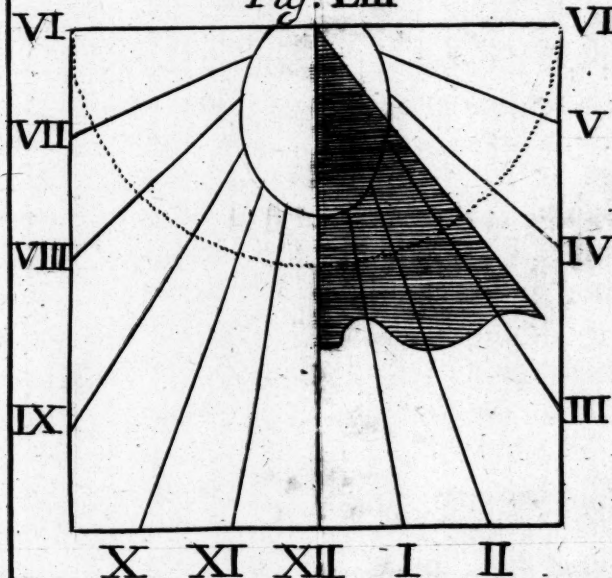


Fig. LIV.

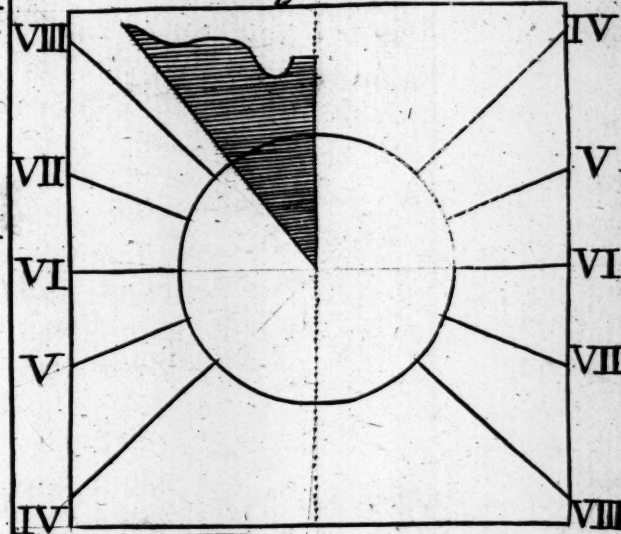
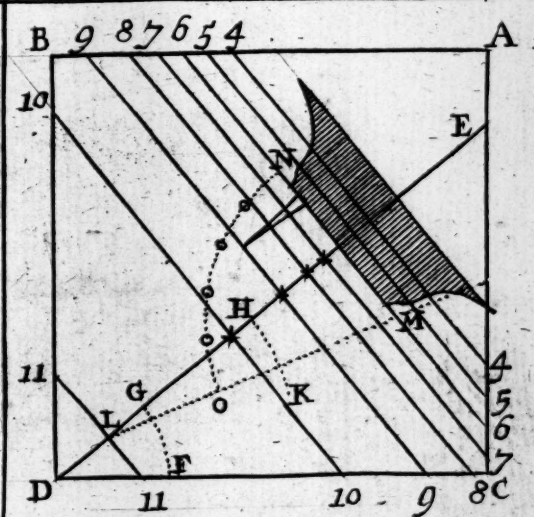
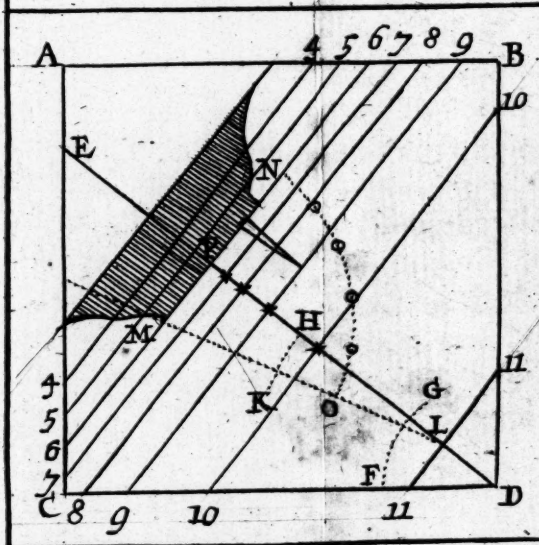


Fig. LV.



Of Dialling.

For

4, 5, 6, 7, 8, 9, 10, 11 in the Morning, in the *East-Dial*.
Must be changed to

8, 7, 6, 5, 4, 3, 2, 1 in the Afternoon in the *West Dial*:
Which is all the difference.

277

Fig.
LV.

C H A P. VIII.

To make an Erect Dial, declining from the South, Eastward, or West-ward; 30 degrees in the Latitude of 51 deg. 30 min.

I. By the Globe.

THE Globe being Rectified to the *Latitude* of the place, the *Quadrant of Altitude* in the *Zenith*, the *Index* of the Hour-Circle at 12, and the *Equinoctial Colure* brought under the *Meridian*;

Fig.
LVI.

1. Count the Declination of the Plain upon the *Horizon*, from the *East* or *West*-points thereof (according as the Plain declines) towards the *South*: namely, 30 degrees; and to that point of the *Horizon* bring the *Quadrant of Altitude*, and there keep it.

2. Turn the *Globe* about till the *Index* of the Hour-wheel cuts 11 of the Clock, or rather (as I said before) till 15 degrees of the *Equinoctial* have passed the *Meridian*, and then shall you find the *Equinoctial Colure* to cut the *Quadrant of Altitude* at 9 deg. 50 min. if you count the degrees from the *Zenith* point downwards.

3. Turn the *Globe* farther about, till 30 degrees of the *Equinoctial* be past the *Meridian*, and then shall you find the *Colure* to cut the *Quadrant of Altitude* at 18 deg. 14 min. counted from the *Zenith* downwards as before.

4. Do the like with all the rest of the Hours, and you shall find that at the several 15 degrees of the *Equinoctial*, the *Equinoctial Colure* will cut such degrees of the *Quadrant of Altitude* as are expressed in this Table, if you count them from the *Zenith* downwards, as is before directed.

This done;

5. Bring the *Quadrant of Altitude* to the other side of the *Meridian*, and set it to 30 degrees, the Plains declination, counted from the *East* or *West*-points *Northward*, as you did

Hours from Noon.	Hour-distances upon the Plain.	
	d.	m.
12	00	00
11 1	09	50
10 2	18	14
9 3	26	19
8 4	34	56
7 5	44	56
6 6	57	49
5 7	75	37

B b b 2

before

Fig. I. VI

before towards the *South*, which will be in the just opposite point of the *Horizon* to which it was before; and also, bring the *Equinoctial Colure* under the *Meridian*. Then,

6. Turn the *Globe* about (the contrary way to what you did before) till 15 degrees of the *Equinoctial* be past the *Meridian*, and then shall you find the *Equinoctial Colure* to cut at 12 deg. 23. min of the *Quadrant of Altitude* counted from the *Zenith*.

Hours from Noon.		Hour-distances on the Plain.
		d. m.
12		00 00
1	11	12 23
2	10	29 19
3	9	52 42
4	8	80 07

And so continuing turning the *Globe* about till 30, 45, and 60 degrees of the *Equinoctial* have passed the *Meridian*, you shall find the *Equinoctial Colure* to cut the *Quadrant of Altitude* at such degrees as are expressed in this Table.

The Hour-distances upon the Plain being thus attained, there are two other requisites in all upright declining *Dials* also to be found by the *Globe*, before the *Dial* can be finished. Namely,

1. The distance of the *Sub-stile* from the *Meridian*.
2. The height of the *Pole* above the *Plain*, or the height of the *Stile* above the *Sub-stile*.

To find both which,

Bring the *Equinoctial Colure* to the Plains declination 30 degrees counted upon the *Horizon* from the *South-Eastward*; and the *Quadrant of Altitude* to 30 degrees counted in the *Horizon* from the *East-Northward*: So shall the *Quadrant* cut the *Colure* at Right Angles. And

The number of degrees of the *Quadrant* contained between this *Intersection* and the *Zenith* (which here is 21 deg. 41 min.) is the distance of the *Sub-stile* from the *Meridian*. And the degrees of the *Colure* contained between this *Intersection* and the *Pole* (which here is 32 deg. 37 min.) is the height of the *Pole* above the *Plain*.

II. The Geometrical projection of this *Dial*, in order to the Trigonometrical Calculation of the Hour distances and other Requisites belonging to such an *Erect Declining Plains*.

I. Upon the Point Q, as a Centre, with 60 de. of a Scale of Chords, describe a Circle, representing your *Dial Plain*: And cross it with two Diameters Z Q N for the *Vertical*; and Q O for the *Horizontal Line* of the *Plain*.

2. Set 30 deg. the *Plains Declination*, from N to *c*, if the Plain Decline *Eastward*, as in this Example; or from N to *e*, if *Westward*: Then lay a Ruler from Z to *c*, and it will cut the *Horizontal Line* of the *Plain* in K, so have you three Points Z, K and N, by which to describe the Arch ZKN, representing the *Meridian* of the *Place*. And to find the *Pole* thereof, set 90 de. from *c*, to *d*, and then a Ruler laid from Z to *d*, will cut the *Horizontal Line* HO, in W, which is the *Pole* of the *Meridian Circle* ZKN: and the *West-point* of the *Horiz.*

3. Set 51 deg. 30 m. the *Latitude* of the *Place*, from O to *a*, and from N to *b*: Then a Ruler laid from W to *a*, will cross the *Meridian* in P, the *Pole* of the *World*: And laid from W to *b*, it will cross the *Meridian* in Æ, the point through which the *Æquinoctial Circle* is to pass: And now you have two points W and Æ, through which the *Æquinoctial Circle* æ Æ æ may be described (by the XXI *Geometrical Problem*, Lib. 1.)

4. Through P, the *Pole* of the *World*, and Q, the *Pole* of the *Plain*, draw the right Line PQ, for the *Axis* of the *World*, and *Sub-stilar Line* of your *Dial*: And in this Line (extended) will the *Center* of the *Æquinoctial Circle* æ Æ æ be found.

5. From P, the *Pole* of the *World*, lay a Ruler to Æ, the intersection of the *Meridian* and *Æquinoctial*, and it will cut the *Plain* B: At this point B, begin to divide the *Semicircle* HNO of the *Plain*, into 12 equal Parts, at the points ☉ ☉ ☉, &c.

6. From Q, (the *Pole* of the *Plain*) lay a Ruler to every of the points ☉ ☉ ☉, &c. and it will cross the *Æquinoctial Circle* æ Æ æ, in the the Points ***, &c.

7. Lay a Ruler to P (the *Pole* of the *World*) and every of the points ***, &c. and it will cut the *Primitive Circle* representing the *Dial Plain*, in the points N, 9, 10, 11, &c. on the *West* side, and N, 1, 2, 3, &c. on the *East* side of the *Meridian*.

8. Lastly, Lines drawn from the Centre Q through these points, shall be the true *Hour-lines* of an *Erect Plain Declining* from the *South-Eastward* 30 de. in the *Latitude* 51 de. 30 m. And now,

Concerning the other *Requisites* belonging to this *Erect Declining Plain*.

These are all of them represented to the Eye in the *Scheme* of the *Projection* of the *Plain*: Wherein, by the intersection of the *several Circles* there is constituted a *Right-angla Spherical Triangle* ZTP, in which,

The

Fig.

LVI.

The side $\begin{cases} Z T \text{ is the distance of the Substile from the Meridian.} \\ Z P \text{ the Complement of the Latitude of the Place.} \\ T P \text{ the height of the Pole above the Plain.} \end{cases}$

The Angle $\begin{cases} P Z T, \text{ the Complement of the Plain's Declination.} \\ Z P T, \text{ the Plain's difference of Longitude.} \\ Z T P, \text{ is a Right Angle.} \end{cases}$

And all these Sides and Angles may be measured upon the Projection it self, by the Precepts deliver'd in the first Book, Sect. III. of Spherical Trigonometry Geometrically performed by Projection. And they will be found to be as here expressed, viz.

	de.	m.		de.	m.
Side $\begin{cases} Z T \\ Z P \\ T P \end{cases}$	21	49	Angle $\begin{cases} P Z T \\ Z P T \\ Z T P \end{cases}$	60	00
	38	30		36	25
	32	36		90	00

And now I shall shew how all these may be found
By Trigonometrical Calculation.

Before any Erect Declining Dial can be made, there are Three things which must be found, besides the Hour-distances, and those are these :

1. The height of the Pole (or Stile) above the Plain.
2. The distance of the Substile from the Meridian.
3. The Plain's difference of Longitude.

All which may be found in the Triangle Z T P : In which there is given (besides the Right Angle at T,)

- (1.) The Angle T Z P, the Complement of the Plain's Declination, 60 deg.
- (2.) The Side Z P, the Complement of the Latitude of the Place 38 de. 30 m.

By which may be found,

I. The Side T P : The Height of the Pole, or Stile, above the Plain:
By CASE I. of R. A. S. T.

As the Radius, S. 90 deg.

Is to the Side Z P, 38 d. 30 m. the Co-Latitude.

So is the Sine of T Z P: the Co-declination 60 de.

To the Sine of T P, 32 deg. 26 m.

Which is the height of the Pole (or Stile) above the Plain.

II. The

II. The *Side* Z T, The *Distance* of the *Sub-stile* from the *Meridian*: By CASE II of R. S. T.

As the Radius Sine 90 de.

Is to the Tangent of Z P, the Co-Latitude 38 d. 30 m.

So is the Co-sine of T Z P, 30 deg.

To the Tangent of Z T, 21 de. 40 m.

Which is the *Distance* of the *Sub-stile* from the *Meridian*.

III. The *Angle* Z P T, The *Plains* difference of *Longitude*: By Case III. of R. A. S. T.

As the Radius,

Is to the Tangent of the Declination T Z P, 60 deg.

So is the Co-sine of Z P, the Co-Latit. 51 de. 30 m.

To the Co-Tangent of T P Z, 35 de. 25 m.

Which is the *Plain's* difference of *Longitude*.

These three Requisites being thus found by Trigonometrical Calculation; The Plain's Difference of Longitude 36 de. 25 m. falling between 30 and 45 deg. (which are the Second and Third Equinoctial Hour-distances) there will be contained therein two compleat Hours, and 6 deg. 25 m. over: which shews that the Sub-stilar Line of the Dial will fall between the Hour-Lines of IX and X in the Morning (in this East Dial) but between the Hour of II and III in the Afternoon if the Plain had Declined Westward:

Having proceeded thus far. Prepare a Table of Hours fit for the Plain, such as is here done.

Fig.
LVI.

A Table of the Hour Distances, for a South-Dial, Declining East or West, 30 deg. In the Latitude of 51 deg. 30 min.

	d.	m.
Stiles height	32.	36
Distance of the Sub-stile and Meridian.	21.	40
Difference of Long.	36.	25

Hours from the East	Hours from the West	Equinoctial Hour Distances.	True Ho. Distances upon the Plain from the Sub-stile.	
			D.	M.
IV	VIII	83	35	78
V	VII	68	35	53
	VI	53	35	36
VII	V	38	35	23
VIII	IV	23	35	13
IX	III	8	35	4
		Sub-stile.		
X	II	6	25	3
XI	I	21	25	11
	XII	36	25	21
I	XI	51	25	34
II	X	66	25	51
III	IX	81	25	74

Then against XII, set the Plain's Difference of Longitude 26 de. 25 min. (in the second Column) and from it subtract 15 deg. and there will remain 21 d. 25 m. which set against XI. and I: And from 21 d. 25 m. subtract 15 deg. and there will remain 6 d. 25 m. which set against X and II. And (because it is less than 15 de.) write the Word Sub-stile over it, and subtract it from 15 deg. and there will remain 8 deg. 35 m. which write over Sub-stile, and against the Hours of IX and III. Then to these 8 d. 35 m. add 15 deg. and it makes 23 de. 25 m. which set against VIII and IV: And thus, by the continual Addition of 15 deg. you shall have such *Equinoctial Distances* as in the Table: Which Table, thus prepared, the next thing will be

To find the true *Hour-distances* upon the *Plain*, from the *Sub-stile*.

For which, This is

The Canon for Calculation.

As the *Radius*

To the Sine of the Stiles height 32 d. 36 m.

So is the Tangent of the *Equino-*

ctial Distance 6 d 25 m.

To the Tangent of 3 deg. 28 min. Which is the *Distance* of the *Hour-lines* of X and II. upon the *Plain* from the *Sub-stile*.

And so will the Tangent of the next *Equinoctial Distance* 21 d. 25 m. be, to the Tangent of 11 de. 56 min. for the *Distance* of the *Hour-lines* of XI and I, from the *Sub-stile*: And so for all the rest, as in the Table. And so you will find them to be in the Figure also.

Fig.
LVII.

And in the making of this *Dial*, you have made four; as in Fig. LVII.

For,

Of Dialling.

283

Fig.
LVII.

For, if you hold the Paper upon which the *South-East declining Dial* is drawn, against the Light, then shall you discover the *Stile* to stand on the *Right-hand* of the *Plain*; whereas it now stands on the *Left-hand*; so the same *Hour-lines*, *Sub-stile*, *Stile* and all, being drawn on the back-side of the Paper, and those that are the *Forenoon-hours* in the *East-decliner* numbred as the *Afternoon-hours* in the *West-decliner*; that is, call 11, 1, and 10, 2, and 9, 3, &c. as in the Tables; so shall the *South-Dial* declining *East* 30 degrees, become a *South-Dial* declining *Westward* 30 degrees.

And if you turn the *South-East-Dial* upside-down, so that the *Stile* may point upwards towards the *North-Pole*, (and leave out the Hours about 12, as 9, 10, 11, and 1, 2, and 3, which in *North Dials* represent 9, 10, and 11 at Night, and 1, 2, and 3 in the Morning; all which time (in those middle *Latitudes*) the Sun is under the *Horizon*) it will become a *North-Dial* declining *Eastward* 30 degrees.

Also if you turn the *South declining West-Dial* upside-down, and leave out the hours about *Midnight*, as 9, 10, 11, 12, 1, 2, and 3, it will then become a *North-Dial* declining *Westward* 30 degrees.

Now for such *South* or *North Dials* as do decline far towards *East* or *West*, as 60, 70, 80, or 85 degrees, there you shall find that the *Hour-distances* will fall so near together, that they will be of no competent distance one from another, except they be extended very far from the Centre; and therefore the old way hath been (in such Cases) to draw the *Dial* upon the Floor of a Room, extending the *Sub-stile*, *Stile* and *Hour-lines* till they appear of a competent distance from each other, and then according to the bigness of your *Dial-plain*, to cut off the *Hour-lines*, *Stile* and *Sub-stile*; and so transfer them from the Floor to the *Plain* upon which the *Dial* is to be made: but this way being to Mechanical for an Artist to exercise, I shall therefore here insert a more artificial way of performing this work *Geometrically*, by which (although the *Dial* should decline 80 or 88 degrees) upon a quarter of a sheet of Paper you may draw your *Dial*, and have the *Stile* of a competent height, and all the *Hour-lines* at a convenient distance one from another. And so let this suffice to be said in this place concerning *Upright declining Dials*; for I intend not here to teach the *Art of Dialling*, but shew the *Use* of the *Globes*: and from thence to Calculate the Requisites from them *Trigonometrically*.

C c c

C H A P.

Fig.
LVII.

C H A P IX.

Concerning such Erect South or North Dial Plains, which decline many degrees towards the East and West.

I. By the Globe.

Fig. LVIII. THE Operation by the *Globe* is altogether the same, as in the last Chapter, and therefore need not be here again repeated. Wherefore, I will proceed to

II. The Trigonometrical Calculation.

This Work also is to be performed in like manner as the Former: And therefore I shall not again repeat it, but give an *Example* ready wrought.

Suppose an *Erect Plain* in the *Latitude* of *London*, 51 de. 30 m. should decline from the *South* towards the *East* 85 deg.

I. For finding the *Requisite*, if you work according to those *Canons* for *Calculation*, in the last Chapter delivered; you will find

		de.	m.
The	Height of the Pole (or Stile) above the Plain	3	06
	Distance of the Sub-stile and Meridian	39	22
	Plain's Difference of Longitude	86	05

Hours		Equinoctial Distance		Hour Distances on the Plain	
Morn.	Aft.	D.	M.	D.	M.
	XII	86	05	38	23
XI	I	71	05	8	58
X	II	56	05	4	36
IX	III	41	05	2	42
VIII	IV	26	05	1	31
VII	V	11	05	0	36
The Place of the Sub-stile.					
VI		3	55	0	13
V	VII	18	55	1	04
IV	VIII	32	55	2	06

These *Requisites* thus found, you may proceed to the making of a *Table* for the *Hour-distances* in all respects, as in the last Chapter: By setting down the *Hours* proper for the *Dial* in order, as in this *Table*.

Then, against XII, set the *Difference of Longitude*, 86 d. 05 m. from which Subtract 15 deg. and there will remain 71 d. 05 m. which set against XI and I. And from 71 d. 05 m. subtract 15 de. the remainder will be 56 d. 05 m. which set against X and II. And so by the continual Sub-

straction.

fraction of 15 deg. you shall find 11 de. 05 m. to stand against the Hours of VII and V ; under which write *The place of the Sub-stile* : And then, subtract 11 d. 05 m. from 15 de. there will remain 3 de. 55 m. which must be set under the *Sub-stile*, against the Hour of VI : And then, by the continual *Addition* of 15 deg. thereto; you shall have such *Æquinoctial distances* as are expressed in the Table, against the Hours of V and VII, and IV and VIII. Then

Fig.
LVIII.

II. For the *Five Hour distances upon the Plain* : If you work by the *Canon* delivered in this last Chapter, you will find them to be such as are exhibited in the *Third Column* of this Table.

By this Table, you see that the *Five Hour Distances* upon the Plain about the *Sub-stile* (and indeed all the rest, except the extreme Hour of XII) do fall so near together, that without a very large extention of them from the *Centre*, there will be no competent distance between *Hour-line* and *Hour-line* : Wherefore, laying aside your Table, proceed to make your *Dial Geometrically*, according to the following Precepts.

III. The *Geometrical Projection*, of this (or the like) *Dial* : when the *Pole* hath but small *Elevation* above the *Plain*.

1. Draw a Right Line A B, perpendicular by one side of your *Plain*, and towards the *Right Hand*, because the *Plain* declineth *Eastward*, And with 60 deg. of a *Scale of Chords*, describe an obscure Arch of a Circle C D E ; and upon it set 38 de. 23 m. the *Sub-stiles distance* from the *Meridian*, from C to D, and draw the Line A D, for the *Sub-stile*.

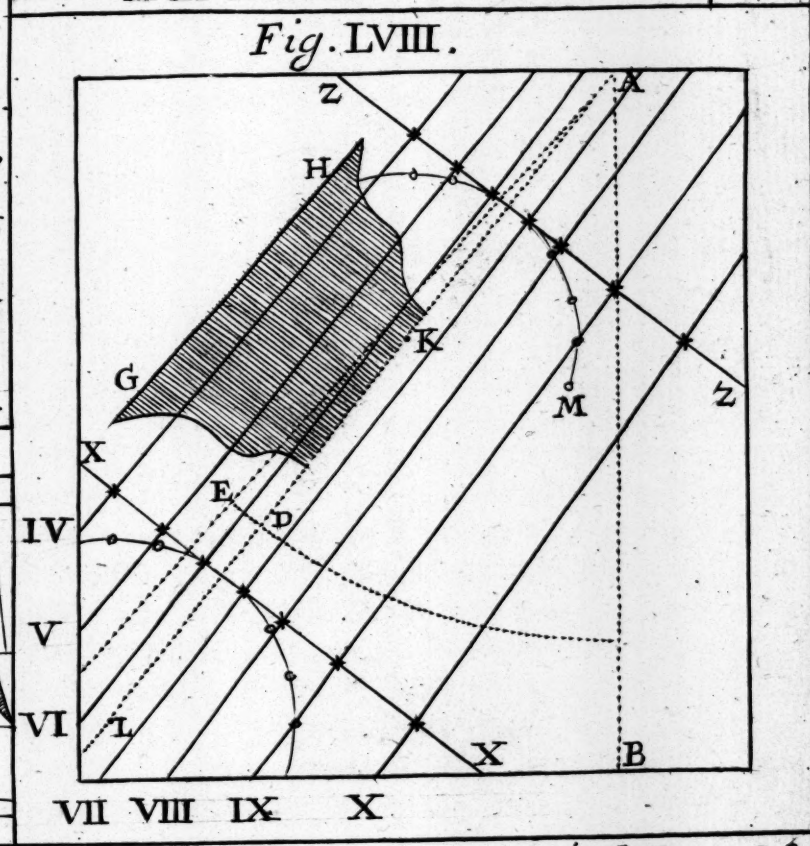
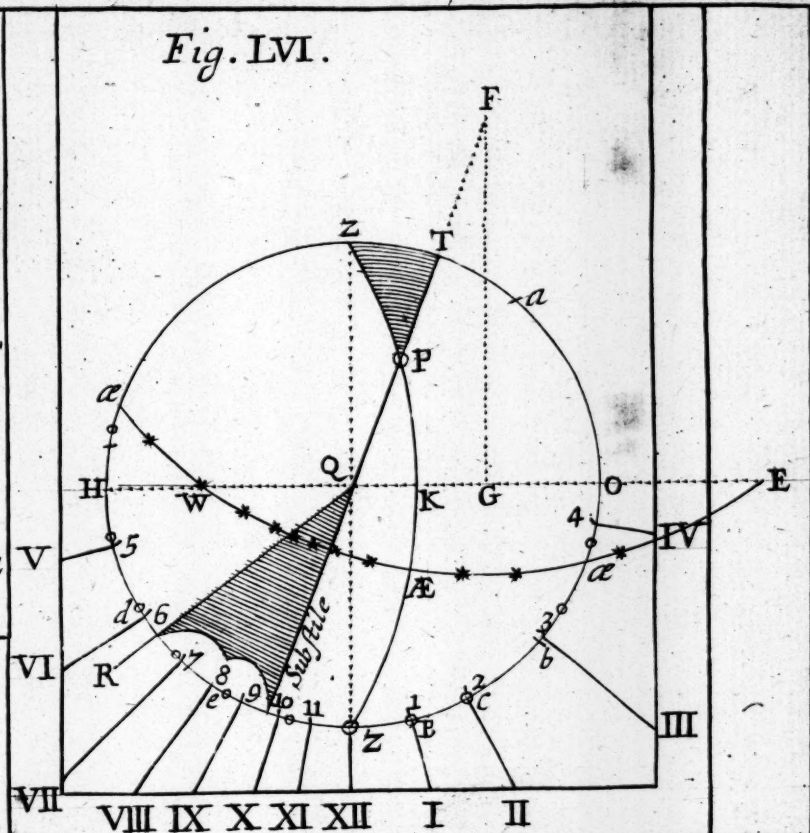
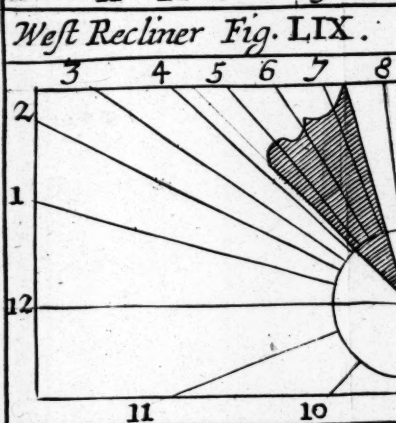
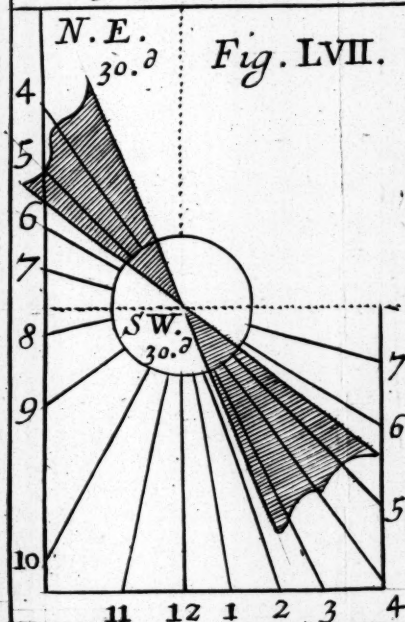
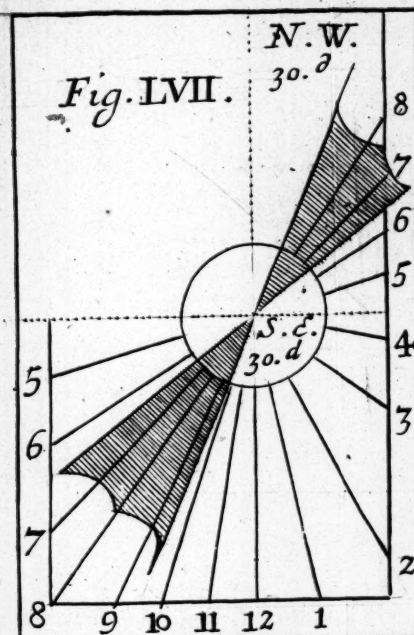
Fig.
LIX.

2. Take 3 deg. 6 m. the *Stiles height* ; and set them from D to E, drawing the Line A E for the *Stile* of the *Dial*.

3. Now (because the *Stile* is but of small *Elevation*, viz. but 3 de. 6 m.) draw another Line G H, parallel to the Line of the *Stile* A E, at such convenient distance as in your judgment will best fit the *Dial Plain*, so that the Hours belonging to it, may come within the Limits of the *Plain* : So shall that Line G H, so drawn, be the *Augmented Stile* of the *Dial*.

And, by the *Sub-stile*, and this *Augmented Stile*, the *Hour-Lines* may be described (at convenient distance) without any regard had to the *Centre* of the *Dial*. For,

4. Assume any two points in the *Sub-stilar Line* of the *Dial* A D, at some convenient distance from each other, as R and S : and through those two points, draw two infinite Right lines, both



C H A P. X.

Of Direct South and North Declining Plains, and how Hour-Lines may be described upon them.

SUCH Plains as do directly behold the $\left\{ \begin{smallmatrix} \text{North} \\ \text{South} \end{smallmatrix} \right\}$ point of the Horizon, but do Recline (or fall backwards) from the Zenith towards the $\left\{ \begin{smallmatrix} \text{South} \\ \text{North} \end{smallmatrix} \right\}$ are called $\left\{ \begin{smallmatrix} \text{North} \\ \text{South} \end{smallmatrix} \right\}$ Direct Plains Reclining: So many Degrees as the Reclination is: And of such Plains there are Six Varieties; Three of South, and Three of North Recliners: All which may be Reduced to New Latitudes, wherein they will become Horizontal Plains: And consequently Dials (or Hour Lines) may be described upon them both by the Globe, Spherical Projection, and Trigonometrical Calculation, according to the Precepts delivered in the Fourth Chapter hereof.

I. Of South Recliners

Examples of all these Varieties of Reclining Plains, in the Latitude of London, 51 deg. 30 m. To find the New Latitudes.

I. Variety. Let there be a Direct South Plain in the Latitude of London, which Reclines from the Zenith thereof 20 deg. In what Latitude will that be an Horizontal Plain?

The Plains Reclination 120 deg. being less than 38 de. 30 m. the Complement of the Latitude of London; Subtract 20 deg. from 38 d. 30 m. the Remainder (or difference) 18 de. 30 m. is the New-Latitude. So that an Horizontal Dial made for that Latitude, shall be a South Recliner 20 deg. in the Latitude of London.

II. Variety. If a South Plain in the Latitude of 51 deg. 30 m. should Recline 60 deg. from the Zenith thereof: In what Latitude will that be an Horizontal Plain?

The Reclination of the Plain 60 deg. being Greater than 38 deg. 30 m. the Complement of the Latitude of London, Subtract 38 de. 30 m. from 60 deg. and the Remainder 21 deg. 30 m. is the New-Latitude: And an Horizontal Dial made for the Latitude of

Fig. 21 de. 30 m. shall serve for a *South Dial Reclining* 60 deg. in
LIX. the Latitude of *London*.

III. Variety. If a *South Plain* in the Latitude of *London* should Recline from the Zenith thereof 38 d. 30 m. Equal to the Complement of the Latitude of *London*. Then

The Difference between the Complement of the Latitude of *London* and the Reclination being nothing; it shews, the *New-Latitude* to be no Latitude, that is neither Pole hath any Elevation above such a Plain: And therefore a Dial for such a Plain must be made in all respects as an *Erect Direct East or West Dial* is made, by the Precepts in Chapter VII. hereof: Only, the *Hour-Line* of VI there, must be the *Hour-Line* of XII in this: And as the *Stile* there was equal to the distance between *Six* and *Three* or *Nine* a Clock. So in this, it must be equal to the distance between XII and IX or III. And may be either a *Plate* of that breadth; or a *Wyre* or *Pin* of that Length.

II. Of North Recliners

I. Variety. If a *North Plain* in the Latitude of 51 de. 30 m. should Recline from the Zenith 20 deg. In what Latitude will that be an *Horizontal Plain*?

The Reclination 20 deg. being less than the Complement of the Latitude of *London*, 38 de. 30 m. Add the Reclination 20 de. and the Complement 38 deg. 30 m. together; their Sum 58 deg 30 m. is The *New Latitude*: And an *Horizontal Dial* for that Latitude shall be a *North Plain Reclining* 20 deg. in the Latitude of 51 de. 30 m.

II. Variety. If a *North-Plain* in the Latitude of *London*, 51 deg 30 m. should Recline from the Zenith 75 deg. In what Latitude will such a Plain be *Horizontal*?

The Reclination 75 deg. being greater than 38 deg. 30 m. Add them together, and they make 113 deg. 30 min. which being above 90 deg. take the Complement thereof to 180 deg. which is 66 deg. 30 m. And that is the *New-Latitude*: So that an *Horizontal Dial* made for the Latitude of 66 de. 30 m. will be a *North Plain Reclining* 75 deg. in the Latitude of *London* 51 deg-30 m.

III. Variety. If a *North Plain* in the Latitude of *London* 51 de. 30 m. should Recline from the Zenith thereof 51 d

30 m. In what Latitude will such a Plain be Horizontal?

Here, the *Reclination* 51 de. 30 m. is *Equal* to the Latitude of London; And the *Sum* of the *Reclination* 51 de. 30 m. and the *Complement* of the Latitude 38 de. 30 m. Added together, their *Sum* is 90 deg. for *The New-Latitude*: And an *Horizontal Dial* made for that Latitude of 90 deg. (which is no other then a Circle divided into 24 *Equal Parts*, for the *Hours*; and a *Wyre* erected perpendicularly, of any *Length* for the *Stile*.)

These are all the *Varieties* of *South* and *North Reclining Plains*, Reduced to *New Latitudes*, wherein they will become *Horizontal Plains*: So that neither *Trigonometrical Calculation*, *Geometrical Projection*, or *Operation* by the *Globe*, need be here Repeated, they being all the same as in the IVth Chapter hereof.

Note also: That in making of any of these *North* or *South Reclining Dials*, you have made also A *Direct North* or *South Dial* inclining from the *Zenith* towards the *Horizon* so many Degrees as is the $\left\{ \begin{array}{l} \text{Re-} \\ \text{In-} \end{array} \right.$ *clination*. So that when you have made a *South-Dial Reclining* from the *Zenith* 60 deg. (as is the *Second Variety* of *South Recliners* in this Chapter) you have made also a *North Dial* inclining to the *Horizon* 60 degrees, either by drawing of the *Hour-lines* and *Stile* through the *Centre*; or by turning the *Reclining Dial* about upon the *Hour-line* of VI. And then, as the *North Pole* is elevated upon the *South Recliner*; so much will the *South Pole* be elevated above the *North Incliner*, &c.

CHAP. XI.

Of *Direct East* or *West Reclining Dials*, and how *Hour-lines* may be described upon them.

All *Direct North* and *South Reclining Dial Plains* were Reduced to *New-Latitudes* wherein they would be *Horizontal Plains*; and therefore made by the *Directions* given in the CHAP. IV. hereof: So all *Direct East* or *West Reclining Dial Plains*.

Fig.
LIX.

Plains in any one *Latitude*, may be *Reduced* to *Erect*, or *Upright*, *Declining* Plains in another *Latitude*: and therefore may be made by the *Precepts* delivered in the VIIIth. C H A P. hereof. Either, By the *Globe*; By *Spherical Projection*; or, By *Trigonometrical Calculation*: So that the Work of this C H A P. shall be only to shew

- I. How to *Reduce* any *Direct East* or *West Reclining Dial Plain* in any *Latitude*, (suppose *London* 51 de. 30 m.) to a *New-Latitude*: wherein the *Reclining Plain* shall become an *Erect* (or *Upright*) *Plain*. And
- II What *Declination*, that *Upright Plain* shall have in that *New-Latitude*.

E X A M P L E.

Suppose then, that a *Direct East* or *West Plain*, in the *Latitude* of 51 de. 30 m. should *Recline* from the *Zenith* 40 deg. In what *Latitude* will that be an *Upright Plain*? And what *Declination* shall it have in that *Latitude*?

R U L E.

The *Complement* of the *known Latitude*, is (always) The *New-Latitude*. And

The *Complement* of the *Reclination* is (always) The *Declination* in that *New-Latitude*.

So that if an *East* or *West Dial*, should *Recline* 40 deg. in the *Latitude* of 51 d. 30 m. That will be An *Upright Plain*, *Declining* 50 deg. in the *Latitude* of 38 de. 30 m

For, 38 de. 30 m. being the *Complement* of the *Latitude* of *London* 51 d. 30 m. is the *New-Latitude*: — And 50 deg. being the *Complement* of 40 de. the *Plains Reclination*, is the *Declination* in the *New-Latitude*.

So that, if (according to the *Precepts* delivered in the VIIIth. C H A P. hereof) you make an *Upright Dial* for the *Latitude* of 38 de. 30 min. to *Decline* 50 deg. Such a *Dial* will serve for an *East* or *West-Dial Reclining* 40 de. in the *Latitude* of (*London*) 51 de. 30 m.

Thus for the *Making* of the *Dial*.

But in the *placing* of the *Dial* (thus made) upon the *Reclining Plain*, this *Difference* is to be observed: For, Whereas in all *Upright Declining Plains*, the *Meridian*, or *Hour-line* of XII is always *Perpendicular* to the *Horizon* of the *Place* for which it is made. But

But this *Declining Plain* when it is applied to the *Reclining Plain*, the *Hour-line* of XII. must lie *Parallel* to the *Horizon* of the *Place*, as in the *Figure*.

And here Note, that all *East Recliners* in the known *Latitude* (as here *London*) are *North-East Decliners* in the *New-Latitude*: And all *West-Recliners*, are *North-West-Decliners*.

And Note farther; That upon all *East* and *West Reclining Plains* in *North-Latitudes*, that the *North Pole* is (always) *Elevated*: And upon the *East* and *West inclining Plains*, opposite to them, the *South Pole* is *Elevated*.

And Lastly, Note that when you have made an *East* or *West Reclining Dial*, you have made Four, viz. An *East* and *West Reclining*, and, An *East* and *West Inclining*.

C H A P. XII.

Of Dial Plains that do both Decline and Recline: How Hour-Lines, &c. are to be described on them.

THESE Plains may also be Reduced to *New-Latitudes* and *New Declinations*, where they may stand as *Upright Decliners*: And so *Dials* may be described on them, by the *Directions* in the VIIIth. C H A P. hereof: And to find the *New-Latitude*, and *New Declination* of any such Plain, The *Rules* following will direct.

E X A M P L E.

Suppose that in the *Latitude* of *London*, 51 d. 30 min: A *South Plain* should Decline towards the *East* or *West* 24 deg. 20 min. And also Recline from the *Zenith* 54 deg.

I. To find the *New-Latitude*

The *Canon* for Calculation.

As the Radius, Sine 90 de.

Is to the Co-sine of the *Old Decl.* 24 de. 20 m.

So is the Co-Tangent of the *Reclination* 54 d.

To a Fourth Tangent, viz. 33 de. 30 m.

This *Fourth Tangent* being thus found, the *Rules* following are to be observed.

10.

9.959596

9.861261

✱9.820857

D d d

I. In

I. In South Recliners.

R U L E I. This *Fourth Tangent* must be compared with the *Old Latitude*, and the *Complement* of their *Difference* is the *N E W L A T I T U D E*.

So, In this *Example*, The *Fourth Arch* 33 de. 30 m. being subtracted from the *Old Latitude* 51 de. 30 m. their difference is 18 deg. whose *Complement* 72 deg. is the *N E W L A T I T U D E*.

R U L E II. If the *Fourth Tangent* fall out to be *Equal* to the *Old Latitude*, Then the *Difference* will be nothing ; And so the *Plain* will be a *Polar declining Plain* : For the *Pole* will have no *Elevation* over it.

So, in the *Latitude* of 51 de. 30 m. If a *South Plain* should *Decline* towards the *East* or *West* 65 deg. 40 m. And *Recline* 18 de. 9 m. By the former *Canon*, the *Fourth Tangent* will be found 51 de. 30 m. *Equal* to the *Old Latitude* : So that the *Difference* is *Nothing*. And the *Hour-Lines* will be *Parallel* one to another as in the *Direct East, West, and Polar Dials*, in *C H A P. VII.* and *C H A P. X.*

R U L E III. If the *Fourth Tangent* prove to be *Greater* than the *Old Latitude*, Then, The *North Pole* is *Elevated* in *South Decliners* : But if the *Fourth Tangent* be *Lesser* than the *Old Latitude*, Then, The *South Pole* is *Elevated* in *North Decliners*.

II. In North Recliners.

R U L E I. The *Fourth Tangent* found as before, is to be compared with the *Complement* of the *Old Latitude*, and their *Difference* is the *N E W L A T I T U D E*.

R U L E II. If the *Fourth Tangent* prove to be *Equal* to the *Complement* of the *Old Latitude* : that *Declining Reclining Plain* will be an *Æquinoctial Plain Declining*.

So, In the *Latitude* of 51 de. 30 m. If a *Plain* should *Decline* from the *North* towards the *East* or *West* 60 deg. And also *Recline* from the *Zenith* 32 deg. 11 m. The *Fourth Tangent* will be found to be 38 de. 30 m. *Equal* to the *Complement* of the *Old Latitude* : And will, therefore, be An *Æquinoctial Declining Plain*.

II. To

Of Dialling

293

II. To find the *New Declination*.

The *Canon* for Calculation.

As the Radius, Sine 90 d.

10.

To the Co-sine of the Reclination 54 d.

9.769218

So is the Sine of the Old Declination, 24 d. 20 m.

9.614944

To the Sine of 41 de. 01 min.

*9.384162

Which 41 de. 1 min. is the *N E W D E C L I N A T I O N*.

And now, if (according to the *Precept* delivered in the VIIIth. C H A P. hereof) you do make an Upright South Dial Declining 14 deg. 1 m. in the Latitude of 72 deg. That Dial shall serve for A South Plain, in the Latitude of 51 de. 30 m. Declining 24 deg. 20 m. and Reclining 54 deg.

And in such a Dial you will find the *Requisites* belonging thereunto, to be as the Table in the Margin. And thus much for the Making of the Dial according to its *New Latitude*, and *New Declination*.

But to apply this Dial to the Reclining Plain in the Old Latitude, you are not to place the *Hour-line* of XII perpendicular to the *Horizon*, as in Upright Plains, but it must make an Angle with the *Horizontal Line* of the Reclining Plain: And therefore the quantity of that Angle must be first found. Therefore,

New Latitude	72. 00
New Declination	14. 01
Stiles heighth	17. 26
Dist. Sub. & Merid.	4. 17
Difference of Long.	13. 59

Hours.		Eq.	Di.	Tr.	HD.
41 VII	V	89	17	89	47
41 VIII	IV	74	17	46	48
41 IX	III	59	17	25	46
41 X	II	44	17	16	17
41 XI	I	29	17	9	32
41 XII		14	17	4	22
19 I	XI	00	43	0	12
19 II	X	15	43	4	49
19 III	IX	30	43	10	06
19 IV	VIII	45	43	17	05
19 V	VII	60	43	28	06
19 VI		75	43	49	39

III. To find the Angle made between the Meridian and the Horizon.

The *Canon* for Calculation.

As the Radius, Sine 90 d.

10.

Is to the Sine of the Reclination 54 d.

9.655347

So is the Tangent of the Old Declination 24 d. 20 m.

9.907957

To the Tangent of 20 deg. 6 m.

*9.563304

D d d 2

Whole

Whole Complement 69 deg. 54 min. is the *Angle* that the *Meridian* (or *Hour-line* of XII.) must make with the *Horizontal Line* of the *Reclining Plain*. Thus for the *Quantity* of the *Angle*. But

IV. To know which way (or to what Coast) the *Meridian Line*, *Ascending* or *Descending Above* or under the *Horizontal Line* of the *Plain*, is to be drawn.

GENERAL RULES.

In {	{	North In-	{	Less than Equino-	{	Above	{	That <i>End</i> of the <i>Ho-</i>
		cliners.				ctial, the Meridian		
				must be drawn				lies <i>contrary</i> to the
								Coast of the Plain's
								Declination.

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	Less than a Polar the Meridian must be drawn.	Above	That End of the Horizontal Line, that looks the same way with the Coast of the Plain's Declination—And this Meridian thus drawn, in North Recliners, represents 12 at Midnight.
In { North Recliners South Incliners	Equal to a Polar, the Meridian must be drawn below the Horizontal Line, at that end which is contrary to the Coast of Declination: And the Six a Clock Hour-line, is (always) the Sub-silar.	Below	
	More than a Polar, the Meridian must be drawn.	Below	And from that End of the Horizontal Line, which lies contrary to the Coast of the Plain's Declination— And, in South Incliners, is only serviceable to help to draw the rest of the Dial.
		Above	

V. How the Dial (being made according to the *New Latitude* and *New Declination*) is to be transferred from the *Paper Draught*, upon the *Reclining or Inclining Plain*.

Having drawn an *Horizontal Line* upon your *Dial Plain*, in the most convenient part thereof, and made choice of a Point therein for the *Centre* of your *Dial*; apply the *Centre* of your *Paper Draught* to this *Centre*, moving the *Paper Draught* about till the *Meridian Line* thereof do make an *Angle* with the *Horizontal Line* drawn upon the *Plain*, equal to what you found it to be by the *Third Section* of this C H A P. And to its *Proper Coast*, as you find it by the *Fourth Section*: Then (if you have not erred in any of your former workings) will the *Stile* of your *Paper Draught* (or rather a pattern of it cut in *Past-board*) being placed upon the *Sub-stile* of the *Paper Draught*, have direct respect to the *coated Pole*.—And thus due respect being had to what is here deliver'd

delivered, you may easily transfer the *Stile* and the rest of the *Hour-lines* to the *Plain*; putting thereon so many as the *Plain* is capable to receive at any time of the *Year*; and leaving out such as are superfluous.

C H A P. XIII.

Of the Inscription of other Great and Small Circles of the Sphere upon all sorts of Dial Plains.

IN the foregoing Chapters is shewed how the *Meridians* (or *Hour-Circles*) of the *Sphere* may be described upon all sorts of *Plain Superficies* howsoever situate: And as those *Hour-Circles* were described, so may all other *Circles* of the *Sphere* be also delineated: And the manner how, I have largely treated of in my particular *Treaties of Dialling* already extant: So that in this place I shall be but brief therein, and yet sufficiently *Plain*. And whereas the *Hour* of the *Day* on all *Sun-Dials* is shewed by the whole *Axis* of the *Stile*; so all other *Circles* so described, are shewed by one single point taken in some convenient place of the said *Axis*, from which Point a Perpendicular let fall to the *Plain* upon the *Sub-stilar Line* shall be called the *Perpendicular Stile*: And the point upon the *Plain*, The *Foot* of the *Perpendicular Stile*; and the Point in the *Axis* (before assumed) the *Gnodus* or *Apex* of the *Stile*. — Now, for the performances of the Work following (besides the *Trigonometrical Calculation*) the Practitioner must be provided with a *Scale* (or a *Sector* rather) which must have upon it *Scales of Sines, Tangents, Secants, Chords, and equal parts*: or if he have by him, *Tables of Natural Sines, Tangents, and Secants*, then a *Line of equal parts* only drawn from the Centre, will perform the work rather better than by the other *Scales*. Being thus provided; The first business must be

How to proportion the *Perpendicular Stile* to the *Plain*.

If your *Dial Plain* be small, consider whether it be *Direct*, or *Declining*; if *Direct* the *Sub-stile* may be placed in the *Middle*: If *Declining*, more to the side contrary to the *Coast* of *Declination*.

The *Sub-stile* well made choice of, and the *Perpendicular Stile*

Stile thereon *Erected*: Make the *Perpendicular Stile* the *Radius* (or Equal to the *Tangent* of 45 degrees) and make that part of the *Sub-stile* which lies beyond the foot of the *Stile* and towards the Centre, equal to the *Tangent Complement* of the height (of the Pole (or *Stile*) above the *Plain*: and the other part of the *Sub-stile*, below the *Foot* of the *Stile*, Equal to the *Tangent Complement* of the *Meridian Altitude* of the *Sun*, when he is in that *Tropick* which is to be most remote from the Centre of the *Dial*.

C H A P. XIV.

Of the Inscription of the Signs or Parallels of the Sun's Course.

A *Sign* is the Twelfth part of the the *Ecliptick*, and therefore contains 30 degrees.

A *Parallel*, is the *Sun's Diurnal Motion* Day by Day; and because there are 47 deg. between the two *Tropicks*, there may be so many *Parallels*, that is, *Circles*, which the *Sun* describeth every 24 Hours: and although there be 47 of these, yet in the *Latitude* of 51 deg. 30 min. we account but nine, viz. those which are the *Day* from *Sun* to *Sun*, when it is 8, 9, 10, 11, 12, 13, 14, 15, or 16 just hours long. The *Description* of these *Parallels* and *Signs* is made the same way; only due respect must be had to the quantity of the *Sun's Declination*: For (in all *Direct Horizontals*) the *Perpendicular Stile* being made *Radius*, the *Tangent Complement* of the *Sun's* height, in any *Sign* or *Parallel*; at any hour of the *Day*, set off from the *Foot* of the *Stile*, and extended to that *Hour-line*, gives a *Mark* upon the *Hour-line*, by which the *Parallel* of that *Day* shall pass: So that this *Work*, repeated so often as the number of *Parallels* to be inscribed, and the *Hour-lines* require; shall give respective *Points* enough, in each *Hour*, to draw each *Parallel* by.

Example. In the *Latitude* of 51 de. 32 m. the *Sun* being in the beginning (or entring) of *Pisces*; The *Sun's* height above the *Horizon* at every *Hour* may be found (by CASE IX of O. A. S. T.) to be as followeth, viz..

Att

Deg. M.

At	{	12	27.	01	The Complement	62.	59
		11	25.	37		64.	23
		10	21.	49		68.	11
		or				74.	03
		9	15.	57		81.	28
		8	8.	32			

Now, the *Perpendicular Stile* being *Radius*, the *Tangents* of the *Complements* of the respective *Altitudes*, as 62 deg. 59 m. the *Complement* of 27 de. 01 m. set from the foot of the *Per. Stile*, on the *Hour-line* of 12 (or *Sub-stile*) shall give a point thereon, by which the *Parallel* of *Pisces* must pass: And so, the *Tangent* of 64 de. 23 m. set from the foot of the *Per. Stile*, upon the *Hour-lines* of 11 and 1 a Clock, shall give you two other points by which the said *Parallel* shall pass: And so for all the rest of the *Hour-lines*, through which points found upon all the *Hour-lines*, a *Line* drawn by an even hand, shall be the *Parallel* required; for along that *Line* will the *Shadow* of the *Top* of the *Per. Stile* (as it creepeth along) pass, when the Sun is in the beginning of *Pisces*, viz. about the 9th of *February*.

And therefore, generally in *Verticals*, as also in all *Recliners*; that is to say, upon all *Plains* whatsoever: Draw an *Horizontal Dial* proper to the *Plain*, and inscribe the *Signs* or *Parallels* upon it, by setting off from the *Foot* of the *per. Stile*, the *Tangents* *Complements* of the *Sun's height* at every hour in the beginning of every *Sign* above that *Plain* (taken as an *Horizontal*, the *Foot* of the *per. Stile* being ever *Radius*) and at the end of these *Tangents* so set off upon every respective *Hour-line*, will be a *Point*: By which *Points*, *Lines* drawn with an even Hand, shall trace out upon the *Dial Plain*, the *Parallels* required.

Example. Suppose a *Plain* Decline 30 deg. and Recline 55 deg. the *Height* of the *Pole* above the *Plain* 19 deg. 25 min. And the *Sun's height* at the beginning of *Taurus* to be at the several *Hours*, as in this Table.

		De.	M.			De.	M.
At	{	12	38.	05	Complements	07.	55
		11	73.	30		16.	30
		10	60.	03		29.	57
		or				43.	59
		9	46.	01		58.	07
		8	31.	53		72.	13
		7	17.	47			

Then,

Then, The Tangents of the Complements of these *Hour-distances* (as 7 deg. 55 m. for 12 : 16 deg. 30 m. for the Hours of 11 and 1) set off from the Foot of the *per. Stile* (the said *Stile* being the *Radius* to those *Tangents*) to the obscure *Horizontal Hours* of 12, 11, 10 : and 1, 2, 3, &c. give the true *distances* between the Foot of the *Stile*, and those *auxiliary Hours*, for the *Parallel* of *Taurus* ; and so points for the describing of other *Parallels* of *Declination* : Having first (by *Trigonometrical Calculation*) found the *Horizontal Distances*, and the *Sun's Altitude* at his entrance into those *Parallels* of *Signs* or *Declination*, in such *Latitude* as you have need of. All which are taught how to do in the foregoing parts of this Book.

C H A P. XV.

Of the Inscription of the Vertical Circles (commonly called Azimuths) upon all Dial Plains.

THESE are great *Circles* of the *Sphere*, whose *Poles* lie in the *Horizon*, and intersect one another in the *Zenith* and *Nadir* Points of the *Place* wherein the *Dial* is to stand.

The whole *Horizon* being divided into 32 equal parts ; these *Circles* passing through those *Divisions*, are called *Points* of the *Compass*, and denominated accordingly, as *South*, *S by E*, *SSE*, &c. But the better way of accounting them is by 10, 20, 30, &c. *Degrees* from the *Meridian* on either side thereof.

First, in all *Horizontal Dials* ; the *Perpendicular Stile* being chosen, making the Foot thereof the *Centre* ; at any convenient distance, describe a *Circle* ; and account from the *Meridian* both ways, *Arches* equal to 10, 20, 30, &c. *Degrees* : From which *Divisions*, right Lines drawn to the Foot of the *Stile* aforesaid, shall represent those *Azimuths* upon that *Dial*.

Secondly, Upon a *Prime Vertical* (or *South*) *Dial* : Through the Foot of the *Per-stile*, draw a Right-Line Parallel to the *Horizon* ; and making the said *Stile Radius* ; upon the *Parallel Line*, set off, both ways from the *Meridian Tangents* of 10, 20, 30, &c. *Degrees* ; through which *Divisions*, Right-lines drawn, all at *Right Angles* with the *Parallel Line*, shall be the *Azimuths*.

E e e

Thirdly,

Thirdly, Upon any *Declining Vertical*, the same being done, shall give the *Azimuths* of 10, 20, 30, &c. degrees from the *Meridian* of the *Plain*; or from the *Meridian* of the *Place*, just allowance being made for the *Difference* of *Meridians*.

Fourthly, In *South Declining Reclining Plains*, the *Per. Stile* being chosen, and made the *Radius*, the *Tangent Complement* of the *Reclination*, applyed from the *Foot* of the *Per. Stile* to the *Meridian* of the *Place*, shall determine the *Zenith* of the *Place*: through which, and the *Foot* of the *Stile*, (that is the *Zenith* of the *Plain*) a right Line drawn, shall be a *Perpendicular* to the *Horizontal Line*, and shall concur with the *Æquator* in the *Hour-line* of 6; and therefore, if from the *Foot* of the *Stile* upon the said *Perpendicular*, towards the *North* (for the former application was made towards the *South*) be set off the *Tangent* of the *Reclination*, a Line drawn from the end thereof, at *Right Angles* with it, shall be the *Horizontal Line*: Upon which, the *Tangents* of 10, 20, 30, &c. (the *Secant* of the *Reclination* being now made *Radius*) set from the said *Right Angle*; Lines drawn from them to the *Zenith* of the *Place*, shall be the *Azimuths*.

Fifthly, The *Distance* between the *Meridians* being known, upon the *Horizontal Line*; the *Azimuths* which were accounted from the *Meridian* of the *Plain*, may be fitted for the account from the *Meridian* of the *Place*, with ease. — For *Example*, let that distance be the *Tangent* of 20 deg. Then that *Azimuth* which is 10, from the one; is 10 from the other also: And that which is 30 on the same side of the *Sub-stile*, is 10 on the other side of the *Meridian* of the *Place*: And the like method serves for any distance.

C H A P. XVI.

Of the Inscription of Almicanter or Circles of the Sun's Altitude upon Dial Plains.

THESE are lesser *Circles* of the *Sphere*; and may be called the *Parallels* of *Declination* from the *Horizon*; they having in all respects, the same relation and habitude to the *Azimuths*, as the *Signs* and *Parallels* of *Declination* have to the *Meridians*; although these be counted by 15 deg. and those usually by 10. And

And therefore, as in the description of the *Signs* and *Parallels*; so in these,

Let an *Horizontal Dial*, proper to the *Plain*, be first (obscurely) described; and then, as it was there shewed, that the points through which the *Signs* or *Parallels* must pass, upon every *Hour-line*, might be had by applying the *Tangents* of the *Complements* of the *Sun's height* of those *Hours* in those *Parallels*, from the *Foot* of the *Per. Stile*, to the respective *Hour-lines*: — So here, making use of that *Azimuth* which is perpendicular to the *Plain*, (which in all *Plains* is that which passeth through the *Foot* of the *Per. Stile*) the rest of the *Azimuths* being also inscribed, the *Tangents Complements* of the *Sun's height* above the *Plain*, when he is in any *Azimuth*, applyed from the *Foot* of the *Per. Stile* to the said *Azimuth*, gives a *Point*, through which that *Circle* or *Almicenter*, upon that *Azimuth* must pass.

Now to know what *Altitude* the *Sun* will have, when he will be upon any *Azimuth*, in any *Parallel* of *Declination* (or degree of the *Ecliptick*) is taught in the *Section* of *Astronomical Problems*; Or, by the Resolving of an *Oblique Angled Spherical Triangle*, wherein is always Given, *Two Sides*, and the *Angle* opposite to one of them to find the third *Side*, (By *CASE V.* of *O. A. S. T.*) Which *third Side* so found, is the *Complement* of the *Altitude* which is in this case required; and must accordingly be set from the *Foot* of the *Per. Stile* unto the *Azimuths*, &c.

CHAP XVII.

How to inscribe the Jewish, Babylonish and Italian Hours, upon all Dial-Plains.

FOR the Inscription of these *Hours* upon *Dial-plains*, there needs no *Trigonometrical Calculation*: For the two *Tropicks*, the *Equator*, and other *Parallels* of *Declination* being already described (or such of them as shall be needful) together with the common *Hour-lines* proper for the *Plain*, Points through which these *Hour-lines* may pass, may be found by these following Directions.

The *Babylonish Hours* are accounted *Equal Hours* from *Sun-rising*, and may be inscribed upon any *Plain*, by help of those two *Pa-*

parallels of Declination which shew the *Longest* and *Shortest Day*, consisting of *whole* (or entire) Hours: As with us 16 and 8; and the *Æquator*. For,

A Line drawn $\left\{ \begin{array}{l} 9 \\ 7 \\ 5 \end{array} \right\}$ In the Parallel of $\left\{ \begin{array}{l} 8 \text{ of Declination} \\ \text{Æquinoctial} \\ 16 \text{ of Declination} \end{array} \right\}$ Shall

be the *Hour* of *One* from the *Sun's rising*. And likewise in the same order; through 6, 8 and 10, shall give the *Second Hour* from *Sun-rising*; and in the like *Order*, all the rest.

In Winter, when the *Parallel* of 8 Hours shall fail, the other two Points will serve to draw it by, because those Hours are *Streight Lines*. But after the first *Six Hours* are inscribed, the *Æquinoctial* also failing, some other *Diurnal Arch*, (as of 9 or 10 Hours) must be described to supply that want.

The *Italian Hours* are accounted by 1, 2, 3, &c. from *Sun-setting*: And for the *Inscription* of these, the same *Parallels* of 8 and 16 Hours, with the *Æquator*, will serve: For a Line drawn through them, in the Hours of 9, 7 and 5, *Afternoon* (observing the same as before) shall be the *hour* of *One*; the like through 7, 5 and 3, shall be the *hour* 23: The *Night hours* of 9, 10, &c. are the *Morning hours* produced.

The *Jewish hours* are reckoned like the *Babylonish*, from *Sun-rising*; but unequally; their *Sixth hour* being (always) *Noon*; and every *hour*, one *Twelfth* part of the *Artificial Day*, of what length soever that be.

For the *Inscription* of them: The *Vulgar Hours* proper for the *Plain* being first drawn, and the *Diurnal Arches* of 15, 12; and 9 hours, divide the degrees in each by 12; and that Quotient by 15; or else (which is all one) divide the said *Arches* by 180, the three Quotients shall give the just *Times*, in hours, and usual parts of hours from 12 a Clock, upon the two *Parallels* and the *Æquator*: through which, *Lines* drawn by a *Ruler*, shall be the *Jewish hours* required.

Example. In Latitude 51 deg. 32 min. the *Diurnal Arch* of 15 hours is in Degrees 225, which divided by 180, the Quotient is $1\frac{1}{4}$ h. and so much the *Jewish hours* of 5 and 7 are distant from *Noon*; one hour and a quarter being a *twelfth part* of the *Diurnal Arch* of 15 hours: And this *hour* and *quarter* being doubled, gives the place for 4 and 8: Tripled, the place of 3 and 9, &c. from *Noon*, upon that *Parallel* of 15 hours.

In like manner, the *Diurnal Arch* of 9 hours is 135 deg. which divided by 180, the Quotient is $\frac{3}{4}$, that is 3 quarters of an hour: Which shews the place of the *Jewish hours* of 7 and 5, to be three quarters after, or before, *Noon*: and that doubled is *One hour and a half*, which gives the place of 8 and 4; all one with our 1 $\frac{1}{2}$ and 10 $\frac{1}{2}$; and so *Tripling* and *Quadrupling* and *Quintupling* 3 quarters, you have the places of the *Jewish hours* upon this *Parallel* of 9 hours length of the Day.

And these parts *Doubled* and *Tripled*, as is said, will always (in this *Parallel* and the former) fall upon *even hours, halves and quarters of hours*: And that is the only reason why these two *Parallels* of 15 and 9, are preferred; there being no necessity of using them, more than the *Tropicks* or other *Parallels*, only this convenience of even parts.

Lastly. In the *Diurnal Arch* of 12; that is, the *Aequator*, the *Common* and the *Jewish hours* concur; that is, the *Jewish hours* of 5 and 7, with our *hours* of 11 and 1: Their 4 and 8, with our 10 and 2, &c. So that a *Line* drawn from 1 $\frac{1}{2}$ in the *Parallel* of 15, to 1 in the *Aequator*, and from thence to $\frac{3}{4}$ in the *Parallel* of 9, is the 7th *Jewish hour*. And so are all the rest to be inscribed.

Trigonometria Practica.

SECTION VI.

OF NAVIGATION.

I Intend not here to treat of *Navigation* in the general; it being an Art that requires (for the true Understanding, either the Theory, or Practice of it) an inspection into divers other *Sciences Mathematical*; of which, that of *TRIGONOMETRIA*, (or the *Doctrine of Triangles*) is the *Principal*; for that the solution of all such *Problems* which are of daily Use at Sea, are performed thereby; and those are such as concern *Longitude*, *Latitude*, *Rumb* (or course) and *Distance*, &c. And therefore, I shall *Define*, First, what is meant by *Longitude*, *Latitude*, *Rumb*, *Distance*, &c. And Secondly, any two of them being *known*; how to find the other two; and that by *Trigonometrical Calculation*: with some other *Problems* pertinent to that Art. And I shall perform them, (1) By Plain Sailing. (2) By Mercators Sailing; And (3) By the Middle Latitude.

DEFINITIONS.

- I. *Longitude*, Is the *Distance* of a *Place* from some known *Meridian* to that *Place*; and is always counted upon the *Æquinoctial Circle*, from that known *Meridian* towards the *East* or *West*.

II. *La-*

II. Latitude, Is the Distance of any Place from the *Æquinoctial* Circle; counted upon that *Meridian* Circle which passeth over that Place, towards either of the *Poles*; either *North* or *South*: and accordingly the *Latitude* is Denominated either *North* or *South Latitude*.

III. Rumb, (or Course) Is that *Angle* which a Ship in her *Sailing* makes with the *Meridian* of the Place from whence the Ship came, and the Place where the Ship then is: But the *Complement* of the *Rumb*, is that *Angle* which the *Rumb* makes with that *Parallel* of *Latitude* in which the Ship is; and is the *Complement* of the *Rumb* to 90 deg. — The *Rumb* is made known to the *Mariners* at all times, by help of his *Compass*.

IV. Distance, Is the Number of *Leagues*, *Miles*, *Centesms*, &c. that a Ship hath *Sailed* upon any *Rumb* or *Course*. — And this is known to the *Mariner* by the *vering* (or running out) of the *Log-Line* in any known quantity of *Time*.

V. If your Rumb (or Course) be directly *East* or *West*, you alter not your *Latitude* at all: — If your *Course* be *Northward*, you continually *Raise* the *North Pole*; and you increase your *Latitude Northward*: Or the *South Pole*, if you *Course* be from the *Æquinoctial* *Southward*.

So that

The *Raising* of the *Pole* is, When you Sail from a *Lesser* *Latitude* to a *Greater*: And the *Depressing* of the *Pole* is, when you Sail from a *Greater* to a *Lesser* *Latitude*.

These things known; before I proceed to the *Solution* of the several *Questions* or *Problems* in *Navigation*, unto which the *Doctrine* of *Triangles* is subservient, it will be necessary to say something concerning the *Situation* of *Places* upon the *Earth* or *Sea* in respect of *Longitude* and *Latitude*: And of their *Distances* one from another in *Leagues*, *Miles*, or *Minutes*: And then, shew how to lay down (upon a *Blank Chart*) any two (or more) *Places*, according to their respective *Latitudes* and *Difference* of *Longitudes*. All which shall be comprehended under these *Heads* following.

I. Of

I. Of the Situation of Places, in respect of Longitude and Latitude,

IN this Problem there are variety of Cases, according as the places may be situate one from the other, in respect of the *First*, or *General Meridian*, as to their *Difference of Longitude*; and in respect of the *Æquinoctial*, as to their difference of *Latitude*: And for the better understanding of these *Varieties*, I shall exhibit all of them in one *Scheme*: In which,

Fig. LXI. N Æ S is the *First Meridian*, or beginning of *Longitude*.
 W Æ E the *Æquinoctial Circle*.
 Æ E the *East part* thereof.
 Æ W the *West part* thereof.
 N the *North Pole*.
 S the *South Pole*.
 D B and T A, two *Parallels of North Latitude*.
 O R and Z X, two *Parallels of South Latitude*.
 N I L M S a *Meridian of West Longitude*.
 N H C S a *Meridian of East Longitude*.

CASE I. If the Places lies in the *Æquinoctial Circle*, as the Places L, G, H, and K; and so have no *Latitude*— Or in the same *Parallel of Latitude*, as the Places D, V, X, and B, in the *Parallel of 40 de. of North Latitude*; and the Places O, M, C and R in the *Parallel of 35. 85 deg. of South Latitude*: Then, (1.) If the Two Places proposed, do lie both on the *East-side* of the *First Meridian*, as H, K— X, B—C R: Or, both on the *West-side*, as D, V— L, G—O, M: Then, The *Lesser Longitude* subtracted from the *Greater*, gives the *Difference of the Longitudes* of those Places. But, (2.) If of the two Places proposed, one do lie on the *East-side*, and the other on the *West-side* of the *First Meridian*: As V and X— L and H, M and C: Then, the two *Longitudes* added together, gives the *Difference of Longitude* between those two Places. But, Note,

If the *Sum* of the two *Longitudes* do exceed 180 deg. subtract the *Sum* of them from 360 deg. and the *Remainder* is the *Difference of Longitude* of those two Places.

CASE

CASE II. If One Place lie in the *Æquinoctial Circle*, as L, and the other under the same *Meridian* N L S, as the Places V, in the *Parallel* of 40 deg. of North, and M, in the *Parallel* of 35. 85 deg. † of South Latitude: Then, the Latitudes themselves are esteemed (or are to be taken) for the Difference of Latitude.

Fig.
LXI.

† In all this Section, the Degrees are not divided into 60 min. but into Centesims, or hundred Parts.

CASE III. If Two Places proposed differ both in Latitude and Longitude, do lie both of them on the North, as V in the *Parallel* of 40 deg. and I in the *Parallel* of 18. 25 deg. the lesser Latitude taken from the greater, leaves 21. 75. deg. for the Difference of Latitude of those Two Places: The like, if both the Places had lain on the South-side of the *Æquinoctial*, as at C and Y;—But if the Two Places proposed had been V, in 40 deg. of North Latitude, and C, in 35. 85 deg. of South Latitude; then, the Sum of them two Latitudes 75. 85 deg. would be the Difference of Latitudes; which can never exceed 90 deg.

How to Reduce Degrees and Minutes, (or Degrees and Centesims, or hundred parts of Degrees) into Miles, or Minutes.

THIS is done by multiplying the Degrees by 60. and adding the odd Minutes, if any be, and the Product will be Miles. Also Degrees, and Centesims of Degrees, multiplied by 60, the Product will be Degrees, and Centesims. So,

26 de. 33 m. multiplied by 60: The Product of 26 by 60, is 1560, to which add the 33 Minutes, and it makes 1593 Miles.

Or, 72. 75 deg. multiplied by 60, will produce 4365 Miles: And 26. 47 deg. by 60, will produce 1588. 20 Miles; that is 1588 Miles, and $\frac{2}{10}$ or $\frac{1}{5}$ of a Mile; and so of any other.

III. How to lay down (upon a Blank Chart) any two Places, which differ both in Longitude and Latitude.

LET the two Places proposed be the Harbour of Jamaica, which lies in 18. 25 deg. of North Latitude, and 78. 35 deg. of West Longitude—And the Cape of Good Hope; which lies in 35. 85 deg. of South Latitude, and 27. 50 deg. of East Longitude.

The Geometrical Construction of the Figure.

First, Draw a Right-Line $\text{Æ} \text{Æ}$ for the *Æquinoctial Circle* of the *Terrestrial Globe*: And another Right Line N S , for the *Principal*, or *First Meridian*, from whence the *Longitude* of *Places* begin to be accounted either *East* or *Westward* from it: which two Lines cross each other at Right Angles in the Point α .

Secondly, The *Longitude* of *Jamaica* being 78. 35 deg. Reduce them into *Miles* (by multiplying them by 60) and they make 4701 *Miles*, which taken from any *Scale of Equal Parts* set upon the *Æquinoctial Circle* from α to a , towards the *West*, because the *Longitude* was *Westward*: And through that point a , draw a Right Line $na s$, parallel to $\text{N} \alpha \text{S}$, the *First Meridian*, so shall that Line $na s$, be the *Meridian* under which the Harbour of *Jamaica* lieth.—Also the *Longitude* of the *Cape of Good Hope*, being 27. 50 deg. Reduce them into *Miles* (by Multiplying them by 60) and they make 1650 *Miles*, which set from Æ to b , and through b , draw a Right Line $nb s$ parallel to $\text{N} \alpha \text{S}$, so shall this Line $nb s$, be the *Meridian* under which the *Cape of Good Hope* lyeth.

Thirdly, The *Latitude* of *Jamaica Harbour* being 18. 25 degrees, reducethem into *Miles*, and they make 1095, which set upon the Line $na s$ from a to H , towards n (because the *Latitude* was *North*) so shall H be the Point representing the *Harbour* of *Jamaica*. And, If you draw a Line through this point H , parallel to the *Æquinoctial*, as the Line Hb , that Line shall represent the *Parallel* of the *Latitude* of the *Harbour* of *Jamaica*, viz. of 18. 25 deg.—Also, The *Latitude* of the *Cape of Good Hope* being 35. 85 deg. Reduce them into *Miles*, and they make 2151, which set upon the Line $nb s$, from b downwards towards s , (because the *Latitude* is *South*) to C , so shall the point C , be the Point representing the *Cape of Good Hope*: Through which, if you draw a Right Line CA , parallel to the *Æquinoctial*, it shall represent the *Parallel* of the *Latitude* of the *Cape of Good Hope*, viz. of 35. 85 deg. of *South Latitude*.

Lastly, Draw the Lines HC , CA and AH , so shall you have constituted a *Right-lined Triangle*, Right-angled at A ; from whence shall be deduced and resolved all such *Problems* and *Navigation*, as concern *Longitude*, *Latitude*, *Rumb* (or *Course*) and *Distance*.

PROB.

P R O B. I.

TWO Places, H in 18. 25 de. of North Latitude, and 78. 35 d. of West Longitude : And C, in 35. 85 deg. of South Latitude; and in 27. 50 de. of East Longitude being Given : To Find,

I. The Rumb leading from one Place to the other, the Angle H C A.

II. Their Distance : upon the Rumb ; H C.

The Difference of the Longitudes, and Latitudes of the two Places being Found, and Reduced into Miles, as is shewed in the two foregoing Problems : Then, in the Triangle H A C Right-angled at A, you have given, (1.) The Perpendicular H A, the Difference of Latitude 3246 Miles : And, (2.) The Base A C, the Difference of Longitude 6351 Miles : By which you may find the Rumb H C A (by C A S E I) and the Distance H C (By C A S E VII) of R. A. P. T.

I. For the Rumb H C A.

As A C, the Dif. of Longitude 6351 M.

3.802842

Is to the Dif. of Latitude H A 3246 M.

13.511348

So is the Radius, Tang. 45 deg

10.

To the Tang. of 27. 07 deg.

9.708506

Which is the Angle H C A, whose Complement H C n — A H C, is the Rumb, 62. 93 deg. that is, North-East-erly 62. 93 deg. from C to H, and South-Westerly 62. 93 deg. from H to C.

2. For the Distance H C.

As the Sine of A H C the Rumb 62. 93 deg.

9.949610

Is to A C, the Dif. of Longitude, 6351 M.

13.802842

So is the Rad. Sine 90 deg.

10.

To H C their Distance, 7132. 4 M.

3.853232

P R O B. II.

THE Rumb C H A, 62. 93 deg. And the Difference of Longitude A C 6351 M. Given, to Find,

I. The Difference of Latitude H A, And

F f f 2

II. The

Fig.
LXII.

II. The Distance H C.

By CASE II. and CASE V : OF R. A. P. T.

1. For the Difference of Latitude H A.

As the Sine of A H C, 62. 93 deg.

Is to the Sine of H C A 27. 07 de.

So is AC, the Diff. of Longitude 6351.

To the Diff. of Latitude H A, 3246 M.

9.949610
9.658086
3.802842
13.460928
3.511318

2. For the Distance H C.

As the Sine of the Rumb A H C 62. 93 de.

Is to A C, the Diff. of Long. A C, 6351

So is Rad. S. 90 d.

To the Distance H C 7132. 4 M.

9.949610
13.802842
10.
3.853232

P R O B. III.

THE Distance H C 7132. 4 M. And the Difference of Longitude A C, 6351 M. Given ; to Find,

I. The Rumb A H C, And

II. The Difference of Latitude A H.

By CASE III and CASE VI, of R. A. P. T.

1. For the Rumb A H C.

As the Distance H C, 7132. 4.

Is to the Dif. of Longitude A C, 6351 M.

So is Rad. S. 90 de

To the Sine of the Rumb 62. 93 de.

3.853174
13.802842
10.
9.949668

2. For the Difference of Latitude H A.

Having found the Rumb, by the Last, Say.

As the Sine of the Rumb A H C, 62. 93 de.

Is to the Dif. of Longitude A C, 6351 M.

So is the Sine of H C A, 27. 07 deg.

To the Dif. of Latitude H A. 3246 M.

9.949610
3.802842
9.658086
13.460928

P R O B.

P R O B. IV.

THE Distance CH , 7132. 4 M. And the Difference of Latitude HA 3246 M. Given: To find,

I. The Rumb HCA ,

II. The Difference of Longitude AC .

By CASE III. and CASE VI. of R. A. P. T.

I. For the Rumb.

As the Distance HC 7132. 4 M.

3.853232

Is to the Dif. Latit. HA 3246

13.511348

So is the Radius Sine 90 de.

10.

To the Sine of 27. 07 de. HCA ,

9.658116

Whose Complement is AHC the Rumb 62. 93 de.

2. For the Difference of Longitude AC .

As the Radius Sine 90 de.

10.

Is to the Distance HC 7132. 4 M.

3.853232

So is the Sine of HCA 27. 07 de.

9.658086

To the Diff. of Longitude AC 6351 M.

13.511318

P R O B. V.

A Ship at H , the Harbour of *Jamaica*, in 18. 25 de. of North Latitude; and West-Longitude 78. 35. de. being bound for the *Lyzard*, which lyes in 50. 34 de. of North Latitude; and 5. 96 de. of West Longitude: And she sails upon these several Courses, viz. (1.) North-East 16. 25 deg. 42. 16 Miles from H to K .—(2.) North-East 60. 00 deg. 25. 36 Miles from K to L .—(3.) Directly North, 32. 00 Miles from L to M .—(4.) North-East, 60. 12 deg. 28. 16 Miles, from M to N .—(5.) North-East, 46. 50 deg. 40. 25 Miles, from N to O .—(6.) Directly East 28. 00 Miles, from O to P .—And now I would know,

I. The Difference of Latitude, PR , and Difference of Longitude, HR , at the end of each of these Courses.

II. How the Ship being at P , bears from the Harbour at H , from whence she came.

III. In what Longitude and Latitude she then is.

IV. The

IV. The nearest *Distance* from P to H.

I. The Geometrical Construction and (gradual) Trigonometrical Calculation, of the whole Travers.

DRAW a Right Line NS for the *Meridian*, and another EW at Right-angles thereto, for the *Parallel* of the *Latitude* of *Jamaica*, crossing each other at H, the point of the *Harbour*, where the *Voyage* begins. Then,

I. The First *Course* being *North-East* 16. 25 de. make the Angle a HK equal thereto: And the *distance* Sailed being 42. 16 Miles, set them from H to K; and through K, draw a K parallel to W E: So shall you have constituted a *Triangle* H a K, right-angled at a, in which there is given, (1.) the *Rumb* (or *Course*) a HK 16. 25 deg. (2.) the *Distance* Sailed upon that *Rumb*, 42. 16 Miles: by which you may find, (1.) the *Difference* of *Longitude* a K, 11. 80 M. And (2.) the *Difference* of *Latitude* H a 40. 47 M. By CASE IV. of R. A. P. T.

1. For the *Difference* of *Longitude* a K.

As the Radius Sine 90 deg.	10.
Is to the <i>Distance</i> sailed HK, 42. 16 M.	1.624900
So is the Sine of the <i>Rumb</i> a HK. 16. 25 d.	9.446892
To a K 11. 80 M. the <i>Difference</i> of <i>Long</i> .	*1.071793

2. For the *Difference* of *Latitude* H a.

As the Radius, Sine 90 deg.	10.
Is to the Co-sine of the <i>Rumb</i> a K H, 73. 75 de.	9.982294
So is the <i>Distance</i> sailed HK, 42. 16 M.	1.624900
To H a 40. 47 M, the Dif. of <i>Latitude</i> .	*1.607194
And in such difference of <i>Longitude</i> and <i>Latitude</i> will the Ship be, when arrived to K.	

II. The Second *Course* being *North-East* 60. 00 de. through the point K, draw a Right Line b e parallel to NS; And on the point K protract an Angle of 60. 00 deg. as b K L, and set the distance sailed 25. 36 M. from K to L; and through L, draw L b parallel to W E: Then in the *Triangle* b K L, there is given the Angle b K L the *Rumb* 60. 00 d. and the Side K L 25. 36 M

Of Navigation.

113

36 M. the distance upon the *Rumb*, whereby may be found *b L* the Differ. of *Long.* and *K b* the Difference of *Latitude*, as in the former *Course*.

Fig.
LXII.

1. For the Difference of *Longitude b L*.

As the Radius

10.

To the *Distance Sailed K L* 25. 36 M.

1.404149

So is the Sine of the *Rumb b K L* 60. 00 d.

9.927520

To the Difference of *Longitude b L* 21. 96 M.

✱1.341679

2. For the Difference of *Latitude K b*.

As the Radius

10.

To the *Distance Sailed K L*, 25. 36 M.

1.404149

So is the Co-sine of the *Rumb b L K* 30 00 d.

9.698970

To the Difference of *Latitude K b* 12. 68 M.

✱1.103119

III. The Third *Course*, being directly North, and the *Distance Sailed* 32. 00 M. Through the Point *L*, draw a Right Line *L c* parallel to *N S*. And because the *Course* was directly North, set 32. 00 M. (the *Distance sailed*) from *L* to *M*; and to that Point is the Ship now arrived; which being in the same *Longitude* as when she was at *L*, the difference of *Longitude* is nothing. But the difference of *Latitude* is the same with the *Distance Sailed*; viz. 32. 00 Miles, or Minutes. So that for this *Course* there is no need of any *Trigonometrical Calculation*:

IV. The Fourth *Course* being North-East 60. 12 deg. continue the Line *L M* to *c*. And on the Point *M*, protract the Angle of the *Rumb*, *c M N* 60 12 m. and set the *distance sailed* 28. 46 M. from *M* to *N*: And through *N*, draw *N c*, parallel to *W E*. Then, in the Triangle *M c N*, there is given the Angle *c M N* 60. 12 de. the *Course* or *Rumb*: And the Side *M N* 28. 46 M. the *Distance Sailed*; by which you may find the Side *c N*, the Difference of *Longitude*; and *M c* the Difference of *Latitude*: In the same manner as in the two first *Courses*.

1. For the Difference of *Longitude c M*.

As the Radius

10.

Is to the *Distance Sailed*, *M N* 26. 46 M.

1.422589

So is the Sine of the *Rumb c N M*, 60 12 d.

9.938054

To the Difference of *Longitude c N* 22. 94 M.

✱1.360643

2. For

Fig.

LXII. 2. For the Difference of *Latitude* M c.

As the Radius

Is to the *Distance* Sailed M N, 26. 46 M.

So is the Co-sine of the *Rumb* c N M 29. 88 M.

To the Difference of *Latitude* M c, 13. 18 M.

10.

1.422589

9.697391

✱1.119980

V. The Fifth *Course* being *North-East* 46. 50 d. through the Point N, draw a Right Line d N f, parallel to the *Meridian* N S: And upon the Point N, protract the *Angle* of the *Rumb*, d N O, 46. 50 deg. And set the *Distance* sailed 40. 25 M. from N to O; and through O, draw d O parallel to W E. Then, in the Triangle N d O, you have given the *Angle* d N O the *Rumb*, 46. 50 deg. And the Side N O, the *Distance* Sailed 40. 25 M. by which may be found, the difference of *Longitude* d O, and the difference of *Latitude* N d, as before.

1. For the Difference of *Longitude*, d O.

As the Radius Sine 90 de.

To the *Distance* sailed, N O 40. 25 M.

So is the Sine of the *Rumb* d N O, 46. 50 d.

To the Difference of *Longitude* d O, 29. 19 M.

10.

1.604766

9.860562

✱1.465328

2. For the Difference of *Latitude* N d.

As the Radius

To the *Distance* Sailed, N O, 40. 25 M.

So is the Co-Sine of the *Rumb* d O N, 43. 50 d.

To the Difference of *Latitude* N d 27. 70 M.

10.

1.604766

9.837812

✱1.442578

VI. The Sixth *Course* being directly *East*: And the *Distance* Sailed 28. 00 Miles, continue the Line or *Parallel* d O, and set the *Distance* sailed upon it, viz. 28. 00 M. from O to P, and to that Point is the Ship now arrived: which being in the same *Parallel* of *Latitude* as when she was at O, the Difference of *Latitude* is nothing: But the Difference of *Longitude* is equal to the *Distance* Sailed; namely 28. 00 M. So that for this *Course*, there needs no *Trigonometrical* Calculation.

Fig.

LXIII.

Having thus finished all the *Courses*, you find the *Difference* of *Longitude* and *Difference* of *Latitude* to be such as are set down in the *Scheme* of the *Travers*. And thus is the *First* part of the *Problem* resolved. Now for the *Second*.

VII. From

VII. From the Point P (in your *Travers*, Fig. LXIII) draw a Right Line to H, the *Harbour* from whence you began, and from the same Point P, let fall the *Perpendicular* P R, upon the Line W E: So shall you have constituted a *Right-angled Plain Triangle* H R P, Right-angled at R: By the Solution whereof, the other parts of the *Problem* may be *Trigonometrically* resolved.

VIII. Collect the several *Differences* of *Longitude* and *Latitude*, into one Sum as is done in the following Table.

		Difference of Longitude is		Difference of Latitude is	
		Miles		Miles	
In the Course from	H to K	a K	11. 80	H a	40. 47
	K to L	b L	21. 96	K b	12. 68
	L to M	M		L M	32. 00
	M to N	c N	22. 94	M c	13. 18
	N to O	d O	29. 19	N d	27. 79
	O to P	O P	28. 00	O	
		H R.	113. 89	P E.	126. 03

By the Table you find the Sum of all the *Differences* of *Longitude* to be 113. 89 Miles, or Minutes: Which reduced into *Degrees* and *Minutes* (by dividing them by 60) they make 1. 91 deg. Which subtracted from the *Longitude* of *Jamaica*, 78. 35 deg. there remains 76. 34 deg. for the *Longitude* in which the Ship is, being arrived to P.

Also, you find by the Table, the Sum of all the *Differences* of *Latitude* to be 126. 33 Miles: which reduced into *Degrees*, makes 2. 10. deg. Which being added to 18. 25 deg. the *Latitude* of *Jamaica*, it makes 20. 35 deg. for the *Latitude* in which the Ship is at P. And thus is the *Second* part of the *Problem* resolved.

Now for the *Third*:

IX. The next thing to be found is the *Rumb*, *Course*, or *Bearing* of the Ship, the being at P, to H, the *Harbour* from whence she came.—In the Triangle P R H, there is given, (1) The Side

G g g

Fig. LXIII. Side H R, the *Difference of Longitude* 113. 89 min. (2) The *Difference of Latitude* P R, 126. 03 min. To find the Angle H P R, the *Rumb*: by CASE I. of R. A. P. T.

As the Side P R, *Dif. Latit.* 126. 03 M. 2.100474
 Is to the Side H R, *Dif. Lon.* 113. 89 M. 12.056485
 So is Radius, *Tang.* 45. 00 de. 10.
 To the *Tangent* of the *Rumb* H P R 42. 11 d. 9.956011
 So that the *Course* from P to H is *South-West*, 42. 11 deg. and
 from H to P, *North-West* as much.

X. The last thing to be found is, the *shortest Distance* (or *Distance* upon the *Rumb*) between H and P: By CASE V. of R. A. P. T.

As the *Sine* of the *Rumb* H P R, 42. 11 d. 9.826435
 Is to the *Difference of Longitude* H R, 113. 89 M. 12.056485
 So is the Radius, *Sine* 90 d. 10.
 To the *Distance* upon the *Rumb*, P H, 169. 83 M. 2.230050

P R O B. VI.

IF from the *Latitude* of 18. 25 deg. at H, I sail upon some *Rumb* between the *North* and the *East*, as H P 169. 83 Miles; and then find that I have departed from the *Meridian* from whence I came 113. 89 min. I would then know,

- I. Upon what *Rumb* I steered. And
- II. How much I have altered my *Latitude*.

THIS Problem may be resolved in the *Triangle* H R P: In which there is *Given*, (1) The Side H P, the *Distance Sailed* upon the unknown *Rumb*, 169. 83 M. (2) The Side H R, the *Departure* from the *Meridian* of H, from whence you sailed, 113. 89 M. by which you may find the Angle P H R, the *Rumb* sailed upon: By CASE III, of R. A. P. T.

As the *Distance* sailed H P, 169. 83 M. 2.229014
 Is to the Radius, *Sine* 90 de. 10.
 So is the *Departure* H R, 113. 89 M. 12.053191
 To the *Sine* of P H R, 42. 11 deg. 9.824177
 Whose *Complement* 47. 89 deg. is the Angle P H R, that is
North East 47. 89 de. the *Rumb*.

Then

Of Navigation.

316

Fig.
LXIII.

Then for the Difference of *Latitude* P R.

As the Co-sine of the <i>Rumb</i> H P R, 42. 11 d.	9.826435
Is to the <i>Departure</i> H R, 113. 89 M.	2.056485
So is the Sine of <i>Rumb</i> P H R, 47. 89 d.	9.870321
	11.926806

To the Side P R 126. 03 M. the *Diff.* of *Latit.* 2.100371
 Which reduced into deg. makes 2. 10 deg. which is the difference of *Latitudes*, which being added to 18. 25 deg. The *Latitude* of H, from whence you came, makes 20. 35 de.
 For the *Latitude* in which you are at P.

The Eight foregoing *PROBLEMS*, were all resolved by *Right-angled Plain Triangles* : These which follow, fall under the *CASES* of *Oblique-angled Plain Triangles*.

P R O B. VII.

A Ship at C, in 22 deg. of *North Latitude*, Sails *North-East* 22. 50 deg. 64. 50 *Miles* to D: And from D, she Sails between the *South* and the *East* 124. 00 *Miles* : and then finds that she is in the same *Latitude* from whence she first came : I would now know.

Fig.
LXIV.

- I. What was the *Rumb*, (or *Course*) from D to F.
- II. The Difference of *Longitude* between C and F.
- III. What *Latitude* the Ship was in, when she was at D.

The Geometrical Construction of the *Figure*.

First, Draw a Right Line N S for the *Meridian*; and at Right Angles thereto another W E, for the *Parallel* of *Latitude* 22. 00 deg. *North*.

Secondly, Upon E, Protract the Angle of the *Rumb North-East* 22. 50 deg. a C D and set thereon 64.50 M. the *distance sailed* from E to D; and through the point D, draw an obscure Line D a, parallel to the Line W E.

Thirdly, Take the *Distance Sailed* from D to F 124.00 *Miles* in your *Compasses*, and setting one foot in D, with the other describe the Arch b c, crossing the *Parallel* of *Latitude* W E, in the point F.

G g g 2

Fourthly,

Fig. LXIV. Fourthly, Joyn CD and FD, so shall you have constituted an Oblique-angled Triangle DCF, In which there is Given (1) The Side CD, 64. 50 M. (2.) The Side DF 124. 00 M. and (3.) The Angle DCF, 67. 50 de. the Complement of the *Rumb* sailed upon from C to D; to find (1.) The Angles at D and F, the *Rumbs*: (2.) The Difference of *Longitude* between C and F: And (3.) The Difference of *Latitude*, Ca.

By Trigonometrical Calculation.

1. For the *Rumb* (or *Course*) DFC, By CASE I. of O. A. P. T.

As the Dist. sailed from D to F 124. 00 M.

Is to the Co-sine of the *Rumb*, from C to D, 22. 50 d.

So is the *Distance* sailed, from C to D 64. 50 M.

2.093421

9.965615

1.809559

11.775174

To the Sine of the Angle DFC 28. 72 d.

9.681753

Whose Complement 61. 28 de. is the *Rumb* from F to D, that is *North-West* 61. 28 de: And *South-East* 61. 28 de. from D to F.

2. For the *Difference of Longitude* CF.

The Angles at C and F being known, the Angle at D is also known, and will be found to be 96. 22 de. Then, As the Sine of DFC, 28. 72 deg.

9.681720

Is to the *Distance* sailed from C to D 64. 50 M.

1.809559

So is the Sine of the Angle CDF, 83. 78 de.

9.997411

11.806970

To the *Diff.* of *Longitude* CF, 133. 43 M.

1.125250

3. For the *Diff.* of *Latitude*, DG, equal to Ca. By CASE IV. of R. A. P. T.

As the Radius Sine 90 de.

10.

To the *Dist.* sailed from C to D, 64. 50 M.

1.809559

So is the Co-sine of the *Rumb* from C to D 67. 50 d.

9.965615

To the *Dif.* of *Latitude* DG = Ca, 59. 59 L.

✱ 1.775174

PROB

P R O B. VIII.

TWO Ships set Sail, from the Port P, One of them Sails
North-West 70. 00 deg. 106. 00 Miles to O; The other Sails
South-West 74. 00 deg. 236. 60 Miles, to M. I demand,

Fig.
LXV.

I. How the two Ships at O and M, do Bear from each other :
And

II. Their Distance M O.

The Geometrical Construction.

Having drawn your Meridian N S, and Parallel W E, crossing each other at Right Angles in the point P, which let be the Port from whence the two Ships sailed: Then,

First, Upon P, describe an Arch of a Circle, as *n o m*: And because the Course from P to O, was North-West 70. 00 deg. set 20 deg. (the Complement thereof) from *o* to *m*; and through *m*, draw a Line, as *P m O*, setting thereon 106. 00 Miles, from P to O.

Secondly, Because the Course from P to M, was South-west 74. 00 deg. set 16. 00 deg. (the Complement thereof) from *o* to *n*, And through *n*, draw *P n*, setting thereon, the Distance sailed, viz. 236. 60 Miles from P to M.

Thirdly, Joyn O and M. So shall you have constituted an Oblique-Angled Triangle P O M: In which there is Given, (1.) The Angle O P M, 36 00 deg. the Sum of the Complements of the Bearings of the two Ships. (2.) The two Sides P O 106. 00 Miles and P M 236. 60 Miles. The several Distances which the two Ships sailed: And by help of these, you may Find. (1) The Bearing of the two Ships one from the other: And (2.) Their Distance.

By Trigonometrical Calculation.

I. For the Bearings, O M P and M O P ;

By CASE II. of O. A. P. T.

The Sum of the Sides, P O and O M, is 342.06. M.

Their Difference. 130.06. M.

The half Sum of the Un-known Angles M and O, is 72.00. De.

Being thus prepared, say,

As

Fig. LXV. As the *Sum* of the *Sides* 342. 06 2.534102
 Is to the *Difference* of the *Sides* 130. 06 2.113144
 So is the *Tangent* of the $\frac{1}{2}$ *Sum* of the *Angles* 72 d. 10.448224
12.561368
 To the *Tangent* of 46. 80 degrees. 10.027266
 Which 46. 80 deg. *Added* to 72. 00 deg. (the *half Sum* of the
Angles O and M) gives 118. 80 d. for the *Angle* M O P;
 And 46. 80 d. *subtracted* from 72 00 d. leaves 25. 20 deg.
 for the *lesser Angle* O M P.

Now for the *Bearing*, *subtract* the *Bearing* of the Ship from P to O, viz. 72. 00 deg. from 118. 80 deg. (the quantity of the M O P) there will remain 48. 80 deg. for the *Angle* M O Q, which is *South-West* 48. 80 d. and so doth the Ship at O *Bear* from the Ship at M. And the Ship M, from O, *North-East* 48. 00 deg.

2. For the distance of the Ships M and O.

As the *Sine* of M O P, 118. 80 d. (61. 20) 9.942658
 Is to the *Distance* sailed from P to M. 236. 60 d. 2.374014
 So is the *Sine* of O P M, 36. 00 deg. 9.769218

12.143232
 To the *Distance* O M 158. 70 M. 2.200576

P R O B. IX.

Fig. LXVI. THERE are Two Islands, as K and M, which lie directly North and South of each other, and are distant 37. 75 Miles; And there is a third Island at L, which is distant from that at M, 59. 64 Miles; and from that at K, 80. 92 Miles: Now I would know, How the *Island* at L, bears from the other two Islands M and K.

The Geometrical Construction.

Having drawn NS for the *Meridian*, and WE for a *Parallel* of *Latitude*, crossing each other in the point K, which let be the place of one of the *Islands*; Then

The *Island* M, lying directly North therefrom, at the Distance of 37. 75 M. Set 37. 75 from K to M. — And because the *Island* at L is distant from the *Island* at K, 80. 92 M, with that distance, set one foot of the *Compasses* in K, and with the other describe the Arch *a b*. — Also, the distance of the *Island* at L, being

being distant from M, 59. 64 Mil. with that distance set one foot in M, and with the other describe the arch *cd*, crossing the former arch *ab*, in L. — Lastly, Join L M, and L K, so shall you have constituted an *Oblique-Angled Triangle* K L M, in which all the three Sides are Given. And the Angles at K and M are Required.

The Trigonometrical Calculation, As the Greatest Side K L, 80. 92 M.

Is to the Sum of K M and L M, 97. 39 M.

So is the Difference of those Sides, 21. 80 M.

To the Part of the Greater Side L O, 26. 34.

This L O, 26. 34, subtracted from K L 80. 92 leaves 54. 58 for the other part of the longest side K O; and the half thereof, 27. 29 is the part K P or P O. And now, the *Oblique-Angled Triangle* K L M, is Reduced into two *Right-angled Triangles* M P K, and M P L; in both which the *Hypotenuses* and *Bases* are given, by which the other Angles may be found, By CASE III. of R. A. P. T. According to which, you shall find the Angle M K P to be 43. 71 de. and the Angle M L K 25. 94 deg. — So that L Bears from K North-East 43. 71 deg. — And M from North-East 69. 65 deg.

PROB. X.

There are two Head-Lands at B and C, which are distant 356.00 Miles; And there is a Port at A, from whence two Ships fail to those two Head-Lands, the Ship from A to C, fails South-East 34. 90 deg. and that from A to B, South-West 72. 10 de. And both the Ships together have sailed 424. 12 Miles. I demand the Distance that each Ship sailed; Or, how far are either of the Head-Lands from the Port at A?

The Geometrical Construction.

ONE Ship sailing South-East 34. 90 deg. and the other South-West 72. 10 deg. the distance of those Rumbs is 107. 00 de. Wherefore,

First,

Fig. LXVII. First, Draw two Lines at pleasure, as A B and A C, making the Angle at A, equal to 107. 00 de.

Secondly, Take 356. 00 Miles, in your Compasses, and enter them between the two Lines at pleasure, as at B and C: constituting the Triangle A B C.

Thirdly, Continue the Side B A to D, making A D equal to A C, and joyn C D, constituting another Oblique-angled Triangle D C A, in which there is given. (1) The Angle at C A B, 73.00 deg. (the Complement of B A C to 180 deg.) And (2) The Angles A C D and A D C, each of them equal to half the Angle B A C, viz. to 53.50 deg. Then in the Oblique-Angled Triangle D C D, say,

As the given Side B C, 356.00 M.	2.551450
Is to the Sine of B D C, 53.50 de.	9.905178
So is the Side B D, 424.12 M.	2.627488
To the Sine of 73.27 deg.	12.532666

Which is the quantity of the Angle D C B: From which subtract the Angle A C D, 53.50 de. there will remain 19.77 d. for the Angle A C B. Then in the Triangle B A C,

As the Sine B A C, 107.00 d. (73.00)	9.980619
Is to the Side B C, 356.00.	2.551450
So is the Sine of A C B, 19.77 de.	9.529231
	12.080681

To B A, 125.90 M.
Which subtracted from B D, 424.12, there remains 298 M, for the Side A C. the Distance that the other Ship failed. So that the Port A, is distant from the Head-Land C 298.22 M. And from that at B, 125.90.

P R O B. XI.

Fig. LXVIII. Two Ships set sail from two Head-lands at B and C, distant from each other 356. 00 Miles; and meeting together at A, they observe that the two Head-lands bear so from their Ship, that they make an Angle of 107. 00 deg. Also, the way that both the Ships have made from B and C to A, doth amount unto 424. 12 Miles: I demand how much of this each Ship failed.

The

The Geometrical Construction.

First, **D**RAW a Right-line at pleasure, as B A D, and upon any point thereof, as A, protract an Angle of 107. 00 deg.

Secondly, The Distance of the two *Head-lands* being 356. 00 Miles enter them between the two sides, at B and C, and joyn B C, making the Triangle B A C.

Thirdly, Make A D equal to A C, and joyn CD, so have you constituted another Triangle A C D.

Now in the Triangle A B C, is Given. (1) The Side B C, 356 00. (2) The Angle B A C 107. 00 deg: And in the Triangle A C D, the Angle B A C being 107. 00 de. the Angle D A C must be 73. 00 de. And the Sides A C and A D, being equal (by construction) the Angles A D C and A C D, must be also equal, viz. (each of them) 53. 50 deg. And being thus far prepared, you may proceed to find the Angle A C B.

By Trigonometrical Calculation.

As the Side B C, 356. 00	2.551449
To the Sine of the Angle A D C, 53. 50 d.	9.905178
So is the whole Side B A D, 424. 12 M.	2.627000
	12.532178
To a fourth Sine, 73 06 d.	9.980729

This 73. 06 d. should be the quantity of the Angle D C B, but (by the construction) the Angle you see is Obtuse; and therefore the Complement of 73. 06 deg. to 180, viz. 106. 94 deg. is the quantity of the whole Angle D C B. From which if you subtract the Angle A C D, 53. 50 deg. there will remain 53. 44 deg. for the Angle A C B.

Then

As the Sine of B A C, 107. (73. 00d.)	9.980619
Is to B C 356. 00.	2.551449
So is Sine A C B, 53. 44 deg.	9.904842
	12.456291

To the Side B A, 299. 00 ———— 2.475672
And so much did the Ship from B, sail to A, and that subtracted from 424. 12 there remains 175. 12, and so much did the Ship from C to A Sail.

H h h

II. Of

II. Of MERCATOR's Sailing, by the True Sea-Chart.

ALL the foregoing *Problems* are performed by that kind of *Navigation*, commonly called *Plain Sailing*, or *Sailing* by the *Plain Sea-Chart*; in which *Chart*, the *Degrees* of *Longitude* and *Latitude*, in all *Places*, are supposed to be equal; which is *Erroneous*, though most practised: But there are two other ways of *Sailing*, both more exact than the former, the one called *Mercator's*, the other, *Sailing* by the *Middle Latitude*.

That of *Mercator's* requires that the *Degrees* of *Latitude* in that *Chart* be *enlarged* as they go farther from the *Æquinoctial* towards either of the *Poles*, which is done by *Reducing* them into *Meridional Parts*: but the *Degrees* of *Longitudes* into *Miles* and *Centesms*, as in *Plain Sailing*: And for the *Reducing* of *Degrees* and *Minutes* of *Latitude* into *Meridional Parts*, I have here inserted a *Table* for the ready performance thereof, to every *Degree* and *Quarter*, or 25 *Centesms* of a *Degree*.

A

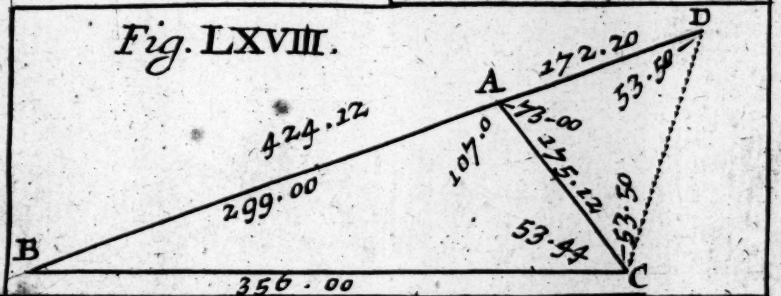
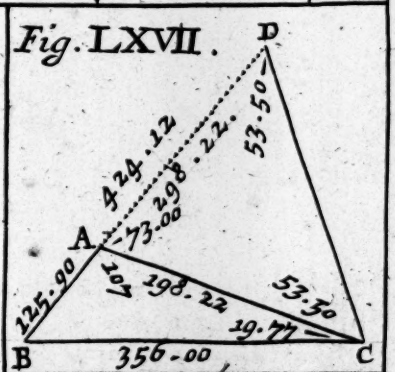
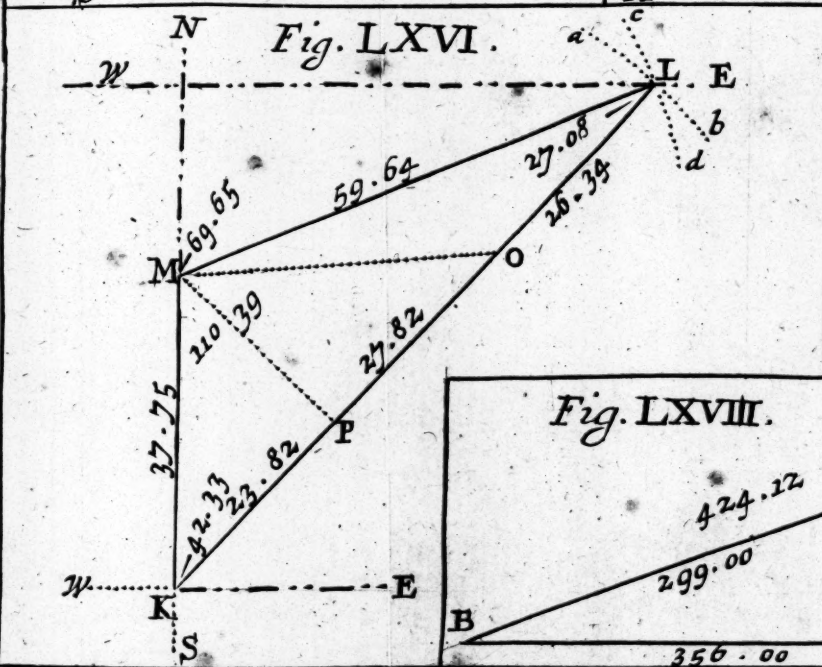
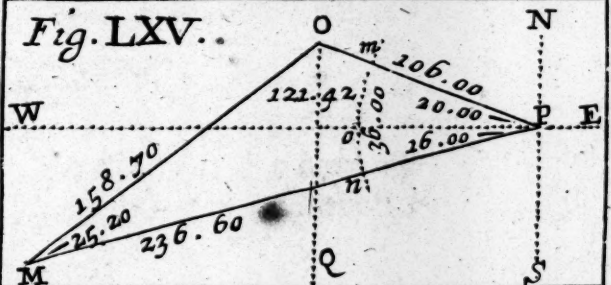
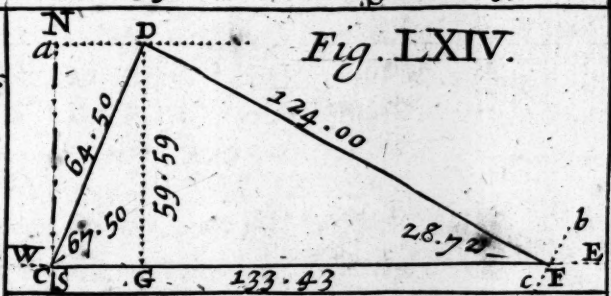
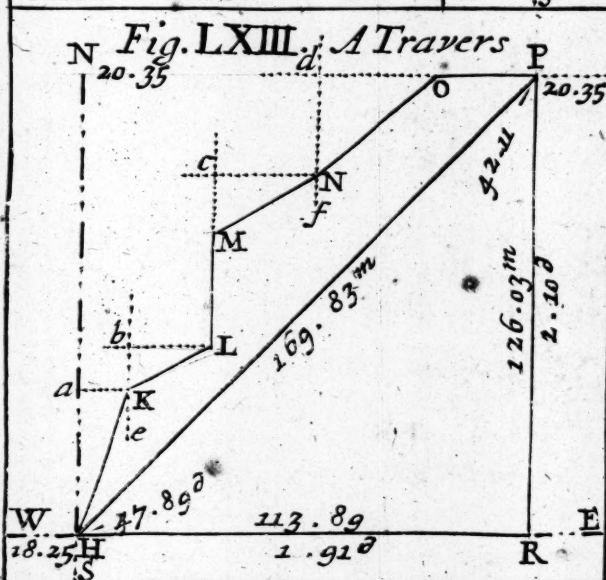
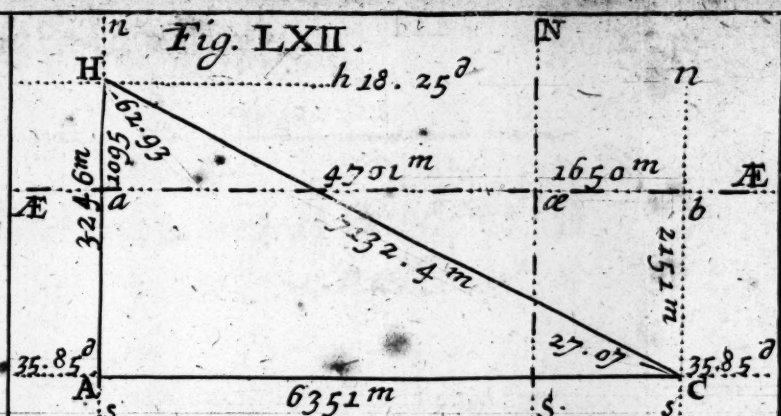
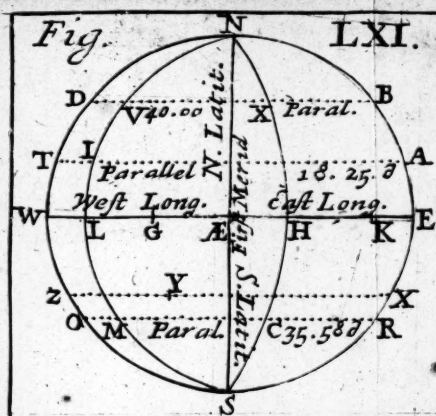


Plate XXI. From Pag. 306. to Pag. 324. Against Pag. 324.

A Table of Meridional Parts.

	Centefines.					Centefines.			
	0	25	50	75		0	25	50	75
0	0	15	30	45	31	1950	1976	1993	2011
1	60	75	90	105	32	2028	2046	2064	2082
2	120	135	150	165	33	2100	2117	2135	2153
3	180	195	210	225	34	2171	2190	2208	2226
4	240	255	270	285	35	2244	2263	2281	2300
5	300	315	330	345	36	2318	2337	2355	2374
6	361	376	391	406	37	2393	2411	2430	2449
7	421	436	451	466	38	2468	2487	2507	2526
8	482	497	512	527	39	2549	2564	2584	2603
9	542	557	573	588	40	2623	2642	2662	2682
10	603	618	634	649	41	2702	2722	2742	2762
11	664	679	695	710	42	2782	2802	2822	2843
12	725	741	756	771	43	2863	2884	2904	2925
13	787	802	818	833	44	2946	2967	2988	3009
14	848	864	879	895	45	3030	3051	3073	3094
15	910	926	942	957	46	3116	3137	3159	3181
16	973	988	1004	1020	47	3203	3225	3247	3269
17	1035	1051	1067	1082	48	3292	3314	3337	3359
18	1098	1114	1130	1146	49	3382	3405	3428	3451
19	1161	1177	1193	1209	50	3475	3498	3522	3545
20	1225	1241	1257	1273	51	3567	3593	3617	3641
21	1289	1305	1321	1338	52	3665	3690	3714	3739
22	1354	1370	1386	1402	53	3764	3789	3814	3839
23	1419	1435	1451	1468	54	3865	3890	3916	3940
24	1484	1509	1517	1533	55	3968	3994	4021	4047
25	1550	1567	1583	1600	56	4074	4101	4128	4155
26	1616	1633	1650	1667	57	4183	4211	4238	4266
27	1684	1700	1717	1734	58	4295	4323	4352	4380
28	1751	1768	1785	1802	59	4409	4439	4468	4498
29	1819	1837	1854	1871	60	4528	4558	4588	4619
30	1888	1906	1923	1941	61	4650	4681	4712	4744

A Table of Meridional Parts.

	Centesmes.			
	0	25	50	75
62	4715	4807	4840	4872
63	4905	4939	4972	5006
64	5040	5074	5109	5144
65	5179	5215	5251	5287
66	5324	5361	5399	5436
67	5475	5513	5552	5592
68	5631	5672	5712	5754
69	5795	5837	5880	5923
70	5967	6011	6056	6101
71	6147	6193	6240	6288
72	6336	6381	6432	6484
73	6535	6587	6640	6693
74	6747	6802	6857	6914
75	6972	7030	7089	7150
76	7211	7274	7338	7403
77	7469	7536	7605	7675
78	7746	7819	7894	7970
79	8048	8127	8209	8292
80	8377	8465	8555	8647
81	8742	8839	8939	9042
82	9148	9258	9371	9488
83	9609	9735	9865	10000
84	10141	10288	10441	10601
85	10770	10946	11133	11330
86	11539	11761	11999	12255
87	12521	12821	14015	13513
88	13920	14381	14914	15545
89	16318	17316	18729	21170
90				

Degrees

The Use of the Table of *Meridional Parts*.

What *Meridional Parts* do answer to 26 deg. of *Latitude*.

Latitude	26. 00 Deg—	1616 Merid. Parts.
	24. 25 Deg—	1509 M. P.
	47. 50 Deg—	3247 M. P.
	63. 00 Deg—	4905 M. P. &c.

2. What *Degrees* and *Cetefms* do answer to.

679 Merid. Parts.	} Answer {	11. 25 Deg.
3522 M. P.		50. 50 Deg.
9488 M. P.		83. 75 Deg.

For the Resolving of the following *Problems*, which (in kind) will be the same with those in *Plain Sailing* as to the *Trigonometrical Work*. But in the performance of them, I will set the places proposed down upon a *Sea-Chart*, made according to the *Projection* of *Mercator*, where the *Degrees* of *Latitude* are *enlarged* according as they tend nearer and nearer to the *Pole*: And on the out-side of such a *Chart*, I have described a *Plain Sea-Chart* also, where the *Meridians* and *Parallels* of *Latitude* are every where of an equal distance; and so wrought the *Questions* according to both *Charts*, by which the *difference* will more plainly appear: And the *Chart* which I have here made to work the following *Examples* upon, begins (at the bottom of it) at about 49.50 deg. of *Latitude*, and extends upwards to 55. 50 deg. of *Latitude*; and at the *Top* and *Bottom* to *Six Degrees Difference* of *Longitude*: But the *Parallels* of *Latitude* in the *Plain Chart*, (which are distinguished by *Pricked Lines*) extend to above 59 deg. of *Latitude* within the same Bounds.

Fig.
LXIX.

P R O B. I.

The *Latitudes* of two *Places*, and their *difference* of *Longitude* being known; To find, (1) The *Rumb* leading from one to the other: And (2) Their *Distance* upon that *Rumb*: according to *Mercators Chart*.

By *Trigonometrical Calculation*

LET one Place be at A in the *Latitude* of 50 deg. and the other at C, in the *Latitude* of 55 deg. but differing in *Longitude* Eastward from B, 6. 50 deg. which two places thus set down in the

Fig. the Chart, draw the Line A C, which is the *Rumb* (or *Course*)
 LXIX. from A to C: And thus have you upon your Chart constituted
 a *Right-angled Plain Triangle* A B C, in which you have given, (1)
 B C, the *Difference of Longitude* 6.50 deg. which you must *Reduce*
 into *Miles* (or *Minutes*) of *Longitude* by multiplying them by 60,
 (as in *Plain Sailing*) and they make 330. 00 *Miles*. — (2) A B,
 the *Difference of Latitude* 5. 00 deg. which must be *Reduced* into
Meridional Parts thus:

The <i>Meridional Parts</i> for 50. 00 d. are	3475
For 55. 00	2968
Their <i>Difference</i> —	493

Which are the *Meridional Parts* answering to the 5. 00 deg. *Difference of Latitude* of the two *Places* A and C: And by these you may find;

- (1.) The *Rumb* B A C.
- (2) The *Meridional Distance* upon the *Rumb* A C.

By Trigonometrical Calculation.

(1.) For the *Rumb* B A C

As the <i>Merid. Dif. of Latitude</i> A B, 493 M. P.	2.692847
Is to the <i>Dif. of Longitude</i> B C 330 <i>Miles</i>	12.518514
So is the <i>Radius</i> , Tangent 45 deg.	10.
To the Tangent of B A C, 33. 80 degrees.	9.825667

Which is the *Rumb*, leading from A to C: whose *Complement* 56. 20 deg. is the Angle B C A, or the *Complement* of the *Rumb*:

(2.) For the *Meridional Distance* on the *Rumb* A C.

As the <i>Co-sine</i> of the <i>Rumb</i> : B C A, 56. 20 d.	9.919592
Is to the <i>Merid. Differ. of Latitude</i> A B 493 M. P.	12.692847
So is <i>Radius Sine</i> 90 deg.	10.

To the <i>Merid. Distance</i> A C, 593.27 M. P.	2.773255
---	----------

Now to find the *Meridional Degrees* answerable to these *Meridional Parts*, you must

First, Subtract the M P O of A B, from the M P of A C, their *Difference* is 100. 27 M. P. the half whereof is 50.13 M. P.

Secondly, Add this half *Difference* 50. 13, to the *Meridional Parts* of the *Greater Latitude* 55. 00 deg. viz. 3968, and it makes 4018. 13, which are the *Merid. Parts* answering to 55.50 deg. of *Latitude*. — Also, subtract this half *Difference* 50. 13, from the *Merid. Parts* of the *Lesser Latitude* 50. 00 deg. viz. 3475, and the

Remainder

Remainder will be 3424. 87, which are the *Merid. Parts* answering to 49. 38 deg. of *Latitude*.

Fig.
LXIX.

Thirdly, The *Difference* of these two *Latitudes* last found, viz. 55. 50 de. and 49. 38 deg. is 6. 12 deg. And that is the true *Distance* upon the *Rumb* between A and C, in *Degrees*.

And according to this Method, may all the other *Problems* (before wrought by the *Plain Sea-Chart*) be performed by *Tri. Calculation*; by the same *Canons*; Only remember, to *Reduce* the *Difference of Latitudes* and *Distance* on the *Rumb*, into *Meridional Parts*, but the *Difference of Longitude* into *Miles*, as in the other.

Now that you may see the *Difference* between these two ways; See the *Figure* of the *Chart*, wherein the two *Places* are laid down by the *Plain Chart*; as in the Triangle A O S, which Triangle being resolved (as is before shewed) you will find

- (1) The *Rumb* O A S, to be 47. 73 deg.
Differing from the truth 13. 93 deg.
- (2) The *Distance* upon the *Rumb* A S, to be 7. 43 de.
Differing from the other 1. 31.

And let thus much suffice for *Sailing* according to *Mercator*.

There is a third way of *Sailing*, which differs not much from this way of *Mercator's*, but may be performed without *Reduction* or use of *Meridional Parts*; which is called *Sailing* by the *Middle Latitude*: of which a little.

III. Of Sailing by the Middle Latitude.

I Shall exemplifie this way of *Sailing* by *Four* of the most usual *Problems* in *Navigation*: And they shall be the same as in the foregoing: whereby the *Difference* will the better appear.

P R O B. I.

The *Longitude* and *Latitude* of two *Places*, A and C, Given to find,

- (1) The *Rumb* B A C.
- (2) The *Distance* upon the *Rumb*: A C.

LET

LET one Place be in 50. 00 de. the other in 55. 00 de. of North Latitude: And 6. 50 deg. difference in Longitude: Then the Middle Latitude between 50. 00 deg. and 55. 00 deg. is 52. 50 de. And its Complement 37. 50 deg. Then

(1) For the Rumb: The Proportion is,
 As the Difference of Lat. 300. 00 Miles. 2.477121
 Is to the Dif. of Longit. 330. 00 Miles. 2.518524
 So is the Co-sine of Middle Latitude, 37. 50 de. 9.784447
12.302971
 To the Tangent of 33. 81 deg. 9.825850
 Which is the Rumb B A C.

(2) For the Distance upon the Rumb.
 The Proportion is,
 As the Co-sine of the Rumb 56.19 de. 9.919542
 To Radius Sine 90 de. 10.
 So is the Difference of Latitude, 300 M. 12.477121
 To the Distance 361.06 M. 2.557579

P R O B. II.

THE Latitudes of two Places, A and C, and the Rumb, B A C, (or Course) between them Given: to Find,

(1) Their Distance A C.

(2) Their Difference of Longitude, B C.

For the Distance it may be found as in the former Problem. But

(2) For their Difference of Longitude, This is
 The Proportion.
 As the Co-sine of the Middle Latitude, 37.50 de. 9.784447
 Is to the Difference of Latitude 300 M. 2.477121
 So is the Tang. of the Rumb, 33.81 de. 9.825876
12.302997
 To the Difference of Longitude B C, 330.01 M. 2.518550

P R O B.

P R O B. III.

THE Distance A C, 361.06 M. And both the Latitudes Given,
To find the Rumb.

The Proportion.

As the Distance A P, 361.06

To the Difference of Latitude 300 M.

So is Radius 90 deg.

To the Sine of 56.19 deg.

Whose Complement 33.81 deg. is the Rumb B A C. from the Meridian.

2.557579

12.477121

10.

9.919542

P R O B. IV.

HAVING one Latitude, the Rumb, and Distance upon the Rumb,
to find the Difference of Latitude.

The Proportion.

As the Radius Sine 90 deg.

To Co-sine of the Rumb, 56.19 deg.

So is the Distance upon the Rumb, 361.06

To the Difference of Latitude, 300 M.

Which Reduced into Degrees, is 5.00 deg.

10.

9.919542

2.557579

12.477121

P R O B. V.

HAVING the Rumb and both Latitudes given, to find the Difference of Longitude.

The Proportion.

As the Co-sine of the Mid. Latitude, 37.50 de.

To the Difference of Latitude, 300.00 M.

So is the Tangent of the Rumb, 33.81 deg.

To the Tangent of 300.03 M.

Which is 5 Deg.

9.784447

2.477121

9.825876

12.302997

2.518550

The Differences in these Three Ways of Sailing : In this Example.

	Plain Sail.	Mercator.	Mid. Latit.
Differ. of Long.	330.00 M.	330.00 M.	330.00 M.
Differ. of Latit.	300.00 M.	300.00 M.	300.00 M.
Rumb.	47.73 De.	33.80 Deg.	33.81 Deg.
Compl. Rumb.	42.27 De.	56.20 Deg.	56.19 Deg.
Distance.	445.97 M.	593.27 M.P.	361.06 M.
	Or	Or	Or
	6.50 Deg.	6.12 Deg.	6.01 Deg.

JANUA MATHEMATICA

VEL,

Trigonometria Practica.

SECTION VII.

OF

Theorical ASTRONOMY.

IN the Fourth Section of this Thrid PART, you have the Doctrine of Triangles applied to Practice in that part of Astronomy which concerns the Diurnal Motion of the Sun and Fixed Stars: Now, in this Section, I shall discourse something of the other Stars or Planets, which have a Secondary Motion of their own; shewing also, how subservient Trigonometry is for the Calculating of their true Places at any time, as also in computing of their Magnitudes, Distances, &c. And in order thereunto I shall first give you a brief description of the Planetary System.

CHAP. I.

Of the Planetary System.

THIS System may be sufficiently represented as in the Figure thereof: Wherein,
First, The Sun being in the Centre, hath only a Rotation from West to East, upon its own Axis, in the space of 26 days, or much

much thereabouts: His Centre being an immoveable Point; to which the *Revolutions* of the other *Planets* are referred.

Secondly, The *Planets* are moved about the *Sun*, from *West* to *East*, in several *Orbs*, which return into themselves, every *Planet* in *Time* proportionable to the *Magnitude* of his *Orb* and *Distance* from the *Sun*; the *Motions* of the *Primary Planets*, η , ν , δ , $\odot$, $\circ$, and ϑ , being *uniform*, perpetually *constant* and *regular*.

The Figure of the System.

Fig.
LXX.

The Circle ϑ BCD ϑ , denotes the *Way* and *Revolution* of the *Planet Mercury*, about the *Sun*, in 88 Days — The Circle $\circ$ EFG $\circ$, the *Revolution* of *Venus* in 225 Days — The Circle $\odot$ HI K $\odot$, the *Revolution* of the *Earth*, with the *Moon* in one Year — The Circle δ LMN δ , the *Revolution* of *Mars* — The Circle ν OPQ ν , the *Revolution* of *Jupiter*, with his four *Companions* (or *Satellites*) in twelve Years — The Circle η RST η , the *Revolution* of *Saturn*, with his *Ring* and *Moon*, in Thirty Years. — The *Moon* also moves round the *Earth*, every Month — *Jupiter's* four *Companions*, move about him according to their *Distances* from him. — The First (and nearest to him) in One Day and 18 Hours. — The Second, in Three Days and 13 Hours. — The Third, in Seven Days and 4 Hours. — The Fourth, in 26 Days and 5 Hours. — *Saturn's Moon* moveth about him in Sixteen Days: And all of them from *West* to *East*, according to their *Planets Revolution* about the *Sun*.

These *Planets*, whose *Revolutions* respect the *Sun* only, as *Saturn*, *Jupiter*, *Mars*, the *Earth*, *Venus* and *Mercury*, are called *Primary Planets*: The other that move about *Saturn*, *Jupiter* and the *Earth*, *Secondary Planets*.

The *Secondary Planets*, are all of them much *Less* in *Magnitude* than their *Primary*; and all the *Planets* together, much *Less* than the *Sun*, from whom they receive their *Light*, *Motion*, &c.

C H A P.

C H A P. II.

Of the Theory of the Sun and the other Primary Planets.

THAT the Motion of the *Primary Planets* about the Sun were *Elliptical*, was first discovered by the learned *Kepler*, which he deduced from the accurate *Observations* of the Noble *Dane Tycho*. And therefore, in such an *Elliptical Figure*, may be described all such *Points, Lines, Arches, &c.* as are requisite to be known, and to inform the *Fancy* in the *Trigonometrical Calculation* of the *Planets Places*. But before I proceed so far, I must shew,

How to describe an *Ellipsis*.

Concerning the *Definition* of this *Figure*, only that it is one of the *Sections* of a *Cone*, and the many ways that it may be Artificially described, I shall say nothing, referring the Reader to *Mindorges*, and others that have largely written of the *Sections* of a *Cone*. But to Describe such a *Figure Mechanically*. Thus:

About any Two *Right-Lines* Given, for the Two *Diameters* of the *Ellipsis*, to describe such an *Ellipsis*.

Let the Line *P Y* be the *Longer*, and *R M* the *Shorter Diameter* given: Let them cross one another at *Right-angles* in *B*. This done, Take half of the *Longest Diameter* *B P* or *B Y*, in your *Compasses*, and setting one foot in *R* or *M*, the other will reach upon the *Longest Diameter* to the *Points Z* and *S*, which will be the *Centres* upon which the *Ellipsis P R Y M* must be described. Wherefore,

In the two *Points Z* and *S*, Fix two *Pins, Nails, or the like*; and about them put a *String* so long, that being doubled it may reach from the *Pin* at *S*, to the end of the *Diameter* at *P*; or from the *Pin* at *Z* to the end *Y*; so then the whole length of the *String*, will be twice as long as the *Distance S P* or *Z Y*; which *String* at that length, joyn at both ends: Then, putting this *String* over the two *Pins*, at *Z* and *S*; with a *Third Pin* (which may be a *Black-Lead Pencil*, or such like) move the *String* about upon the two *Fixed Pins Z* and *S*; and it will by its *Motion* describe an *Ellipsis*, such as is the *Figure P C R E Y F M D P*.

C H A P.

Fig.
LXXI.

C H A P. III.

Of the Motion of the Planets in the Ellipsis.

IN this *Elliptical Figure*, the Line PY is called the *Transverse*, (or *Longer*,) And RM , the *Conjugate* (or *Shorter*) *Diameter* of the *Ellipsis*.--- S the *Lower*, and Z the *Upper Focus* of the *Ellipsis*; upon which two Points the *Elliptical Figure* was described--- And the *Elliptical Figure* it self $PCREYFMDP$ be the *Orbit* of the *Earth*, or any other of the *Primary Planets*:--- S the *Place* of the *Sun*; (the common *Node*, and *Centre* of the *Planetary Orbs*, to which the *true Motions* of the *Planets* are referred,) in the *Lower Focus* of the *Ellipsis*.--- Now, a *Planet* moving in the *Ellipsis*; when it shall be in P , it is in *Aphelion*, or at its *Greatest Distance* from the *Sun*--- When in Y *Perihelion*, or at its *Nearest Distance* to the *Sun*--- About the upper *Foci* of the *Ellipsis* Z , the *mean Motion* of the *Planet* is regulated: B is the common *Centre* of the *Figure*: BZ or BS , the *Excentricity*: And SZ the double *Excentricity*.

The *Figure* thus described, the next thing to be known is, after what manner the *true Place* of a *Planet* may be determined therein.

First, The *mean Anomaly* of a *Planet*, is its *equal Distance* from P , the *Aphelion Point* of that *Planet*: And this *Anomaly* is to be accounted from P , to P again. And here it is to be observed, that a *Planet* moving in the *Elliptical Arch* $PRYM$, upon the *Focus* Z , is as long time tracing the lesser *Arch* CPD , as he is the greater CYM ; the Reason is, for that (the *mean Motion* being made upon Z) the *Planet* describeth *equal Angles* in *equal Intervals* of *Time*: And it is also evident, That if the *Motion* be equal upon one *Focus* Z , of the *Ellipsis*, it must be unequal upon the other S , in which the *Sun* is seated.

For instance: Suppose the *Planet* to be in C , which is 90 deg. from the *Aphelion Point* P , then will the *Angle* of its *mean Anomaly* be PZC ; but the *Co-equated Anomaly* will be the *Angle* PBC : And the *Angle* at the *Sun* PSC , which is the *Angle* to be found.

CHAP. IV.

How to find the Angle that a Planet (in any part of the Fig. Ellipsis) makes with the Sun; and also, the Planets true Place in the Ellipsis. LXXII.

Let S be the Sun, X the Centre of the Planets Mean Motion; the Points H, A, T, several Places of the Planet in its Orb, from the Aphelion point P, then will the several Angles P X H, P X A, P X T, represent the Anomaly of the Planet: And the Angles P S N, and P S M; the Angles at the Sun. Now these Angles may be found as followeth.

Suppose the Mean Anomaly of the Earth being at H, to be 30 deg. this is represented by the Angle P X H, whose Complement B X H, is 150 deg. By help whereof, with the Common Radius, and Excentricity, the true Place of the Planet may be found by Trigonometrical Calculation. For,

First, In the Triangle H X B, there is Given, (1) The Angle H X B, 150.00 d. the Complement of the Planets Mean Anomaly to 180 deg. (2) The Side B H, the Common Radius 100000. And (3) The Side B X, 1685, equal to half X S, the Excentricity. By which you may find the Angle B H X, By CASE I. of O. A. P. T. Then,

(1) As the Common Radius, B H, 100000 Parts	5.000000
To X B (half the Excentricity) 1685 P.	3.226599
So is the Sine of H X B, the Complement of the } Anomaly H Z P, 30 d. to 180 deg.	9.698970

	12.925569
To the Sine of B H X 0.483 deg.	7.925569

And therefore the Angle X B H is 29.518 deg. the double whereof, 59.036 deg. is the Anomaly of Variation.

Then say, Secondly,

(2) As the Radius, Sine 90 d.	10
Is to the Sine of the Greatest Variation } (which is) 0.24 deg.	6.594172
So is the Variation last found, 59.034 d.	9.933210
To the Sine of 0.019 deg.	6.527382

Which is the Variation required.

Now,

Fig. Now, forasmuch as the *Planets* are moved in *Ellipses*, and not in perfect *Circles*, the next thing, therefore, to be enquired is, the *Planets* Place in the *Ellipsis*: And to that purpose, about Z, let there be described a little Circle K O N, upon which, from K to N, I number the *Anomaly* of the *Epicicle* 59.034 deg. before found, and then will the Angle P B Z, be equal to the *corrected Anomaly*; and so is N the place of the *Planet* in the *Ellipsis*; and the Angle Z B N, the *Equation* of the *Epicicle*; or (which is the same) the *Difference* between the *Place* of a *Planet* in the *Circle*, and in the *Ellipsis*: And may be found *Trigonometrically*, thus.

For,

Thirdly, In the Triangle B Z N, you have given, (1) The Side B Z, the *common Radius* 100000 P. (2.) The Side Z N, 00008 P, the *Semidiameter* of the *Epicicle*. (3.) The Angle B Z N 120.98 Deg. by which you may find, the Angles B N Z and Z B N, (by CASE II. of O. A. R. T.) Thus,

The Angle N Z B being 120.98 deg. the Complement to 180 deg. is 59.02 deg. the half whereof is 29.51 deg. The Side B Z is 100000 P. the Side Z N 00008 P, their Sum 100008 P, their Difference 99992. Then,

(3) As the Sum of the Sides B Z and Z N, 100008	5.000035
Is to their Difference 99992	4.999965
So is the Tang. of half Z B N and Z N B 29.505 d.	9.752818
	14.752783
To the Tang. of the half Difference 29.502 d.	9.752748

This half Difference, added to the half Sum, gives 59.007 deg. for the Angle Z N B: And subtracted from it, leaves 0.003 deg. for the Angle N B Z.

Then again say,

As the Sine of B N Z, 59.007	9.993121
Is to B Z 100000 P.	5.000000
So is the Sine of B Z N, 59.02 d.	9.933140
	15.933143
To the Side B N 100004 P.	5.000022

From the Angle B P Z 29.502

Subtract N B Z 005

There remains P B N 29.497 Whose Complement to 180 deg. is 150.503 deg. and is the Angle S B N.

Fourthly,

Fourthly, In the Triangle B S N, there is given, (1) B N 100004. (2) B S 1685. (3) The Angle contained by them, SBN 150.50 deg.

Then,

As the Sum of B N and B S, 101689. 5.007278

Is to their Difference 98319 4.992637

So is the Tangent of half B S N and B N S 14.77d. 9.421029

14.413666

To the Tangent of 14.30 d. the half difference. 9.406388

Which added and subtracted from the half Sun, gives

29.07 deg. for B S N the Angle at the Sun.

And 0.47 deg. for the Opt. Equation B N S.

Then from the Mean Anomaly P X H 30.00 deg.

Subtract the Angle at the Sun B S N, 29.07

There remains the Prosthapheris. 0.93

Again,

As the Sine of B S N, 29.04 de. 9.686117

Is to B N, 100004. 5.000021

So is the Sine of B S N, 150.50 deg. (or 29.50.) 9.692338

14.692359

To S N, 101564. 5.006232

Note that when a Planet is in any of the Points P, A, or Y, there is no Variation, but being out of any of these Points, it hath Variation more or less, and is at its greatest when the Elongation is 45 deg. from any of those Points. — And note also, That this Variation augments the Places of all the Planets (Venus and the Moon excepted) in the First and Third Quadrants of the Orbit; But in the Second and Fourth, it diminishes the Planets place; according to the quantity of the Angle of Variation. And as this Example was wrought, the Earth or other Planet being supposed in the First Quarter of the Ellipsis; the same is to be understood and performed, if the Planet were in the Second, Third or Fourth Quadrant thereof, as is plain by the Figure.

C H A P. V.

Of the Two-fold Inequality of the other Primary Planets.

THE *Sun* seems to be moved under the Plain of the *Ecliptick*; But it is the *Earth* that performeth this *Motion* about the *Sun*, who is seated in the *Focie* of the *Earths Ellipsis*: So that, as the *Earth* is moved under the *Ecliptick Orb*, so much the *Sun* appears to move on the contrary part thereof: And from hence may be concluded, that the *Sun* (or *Earth*) can be subject to no other *Inequality*, than what is produced by their own *Simple Motion* in the *Ellipsis*: And this is true also, in all the *Primary Planets* as in respect to the *Sun*; who is seen, from them, to change his *Place*, according to the quantity of their *Motions*.

Now, For as much as the *Earth* is far distant from the common *Centre* of their *Orbs* (which is the *Sun*) therefore it is, that (by their different *Motions*, and various positions of their *Orbs*) they seem to us to be subject to a *second Inequality*; which is not essential in their *Motions*, but accidental only; being caused through the *great distance* of the *Point* to which their *Places* are referred: So that the *Planets* seem to us to have a different species of *Motion* from the *Natural Motion* in their respective *Orbs*: they appearing at one time *Direct*, at other times *Retrograde*, and sometimes not to move at all, but appear as *Stationary*: To be nearer to the *Earth* at one time than at another; and appearing to the *Eye* of a *Greater* and *Lesser Magnitude*: The cause of all which various *Passions* will manifestly appear from the *Calculation* of their *Places*, which shall be the *Work* of the following *Chapter*.

C H A P. VI.

To Calculate the true Place of a Planet Trigonometrically.

RETAINING the same Method deliver'd in the last Chapter, I shall give an Example in the finding out of the place of the Planet *Mars*. The *Sun* being in 5.85 deg. of *Scorpio*, and the mean Anomaly of *Mars* from his Aphelion point 55.47 deg.
The

Of Theoretical Astronomy.

341

The rest, as in the following Table. And from these things given, the true place of the Planet *Mars* may be found.

Fig.
LXXII.

A Table of the Semidiameters of the Orbits and Epicles; the Excentricities; the Greatest Variations and Inclinations of the Planets, in order to the Trigonometrical Calculation of their Places.

	♄	♃	♂	☉	♀	♂
Semidiam. of the Circle P A Y.	952500	521300	152040	100000	72405	38192
Semidiam. of Epicicle Z N.	788	299	327	8	1	429
Excentricity B X=B S.	54800	24960	14115	1788	530	8100
The Greatest Variation of the Ano.	0.09 d	0.05 d	0.25 d	0.02 d	0.02 d	1.30 d
Angle of the greatest Inclination H B V.	2.51 d	1.36 d	1.85 d	0.00 d	3.38 d	6.90 d

The Trigonometrical Calculation.

IN the Figure the Semicircle P H Y, is half the Orbit of *Mars*, P his *Aphelion* point, H his place in his Orbit, B the Centre, B X the Excentricity: H B the Semidiameter of the Orbit. Fig. LXXII.]

I. Then, In the Triangle X H B, there is given, (1) The Side H B, the Semidiameter of the Orbit, 15204. (2) The Side X B, the Excentricity 1411. (3) The Angle H X B. the Complement of the mean Anomaly P H: 55.47 to 180 deg. viz. 124.53 deg.

By which you may find the Angles B H X, and X B H.
(By CASE II. of O. A. P. T.

As the Sum of the Sides H B and X B 16615.

Is to their Difference 13793.

So is the Tang. of half X H B and X B H 27.73 d.

4.2205003

4.1396587

9.7207213

13.8603800

To the Tangent of 23.50 d.

9.6398797

Which subtracted from 27.73 (the half Sum) leaves 4.15 de. for the Angle at H: And added to 27.73 d. gives 51.31 deg. for the Angle X B H. And that doubled, makes 102.62 d. Which is the Anomaly of Variation.

K k k 2

Which

Fig.

Which found ; say,

LXXII. As Radius

10.

To the Sine of the greatest Variation of *Mars*, 0.25 d. 7.639816

So is the Sine of Variat. last found 102.62 (77.38) 9.989379

To the Sine of H B Z, 0.244 deg. the Variat. x 7.629195

To the Angle X B H 51.32 deg.

Add the Angle H B Z 0.244 the Variation.

The Sum is X B Z 51.564 deg.

The double whereof is 103.128. And is the *Motion* of the *Epicicle* a Z N.

Describe the *Epicicle*, and upon it set the *Motion* of the *Epicicle* 103.128 deg. from a to N, so will the Angle a O N be equal to the corrected *Anomaly* P B Z ; so that N is the Place of the Planet *Mars* in the *Ellipsis* : And the Angle Z B N the *Equation* of the *Epicicle* ; which is the difference between the place of a Planet in the *Circle*, and in his *Ellipsis* ; which may be thus found. For,

In the Triangle B Z N there is given, (1) The Side Z B, the Semidiameter of the Circle 100000, (2) The Side Z N, the Semidiameter of the *Epicicle* 33. (3) The Angle B Z N, the Complement of the Motion of the *Epicicle* N Z a (103.64). viz. 77.36 de.

And by these may be found, the Angle N B Z.

Thus :

As the Sum of the Sides Z B and Z N, 100008 5.00008

Is to their Difference 99992 4.99996

So is the Tang. of half Z N B and Z B N, 51.32 de. 10.09659

To the Tangent of 51.31 d. 15.09655

Which added to 51.32, gives 102.63 de. for the Angle 10.09647

Z N B ; and subtracted therefrom, leaves 0.01 deg. for the Angle N B Z.

Then for the Side N B.

As the Sine of Z N B 102.63 de. (77.37) 9.989362

Is to the Sine of N Z B 77.36 deg. 9.989345

So is the Side Z B, 100000 5.000000

To the Side B N, 99996 14.989345

4999983

From

Of Theoretical Astronomy.

343

Fig.
LXXII.

From the Angle P B Z 51.32 deg.
 Subtract N B Z 0.244
 There remains the Angle X B N 51.076 whose Com-
 plement to 180 deg. is 128.924 for the An-
 gle N B S.

And then,

In the Triangle B N S, there is Given, (1) The Side B N
 99996. (2) The Side B S, 1411. And (3) The included Angle
 N B S 128.92 deg. From whence may be found the Angle at the
 Sun B S N; and the Side S N.

Thus,

As the Sum of the Sides N B and S B, 101407. 5.006082
 Is to the Difference of those Sides 98585. 4.993811
 So Tang. of half the Angles B S N and B N S, 25.54 d. 9.679276
14.673087
 To the Tangent of 24.92 deg. 9.667005
 Which added to 25.54 de. gives, 51.08 deg for the Angle B S N;
 and subtracted, leaves 0.62 de. for the Angle B N S.

Then, For the Side N S,

As the Sine of S N B, 0.62 deg. 8.034260
 Is to the Side B S; 1411. 3.149219
 So is Sine N B S 128. 92 de. (51.08) 9.891115
13.040334
 To the Side S N 101405. 5.006074

Which is the Distance of ☿ from ☉.

Place of the Aphelion.	150.35 deg.
The Angle at the Sun.	<u>51.08.</u>
Heliocentrick Place of Mars.	201.43
The North Node Sub.	<u>47.55</u>
The Argument of Latitude.	153.88.

C H A P. VII.

To find the Place of a Planet in the Ecliptick, &c.

Fig. LXXIII. FOR that a Planets Place in Longitude in the Ecliptick, differeth something from the Longitude in its own Orb, I shall therefore shew (by Trigonometry,) How to find the *Reduction*, and the *Ecliptick Place*, with the *Curtation* and *Parallax* of the Orb, and consequently, the *Geocentrick Place* and *Latitude*.

In order whereunto, in the Scheme; Let S represent the *Sun*, H the Place of *Mars* in his *Orbit*; V his place in the *Ecliptick*; E the *Earth*: A the *North Node* of *Mars*; B the *South Node*; (the two Intersections of the *Planets Orb* with the *Ecliptick*; A D B, a *Semicircle* of the *Ecliptick*: D R, the limit of the Planets *Greatest North Latitude*. These things premised, I proceed to

The Trigonometrical Calculation.

1. In the Spherical Triangle B H V, Right-angled at V, there is given. (1) The Side B H 26.65 de. (the Complement of the *Argument of Latitude*.) — (2) The Angle H B V, the *Greatest Inclination* of *Mars*, 1.85 de. By which you may find the Side B V: (By CASE I. of R. A. S. T. Thus
As Radius, 90 de.

To Co-sine H B V, 88. 15 de.	10.
So is the Tangent of B H, 26.63 de.	9.999773
To the Tangent of B V 26.64 de.	9.700577
The <i>Difference</i> between B H and B V, is 0.01 de. is to be subtracted from 201.43 d. the place of <i>Mars</i> , because the Arch B V is lesser then the Arch B H.	9.700350

Heliocen. Place of *Mars*, in his Ellipsis. 201.43 deg.

Reduction	Subst.	.01
-----------	--------	-----

Heliocen : Place Corrected	201.42
----------------------------	--------

Place of the Sun	215.85
------------------	--------

Difference	14.43
------------	-------

Which is the *Anomaly of Commutation*.

2. In the same Triangle, there is given as before, whereby the *Inclination* of the *Orbit* from the *Plain* of the *Ecliptick* H V, may be found (by CASE II. of R. A. S. T. Thus

As

Of Theorical Astronomy.

345

Fig.

LXXIII.

As Radius, 90.00 de.
Is to the Sine of B H, 26.65 de.
So is the Sine of B H V, 1.85 de.
To the Sine of H V, 0.83 de.

10.
9.651800
8.508974
8.160774

Unto which, the Angle H S V is equal.

3. For the *Curtated Distance* S V. In the Right-angled Plain Triangle H S V, there is given, (1) The Angle V S H, 0.83 d. (2) The Side S H 15204, the *distance* of Mars in his Orbit, from the Sun : By which you may find the Side S V, (By CASE IV. of R. A. S. T.) Thus

As Radius, Sine 90.00 de H V S,
Is to the Side HS, 15204.
So is the Sine of V H S, 89.17 d.
To the Side S V, 15202.

10.
4.181958
9.999954
4.181912

The *Ecliptic Place* of the Planet Mars, with his *Inclination* and *Curtation* thus attained : The next thing to be enquired after is, the *Parallax* of the Earths Orb ; and his *Geocentrick Place*, in *Longitude* and *Latitude*.

4. In order whereunto, in Figure LXXIII. the Circle O X E Z, representing the Earths Orbit ; Number the quantity of the *Anomaly* of *Commutation* 14.43 de. from X (the opposite Place of Mars from the Sun) to E, drawing the Line E V ; which will constitute an Oblique-angled Plain Triangle S V E : in which there is Given, (1) The Side S V 15202. (2) The Side S E, 10000, (3) The included Angle V S E 165.57 de. (the Complement of the *Angle of Commutation* to 180 deg.) By which you may find (1.) the Angles S V E and V E S. And (2) the Side V E.

As the Sum of V S and S E 25202.
Is to their Difference 5202.
So is the Tang. of half the Angles at E and V, 8.22 d.
To the Tangent of 1.71 deg. their difference.
Which added to the half Sum of the Angles at V and E (8.22 d.) gives 9.03 de. for the Angle V E S : And subtracted therefrom, leaves 6.51 de. for the Angle S V E : Which is the *Parallax* of the Orb.

4.401435
3.716170
9.159743
12.875913
8.474478

Then

Fig.
LXXIII.

Then for the Side V E,

As the Sine of V E S 9.93 de.
Is to the Sine of V S E 163.56 (16.44)
So is the Side V S, 15202

9.236650
4.181900
9.451803
13.633703
4.397053

To the side V E 24959

Then,

To the *Heliocentric* Place of *Mars*. 201.43
Add the *Parallax* of the Orb S V E 6.51
The Sum is 207.94

Which is the *Geocentric Long.* of *Mars*.That is 6 s. 27 deg : Or 27 deg. of $\approx$.

C H A P VIII.

*Of the Proportions of the Semidiameters of the Sun, Earth,
Moon, and the other Planets.*

I. Of the *Sun, Earth, &c. Moon;*

BY the best Telescope-Observations.

To the mean distance of the *Earth* from the *Sun* 10.00000
The Semidiameter of the *Sun* is of those Parts 46300
The Semidiameter of the *Earth* 727

To the mean distance of the *Moon* from the *Earth* 1.00000
The Semidiameter of the *Earth* is 1650
The Semidiameter of the *Moon* 446

And from hence, at all times, the Distance of the *Lumina-
ries* being first found, their apparent Semidiameters may
be obtained : And for the Semidiameter of the *Earth's
Shadow*, in *Lunar Eclipses* ; I have here inserted the Dia-
gram of *Hyparcus*.

In which Diagram.

Fig.
LXXIV.

A denotes the *Centre* of the *Sun*.
A D his *Semidiameter*.
B the *Centre* of the *Earth*.

B D

B E her Semidiameter.

A E D, or A B D, the apparent Semidiameter of the Sun.

A E H, or B D E, the Horizontal Parallax.

C G F equal to H E D, the Semi-Angle of the Cone of the Earths Shadow.

B C and B F, being equal to the Distance of the Moon from the Earth.

B T E, her Horizontal Parallax.

C B F, the apparent Semidiameter of the Earths Shadow.

From hence,

1. The Semidiameter of the Sun, the Horizontal Parallax being subtracted, is equal to the Semi-angle of the Cone of the Earths Shadow.

$$\text{So } A E D - A E H = H E D$$

2. The Horizontal Parallax of the Moon, the Semi-angle of the Cone of the Earths Shadow being subtracted, is equal to the apparent Semidiameter of the Shadow.

$$\text{So } B F E - C G E = G B F.$$

3. The Sum of the Horizontal Parallaxes of the Sun and Moon, is equal to the Sum of the apparent Semidiameter of the Sun and Shadow of the Earth.

$$\text{So, } B D F + B F D = A B D + C B F.$$

Therefore, From the Sum of the Horizontal Parallax of the Sun and Moon, subtract the apparent Semidiameter of the Sun; and there will remain the apparent Semidiameter of the Earths Shadow.

$$\text{So, } B F D + B D F - A B D = C B F.$$

II. For the Proportional Magnitudes of these Three Bodies.

The Semidiameter of the Sun 46300 Log.

4.665581

Earth 727

2.861534

The Log. Difference

1.804047

Multiply by

3

The Logar. of 258309

5.412141

So that the Body of the Sun exceeds the Body of the Earth 258309 times.

Ancilla Mathematica.

The *Semidiameter* of the *Earth* 1650 Log. 3.217484
 The *Semidiameter* of the *Moon* 446 Log. 2.649335

The Logar. Difference 0.568149
 Multiply by 3

To the Logar. of 50.63 1.704447
 So that the *Earth* is greater than the *Moon* 50 times, and
 $\frac{1}{10}$ parts.

III. Of the *Semidiameters*, and *Proportions* of the other *Primary Planets* to the *Earth*.

From the accurate Observations of *Hugenius*, *Gassendus*, and *Horrox* are determined

	D.	D.		
The apparent <i>Semidia-</i>	h	10	30	which in such Parts
meters of these 5 Planets	h	24	0	as the <i>Semidiameter</i> of
in their mean distance	h	4	0	the <i>Earth</i> is 727
from the <i>Earth</i> .	h	10	30	gives the <i>Semidia-</i>
	h	5	0	meter of
	h	48	55	
	h	60	54	
	h	29	6	
	h	50	9	
	h	24	2	

From hence may be gathered, That the *Body* of *Saturn* is
 Greater than the *Earth* 298 times.

The *Body* of *Jupiter* 577 times.

But the *Earth* is Greater than the other Three: For it

Exceeds } 15 } Times.
 } 3 }
 } 27 }

Fig. LXX.

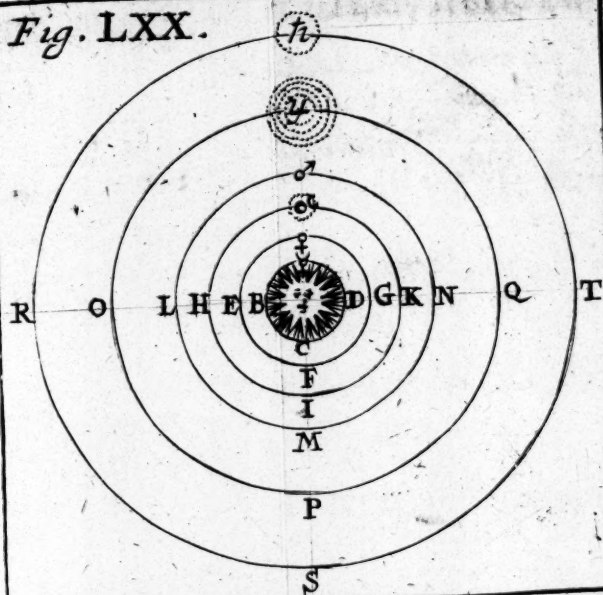


Fig. LXXI.

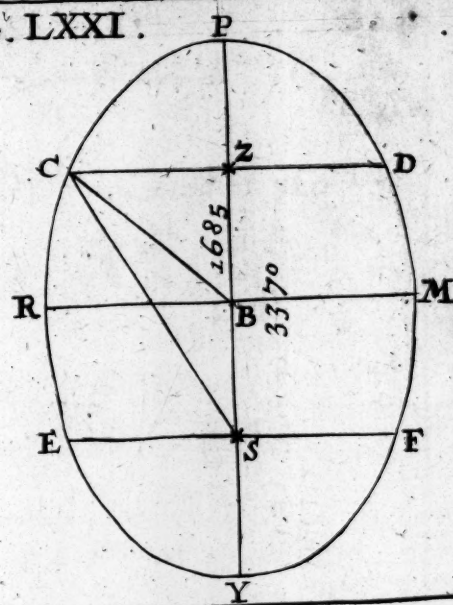


Fig. LXXII.

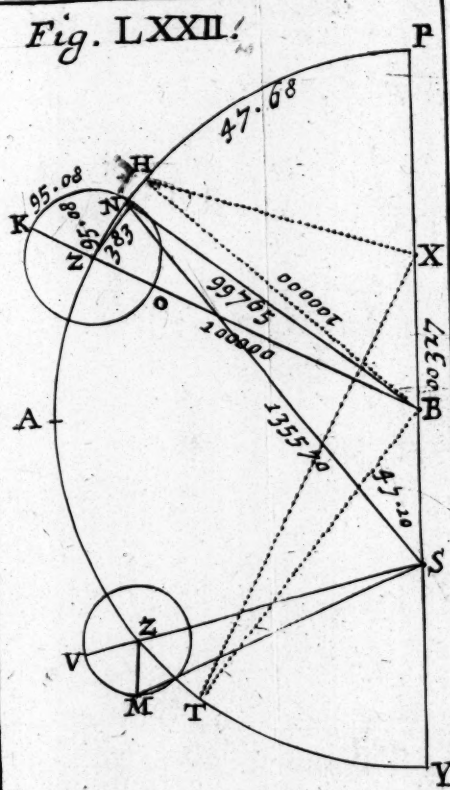


Fig. LXXIII.

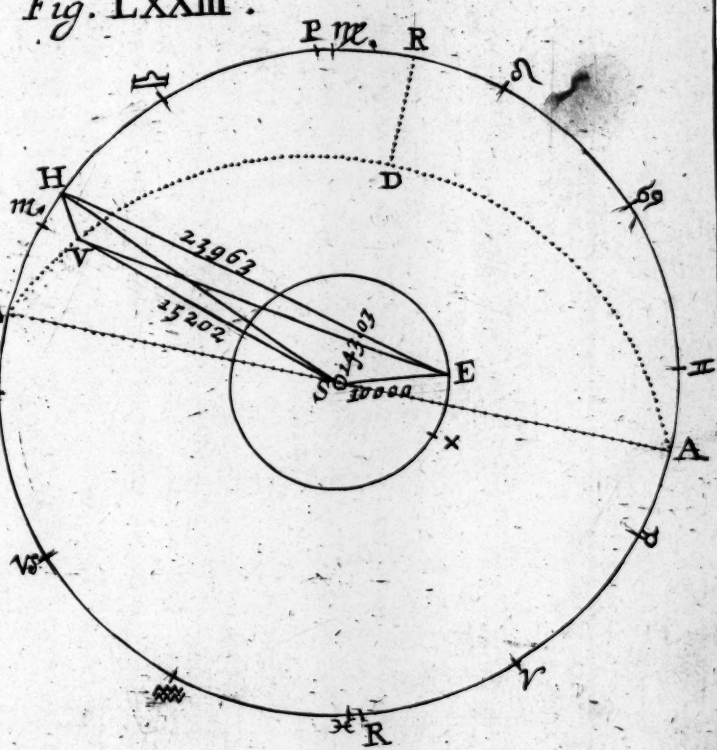
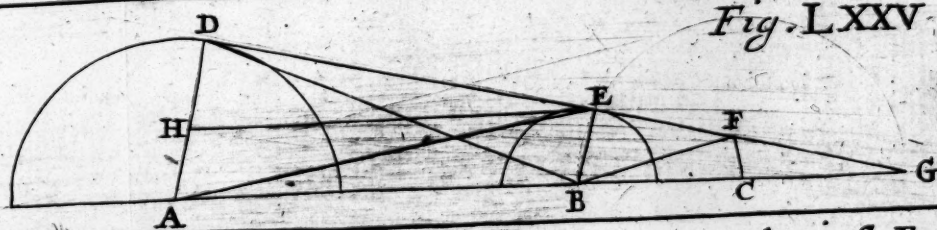


Fig. LXXV.



GEOMETRICAL ASTRONOMY,

S H E W I N G

A Plain and Easie Way to Project Theories of the Planets (in Plano :) And by them to find their true Places, both in Longitude and Latitude, at any time.

And their Distances from the Sun or Earth.

C H A P. I.

How to lay down (in Plano) the Theory of the Earth, or other Planet.

FOR the performance hereof you must have a Scale of Chords, and a Scale of 1000 equal Parts to the same Radius: (Or, rather, a Sector, with such Scales upon it.) Then,

I. How to describe and divide the *Ecliptick*.

With 60 deg. of your Scale of Chords, describe a Circle; which divide into 12 equal Parts, for the 12 Signs, noting them with their respective Characters, γ , δ , Π , &c. This Circle thus described and characterized, represents the *Ecliptick Circle* in the Heavens; whose Centre is $\odot$, the fixed place of the Sun.

In this *Ecliptick Circle*, every of the Planets hath a Point, which is called the *Aphelion* of that Planet: For when the Planet is in that Point, it is at its greatest distance from the Sun. Now, the *Aphelial Points* of each Planet, are these which are exhibited in this little Table.

L 11 2

The

		Deg.
The Aphelial Point of	♄ Saturn	27.50 ♄
	♃ Jupiter	7.82 ♃
	♂ Mars	0.35 ♂
	♀ Venus	2.82 ♀
	☿ Mercury	14.95 ☿
	☾ The Earth	7.00 ☾

II. To draw the *Aphelial Line*.

From the *Aphelial Point* of the Planet (as in our Example MARS,) 0.35 deg. ♂ draw a Right-line to the Centre ☉, as the Line *Ap.* ♂ ☉, and that shall be the *Aphelial Line* of MARS.

For the *Aphelial Line* of the Earth: The EARTH'S *Aphelial Point* being in 7.00 deg. of ♄ Capricorn, from that Point draw a Right-line to ☉ the Centre, as the Line ☉ *Ap.* ☾, and that shall be the Earth's *Aphelial Line*, in the Theories of all the Planets.

III. To find the *Excentric Points* of any of the 3 Superior Planets ♄, ♃, ♂, they will be found by the numbers in this little Table. For,

If from the *Aphelial Point* of

♄	to the Centre ☉ be 1000,	946
♃	the <i>Excentric Point</i> shall be	954
♂	from the <i>Aphelial Point</i>	915

Wherefore, take the length of the *Aphelial Line* *Ap.* ♂ ☉ in your Compasses, and open the *Seſſor* in 10, and 10 of the equal Parts. Then, if from the *Seſſor* you take 915, (the *Excentric* of Mars) it will reach from the *Aphelial Point* *Ap.* ♂, upon the *Aphelial Line* of Mars, to X, so shall X be the *Excentric* of MARS; upon which Point (at the distance of *Ap.* ♂ X) if you describe a Circle, as *Ap.* ♂, ☉, ♂, that Circle shall be the *Orbit* of the Planet MARS, in which it moveth.

For the *Aphelial* and *Excentric Points* of the EARTH in the Theories of the three Superiour Planets, they will be at the Numbers exhibited in this little Table. For,

The

The distance of the *Aphelial Point* of the $\odot$ from the Centre, shall be in the Theory of

$\frac{1}{2}$	101
$\frac{1}{4}$	186
$\frac{3}{8}$	611

And the distance of the *Excentric* of the $\odot$ from the *Aphelion* of the $\odot$, shall be in the Theory of

$\frac{1}{2}$	099
$\frac{1}{4}$	183
$\frac{3}{8}$	600

Wherefore, Out of your *Sector*, (it being still opened to the former distance) take the distance of the *Aphelial Point* of the *Earth* (for MARS 611) and set it (upon the *Earths Aphelial Line*) from the Centre $\odot$ to *Ap.* $\odot$, so shall the point *Ap.* $\odot$ be the *Aphelial Point* of the *Earth* — And the distance of the *Excentric* of the *Earth*, from the *Aphelial Point* of the *Earth* being 600, take 600 out of the *Sector*, and set it from the *Earths Aphelial Point Ap.* $\odot$ downwards towards the Centre $\odot$, and that shall be the Centre whereon to describe the *Orbit* of the *Earth* on the *Theory* of MARS: Which Point falling so near the Centre $\odot$, you may describe the *Earths Orbit* upon the Centre $\odot$ it self; the difference being only $\frac{1}{1000}$ Parts: And in the *Theories* of the other two *Superior Planets*, the difference will be much less; for in the *Theory* of *Saturn* it will be $\frac{1}{1000}$ Parts: And in the *Theory* of *Jupiter*, but $\frac{1}{1000}$ Parts: So that in small *Theories* they need not be much regarded.

4. For the *Aphelial* and *Excentric Points* of the *Earth*, and the two *Inferior Planets* $\circ$ and $\circ$, the *Tables* following will exhibit.

1. For the *Aphelial Points*.

If the distance of the *Aphelial Point* of the *Earth* to $\odot$ be 1000, the *Excentric Point* of

$\odot$	}	Shall be from the Centre $\odot$	1.000
$\circ$			.716
$\circ$			.461

2. For the *Excentric Points*.

And from the *Aphelial Point* of

$\odot$	}	to the Excentric Point of	$\odot$	}	shall be	1.000
$\circ$			$\circ$			.982
$\circ$			$\circ$			.711

5. For

5. For the Nodes of the Planets.

The Ascendant Nodes of the several Planets (for the Earth hath none :) This Table will tell you to what Points in the Ecliptic they are to be drawn. For

The Node of	$\left\{ \begin{array}{c} \text{h} \\ \text{v} \\ \text{♂} \\ \text{♂} \\ \text{♂} \end{array} \right\}$	Is in	$\left\{ \begin{array}{c} \text{d.} \\ 22.45 \\ 5.50 \\ 17.55 \\ 14.00 \\ 14.15 \end{array} \right\}$	$\left\{ \begin{array}{c} \text{♄} \\ \text{♄} \\ \text{♄} \\ \text{♄} \\ \text{♄} \end{array} \right\}$

Wherefore, the Node of the Planet ♂ being in 17.55 de. of ♄ , lay a Ruler from the Centre $\odot$ to 17.55 de. of *Taurus*, and there draw an obscure Line: This Line is the common Section of plain Planets Excentric; with the Plain of the *Ecliptic*; and where this Line crosseth the Orbit of *Mars*, there write ♄ *Dragons Head*; at which Point, when any Planet is in its Orbit, it goes into *North Latitude*: And at the opposite Point, through the Centre, is the place of ♄ the *Dragons Tail*, where it goes into *South Latitude*.

Of the Inclination of the Planets.

The Greatest Inclination of the Planets are such as are expressed in this Table.

For—	$\left\{ \begin{array}{c} \text{h} \\ \text{v} \\ \text{♂} \\ \text{♂} \\ \text{♂} \end{array} \right\}$	$\left\{ \begin{array}{c} \text{—} 2.53 \\ \text{—} 1.32 \\ \text{—} 1.84 \\ \text{—} 3.37 \\ \text{—} 6.90 \end{array} \right\}$	Degrees.

The *Aphelian Lines* and *Orbits* of the *Earth*, and any of the *Planets* being drawn within the *Ecliptick Circle*, the next thing to be done is, to shew how to place the *Earth* and any *Planet* in their respective *Orbits*, for any *Time* proposed: And also, How to *Reduce* their Places so found in their *Orbits* at that time, to their respective Places in the *Ecliptick Circle*.

In order whereunto it will be necessary to shew,

I. How *Time* is to be *Accounted*.

II. To

Of Geometrical Astronomy.

353

II. To Collect the *Anomalies* of the *Earth* and *Planet* for any *Time* proposed.

Fig.
LXXVI.

III. How *Degrees* of *Anomaly* in the *Orbits*, are to be reduced to *Degrees* in the *Ecliptick*.

CHAP. II.

How Time is to be Accounted.

TIME is to be thus Accounted: Any *Day* begins upon its own *Noon*: So that 12 at *Noon* of the first *Day* of *January*, is the *Common Term* of the *Old* and *New Years*; it being the *End* of the *Former*, and the *Beginning* of the *Latter*.

Before the *Longitude* or *Place* of a *Planet* in the *Ecliptic* can be found; the *equal Motion* of the *Anomaly* for the *Time* proposed must be known; and that *Motion* for any proposed *Time* may be collected out of these *TABLES*.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35																																																																	

Epoches of the ANOMALIES for the YEARS.

Epoches for	Earth	♂	♀	♂	♀	♂
Years.	De. Pts.	De. Pts.	De. Pts.	De. Pts.	De. Pts.	De. Pts.
1692	194. 34	346. 17	246. 08	126. 89	247. 04	167. 86
1700	194. 26	33. 88	128. 88	218. 08	248. 42	245. 58
08	194. 18	181. 59	11. 68	309. 26	249. 80	323. 30
16	194. 11	279. 30	254. 48	40. 45	251. 17	41. 01
24	194. 03	17. 01	137. 28	131. 63	252. 55	118. 73
32	193. 95	114. 72	20. 08	222. 81	253. 93	196. 45
40	193. 87	212. 43	262. 88	314. 00	255. 31	274. 17
48	193. 80	310. 14	145. 68	45. 18	256. 68	351. 89
56	193. 72	47. 86	28. 48	136. 40	258. 06	69. 61
64	193. 64	145. 57	271. 28	227. 58	259. 43	147. 33
72	193. 57	243. 28	154. 08	318. 76	260. 81	225. 05
80	193. 49	341. 08	36. 88	49. 94	262. 19	302. 77
88	193. 41	341. 00	279. 68	141. 12	263. 57	20. 49
96	193. 34	78. 71	162. 48	232. 30	264. 94	98. 21
1800	193. 30	225. 27	283. 88	277. 90	85. 63	317. 35
1	359. 74	12. 21	30. 33	191. 27	224. 27	53. 69
2	359. 49	24. 41	60. 66	22. 53	89. 54	107. 38
3	359. 23	36. 62	90. 99	213. 80	314. 32	161. 08
4	359. 96	48. 86	121. 40	45. 59	180. 69	218. 86
5	359. 71	61. 06	151. 73	236. 86	45. 46	272. 55
6	359. 45	73. 27	182. 06	68. 13	270. 23	326. 24
7	359. 19	85. 47	212. 39	259. 39	135. 00	19. 93

Motion.

Motion of the Anomaly, in Months Of the Common Y E A R.

	Earth	h	♈	♉	♊	♋
	100	100	100	100	100	100
	De. Pts.	De. Pts.	De. Pts.	De. Pts.	De. Pts.	De. Pts.
Janua.	30. 55	1. 04	2. 58	16. 24	49. 67	126. 86
Febru.	58. 15	1. 97	4. 90	30. 72	94. 52	241. 45
March	88. 17	3. 01	7. 48	47. 16	144. 19	8. 31
April	118. 27	4. 01	9. 97	62. 88	192. 25	131. 08
May	148. 03	5. 05	12. 55	79. 13	241. 92	257. 94
June	178. 58	6. 05	15. 04	94. 85	289. 98	20. 71
July	208. 95	7. 09	17. 62	111. 09	339. 65	147. 57
August	239. 50	8. 13	20. 19	127. 34	29. 31	274. 43
Septem.	269. 07	9. 13	22. 68	143. 06	77. 38	37. 20
Octob.	299. 62	10. 17	25. 26	159. 30	127. 04	164. 06
Novem.	329. 19	11. 17	27. 75	175. 02	175. 11	286. 83
Decem.	359. 74	12. 21	30. 33	191. 27	224. 77	53. 69

Months in the Leap-Year.

	Earth.	h	♈	♉	♊	♋
Janua.	30. 55	1. 04	2. 58	16. 24	49. 67	126. 86
Febru.	59. 14	2. 01	4. 99	31. 44	96. 13	245. 54
March	89. 69	3. 04	7. 56	47. 69	145. 79	12. 40
April	119. 26	4. 05	9. 95	63. 41	193. 86	135. 17
May	149. 81	5. 08	12. 63	79. 65	243. 52	202. 03
June	179. 38	6. 09	15. 12	95. 37	291. 58	24. 80
July	209. 93	7. 12	17. 70	111. 62	341. 25	151. 66
August	240. 49	8. 16	20. 27	127. 86	30. 92	278. 52
Septem.	270. 05	9. 16	22. 77	143. 58	78. 98	41. 29
Octob.	300. 61	10. 20	25. 34	159. 83	128. 64	168. 15
Novem.	330. 18	11. 20	27. 84	175. 55	176. 71	290. 92
Decem.	360. 73	12. 24	30. 41	191. 79	226. 37	57. 78

M m m

DAYS

D A Y S.

Days	[☉] D. P.	[♂] D. P.	[♂] D. P.	[♂] D. P.	[♀] D. P.	[♀] D. P.
1	0. 99	0. 03	0. 08	0. 52	1. 60	4. 09
2	1. 97	0. 07	0. 17	1. 05	3. 20	8. 18
3	2. 96	0. 10	0. 25	1. 57	4. 81	12. 28
4	3. 94	0. 13	0. 33	2. 10	6. 41	16. 37
5	4. 93	0. 17	0. 24	2. 62	8. 01	20. 46
6	5. 91	0. 20	0. 50	3. 14	9. 61	24. 55
7	6. 90	0. 23	0. 58	3. 67	11. 21	28. 65
8	7. 88	0. 27	0. 66	4. 19	12. 82	32. 74
9	8. 87	0. 30	0. 75	4. 72	14. 42	36. 83
10	9. 86	0. 33	0. 89	5. 24	16. 02	40. 92
11	10. 84	0. 37	0. 91	5. 76	17. 62	45. 00
12	11. 83	0. 40	1. 00	6. 29	19. 23	49. 11
13	12. 81	0. 43	1. 08	6. 81	20. 83	53. 20
14	13. 80	0. 47	1. 16	7. 34	22. 43	57. 29
15	14. 78	0. 50	1. 25	7. 86	24. 03	61. 38
16	15. 77	0. 53	1. 33	8. 38	25. 63	65. 48
17	16. 76	0. 57	1. 41	8. 91	27. 24	69. 57
18	17. 74	0. 60	1. 50	9. 43	28. 84	73. 66
19	18. 73	0. 63	1. 58	9. 96	30. 44	77. 75
20	19. 71	0. 67	1. 66	10. 48	32. 04	81. 85
21	20. 70	0. 70	1. 75	11. 00	33. 64	85. 94
22	21. 68	0. 73	1. 83	11. 53	35. 25	90. 03
23	22. 67	0. 77	1. 91	12. 05	36. 85	94. 17
24	23. 65	0. 80	1. 99	12. 58	38. 45	98. 22
25	24. 64	0. 83	2. 08	13. 10	40. 05	102. 31
26	25. 63	0. 87	2. 16	13. 62	41. 66	106. 40
27	26. 61	0. 90	2. 24	14. 15	43. 26	110. 49
28	27. 60	0. 93	2. 33	14. 67	44. 86	114. 58
29	28. 58	0. 97	2. 41	15. 20	46. 46	118. 68
30	29. 57	1. 00	2. 49	15. 72	48. 06	122. 77
31	30. 55	1. 04	2. 58	16. 24	49. 67	126. 86

The

The manner of Collecting the equal Anomalies from these TABLES, is thus :

First, Exscribe the *Epocha* which belongs to that Year which most nearly precedeth the Year wherein you seek the Place of any Planet.

Secondly, Under that *Epocha* (or number) write the Motions belonging to so many *Years*, *Months* and *Days*, as are compleatly expired since the *Tear* of the *Epocha* ; all which Numbers must be taken out of their proper *Tables*, and set orderly one under another ; which the disjunction of the Numbers will give Direction enough how to do.

Thirdly, All these Numbers must be added into One ; and their *Sum* shall give the *Anomaly* for the *Time* proposed — And if the *Sum* rise to be above a whole *Circle*, or 360 Degrees ; you must then cast away the number of 360 as oft as you may, and the remaining Number must be taken for the *Anomaly*.

Fourthly, These things are to be done both in the *Earth* and the Planet severally ; as is done in the Example following.

CHAP. III.

How to Collect the Anomalies of Saturn and the Earth, for the 16th Day of May, at Noon, in the Year of Christ 1712, being Bissextile, or Leap-Year.

	Anomaly. ☉	Anomaly. ♄
	Deg.	Deg.
Epocha for the Year 1700	194. 26	83. 88
Motion in Years Compleat { 4	359. 96	48. 86
7	359. 19	85. 47
April Compleat B.	119. 26	4. 05
Days Compleat 15	14. 78	. 50
The Sum of the Anomalies	1047. 45	222. 76
The 2 Circles Subst.	720.	
Equal Anomalies for the Time.	327. 45	222. 76
Which reduced to the Ecliptick, are	♑ 15. 50	♈ 15. 00

First, Out of the Table of Years, exscribe the *Æpocha* for 1700: And under it 4 Years and 7 Years, (which together make 11 compleat Years :) Under them exscribe *April Compleat* (putting B to it, for that it is a *Leap-Year* :) And under that exscribe *Days Compleat* 15. Thus is the first part of your Work performed.

Secondly, Repairing to your Tables, you shall find the *Anomaly* of the *Earth* which stands against 1700, to be 194.26 de. and the *Anomaly* of $\odot$ to be 83.88 deg. both which set in their proper Columns under $\odot$ and $\odot$, against 1700. Also, Take out of their proper Tables, the *Anomalies* belonging to 4 Years and 7 Years; and write them in their proper Columns against their respective Times: As 359.96 deg. for the $\odot$, and 48.86 deg. for $\odot$; which set down against 4 Years: Do the like for 7 Years: And for the Month of April, which must be taken out of the Table of Months in the Leap-Years, because 1712 will be Leap-Year: And for the 15 Days, take the *Anomalies* answering to them out of the Table of *Anomalies* for Days.

Thirdly, all the *Anomalies* being thus exscribed out of the Tables; and set orderly one under another; you must add them together, and you shall find the Sum of the $\odot$ *Anomaly* to be 1047.45 deg. And the *Anomaly* of $\odot$ to be 222.76 deg. But, because the Sum of the *Earths Anomalies* is above 360, you must take that Number, as oft as you can (which is twice, viz. 720 deg.) and the Remainder 327.45 deg. is the *Anomaly* of the $\odot$ Reduced: But, the *Anomaly* of $\odot$ being less than 360 deg. needs no such Reduction. And thus have you computed the *Anomalies* of the $\odot$ and $\odot$, for the 16th Day of May, in the Year 1712 at Noon.

C H A P. IV.

How the Degrees of Anomaly of the Earth, or any of the Planets (found as before) in their Orbits; are to be Reduced to Degrees in the Ecliptick.

FOR the performance hereof, the following Table is subservient, for, it will readily (by inspection only) answer your desire.

A TABLE, shewing what Point of the *Ecliptick* answers to any Degrees of Anomaly, that the *Earth*, or any of the other *Planets* is found to have in its own *Orbit*.

Anomalial Degrees.	S. D.	S. D.	S. D.	S. D.	S. D.	S. D.
0	♈ 7. 00	♌ 27. 50	♉ 7. 82	♊ 0. 35	♋ 2. 82	♈ 14. 97
10	16. 65	♍ 6. 44	16. 92	8. 70	12. 69	21. 64
20	26. 32	15. 42	26. 04	17. 09	22. 59	28. 37
30	♋ 6. 00	24. 44	♌ 5. 20	25. 54	♈ 2. 42	♍ 5. 26
40	15. 70	3. 50	14. 42	0. 10	12. 19	12. 19
50	25. 45	12. 72	23. 75	12. 80	22. 23	19. 39
60	♈ 5. 24	22. 05	♌ 3. 20	21. 69	♋ 2. 14	26. 84
70	15. 09	♈ 1. 50	12. 74	0. 80	12. 07	3. 64
80	25. 00	11. 07	22. 44	10. 15	22. 03	12. 85
90	♋ 4. 94	21. 0	♍ 2. 30	19. 80	♈ 2. 01	21. 55
100	15. 00	♋ 1. 00	12. 34	29. 75	12. 07	♈ 0. 85
110	25. 05	11. 22	22. 05	♌ 10. 02	22. 07	10. 85
120	♈ 5. 20	21. 65	3. 89	20. 64	♋ 2. 14	21. 65
130	15. 40	♈ 2. 26	13. 42	♍ 1. 60	12. 20	♋ 3. 35
140	25. 65	13. 07	24. 10	12. 87	22. 30	16. 00
150	♋ 6. 00	24. 01	♈ 4. 90	24. 48	♋ 2. 42	29. 62
160	16. 27	♋ 5. 12	15. 82	6. 27	12. 55	♈ 14. 18
170	26. 64	16. 29	26. 80	18. 26	22. 68	29. 37
180	♋ 7. 00	27. 50	♋ 7. 32	♈ 0. 35	♋ 2. 82	♋ 15. 00
190	17. 37	8. 72	18. 84	12. 44	13. 00	♋ 0. 55
200	27. 73	19. 88	29. 82	24. 42	23. 09	15. 76
210	♋ 8. 05	0. 99	♈ 10. 74	♋ 6. 24	♋ 3. 23	♋ 0. 29
220	18. 35	11. 94	21. 54	17. 82	13. 34	13. 90
230	28. 60	22. 74	♋ 2. 22	29. 10	23. 46	26. 55
240	♋ 8. 80	3. 55	12. 75	♈ 10. 07	3. 50	♋ 8. 25
250	19. 00	13. 79	23. 9	20. 69	13. 57	19. 05
260	29. 04	24. 00	3. 30	♋ 1. 00	23. 60	29. 05
270	9. 07	♋ 4. 02	13. 33	10. 90	♋ 3. 62	8. 35
280	19. 02	13. 84	23. 20	20. 55	13. 60	17. 05
290	28. 92	23. 42	♋ 2. 90	29. 90	23. 57	25. 26
300	♋ 8. 76	♋ 2. 95	22. 45	♋ 9. 01	♋ 3. 50	♋ 3. 07
310	18. 55	12. 29	21. 89	15. 90	13. 42	10. 52
320	28. 30	21. 49	♋ 1. 23	26. 60	23. 32	17. 73
330	♋ 8. 02	♋ 0. 34	10. 44	♋ 5. 17	♋ 3. 23	24. 72
340	17. 69	9. 60	19. 66	13. 62	13. 09	♋ 1. 52
350	27. 35	18. 57	28. 43	22. 0	23. 60	8. 26
360	♋ 7. 00	♋ 27. 50	7. 82	♋ 0. 35	2. 82	14. 95

Anomalies in their Orbits.

The

The Use of the Table

The Table consists of Seven Columns : In the first, towards the Left-hand, is placed the Degrees of *Anomaly*, from 1 to 360, by every 10th Degree. And in the other Six Columns noted at the Head with ☉, ♄, ♀, ♂, ♁, ♃, you have the Degrees, and 100 Parts of the *Ecliptick* which answer thereunto. So that if you seek the Degrees of *Anomaly* of any Planet (found as by the former Table) in the first Column, you shall find against those Degrees of *Anomaly*, the Sign, Degree, and 100 Part of the *Ecliptick* answerable thereunto.

So in the foregoing Example of the Earth ☉ and Saturn ♄, where the *Anomaly* of the Earth was found to be 328, and of Saturn 223 (nearest)——Now, if you look for these Numbers in the first Column of this Table, you shall find, that against 330, (which is the nearest Number to 328) in the Earths Column, ♄ 8.02 de. And that Degree of the *Ecliptick* answers to 330 deg. of the Earths *Anomaly* in its Orbit——Also, against 220 in the first Column (which is the nearest to 223, the *Anomaly* of Saturn) you shall find in the Column for Saturn, ♁ 11.93 deg. And that Degree of the *Ecliptick* answers to 320 deg. of Saturns *Anomaly* in his Orbit : And so of any other.

But in the Use of this Table you are to note, That there being but every Tenth Degree of *Anomaly* in the first Column, the Degrees in the other Columns for the Planets give those Signs and Degrees of the *Ecliptick*, of every Tenth Degree of *Anomaly* : And therefore you must make Proportion (which how to perform, we suppose to be known) and therefore in our Example.

D. M.

The Anomaly of	{	the Earth	}	being	{	328	{	the Ecliptick	}	15.52 ♄
		Saturn				223		Degrees an- swering, are		15.02 ♁

C H A P.

C H A P. V.

The Use of the Theories, and by them to find the Places of the Earth and Planets, both in Longitude and Latitude, &c. for any time proposed. As for MARS and the EARTH, on the 18th of October 1705.

THE Orbits of the EARTH and MARS being described in the Theory, as is before shewed how to do;
First, Collect the equal Anomalies for the Earth and Mars, for the Time proposed, and their Reductions to the Ecliptick, as is here done.

The Equal Anomalies of the Earth and Mars, for the 18th Day of October 1705 at Noon.

	Anomaly.	Anomaly.
Æpoche for Years 1700	194. 25	218. 08
Years Compleat 4	359. 96	45. 59
Month of September Compleat	269. 07	143. 06
Days Compleat 17	16. 76	8. 91
The Sum	840. 05	415. 64
Circle Substr.	720	360.
The Equal Anomalies	120. 05	55. 64
Their Reduction to the Ecliptick	5. 12 8	17. 68 =

Secondly, The Equal Anomaly of the Earth, reduced to the Ecliptick, being in 5. 12 de. of Taurus, lay a Ruler to the Centre ☉, and 5. 12 deg. 8, (as at c) and where it crosseth the Orbit of the Earth, make ☉, for that is the Place of the Earth in his Orbit at that time.

Thirdly, The Equal Anomaly of MARS, reduced to the Ecliptick Circle, being in 17. 68 deg. of Libra: Lay a Ruler to the Centre

Centre $\odot$, and $17.68 \text{ deg} \approx$ (as at d), and where it crosses the Orbit of MARS, make δ ; for that is the place of MARS in his Orbit at that time.

Fourthly, Draw the Right-Lines, $\odot \ominus$, $\odot \delta$ and $\delta \ominus$, constituting the Right-lined Triangle $\odot \delta \ominus$.

The Theory thus finished, I come now to shew,

I. To find the Sun's true Place in the *Ecliptick*, for the time proposed.

Lay a Ruler upon the Line $\odot \ominus$, and it will cut the *Ecliptick Circle* in $b \odot b$, in $5.51 \text{ deg. of Scorpio}$; and that is the Sun's Longitude (or Place in the *Ecliptick*) at the time proposed.

II. To find the Longitude (or true Place in the *Ecliptick*) of MARS for the same time.

Through the Point $\odot$ in the Centre, draw a Right-Line parallel to the Line $\delta \ominus$, as the short Line $a \delta a$, which will cross the *Ecliptick Circle* in $25.00 \text{ deg. of Libra} \approx$, where make the Character of MARS; for that is his true Place of Longitude in the *Ecliptick*, for the time proposed.

Mark now the Triangle $\odot \delta \ominus$, made upon the Theory, by the Centre $\odot$, and the Point of $\ominus$ and $\odot$ in their Orbits: For in that Triangle,

The Side $\left\{ \begin{array}{c} \odot \delta \\ \odot \ominus \\ \ominus \delta \end{array} \right\}$ is the distance of $\left\{ \begin{array}{c} \delta \\ \ominus \end{array} \right\}$ from the $\left\{ \begin{array}{c} \text{Sun.} \\ \text{Sun.} \\ \text{Earth.} \end{array} \right\}$.

All which are to be Measured upon Proper Scales for the several Planets. And

The Angle $\left\{ \begin{array}{c} \delta \odot \ominus \\ \odot \delta \ominus \\ \ominus \delta \odot \end{array} \right\}$ is the Angle at the $\left\{ \begin{array}{c} \text{Sun} \\ \text{Earth} \\ \text{Planet} \end{array} \right\}$ or of $\left\{ \begin{array}{c} \text{Committation.} \\ \text{Elongation.} \\ \text{Paral. of the Orb.} \end{array} \right\}$.

These Angles may be Measured (very near the Truth) by help of your Scale of Chords; but more accurately (when the Sides are known) by Trigonometrical Calculation: Of which hereafter.

But

But for the measuring of the *Sides*, *Proper Scales* must be made for each *Planet*, and the *Earth*. And, Fig. LXXVI.

The *Proportions* between the *Length* of the *Aphelial Lines* of the several *Planets*, (taken from the *Center* of the *Theories*, to the *Aphelial Points*) and the *Scales* by which to measure their *Distances* from the *Sun* or *Earth*; are these exhibited in this *Table*, viz.

		Parts.
For	{ Saturn	{ 85. 63
	{ Jupiter	{ 22. 87
	{ Mars	{ 56. 73
	{ Earth	{ 69. 38
	as 100 to	

Which Numbers will make *Proper Scales* for measuring the *Distances* in each *Theory*. And the *Scale* for the *Earth*, will serve, also, for the *Theories* of *Venus* and *Mercury*.

And note farther, That all *Distances* measured upon these *Proper Scales*, are

In	{ Saturn	{ 400
	{ Jupiter	{ 200
	{ Mars	{ 100
	{ Venus	{ 50
	{ Mercury	{ 50
	to be multiplied by	

And these *Products* shall *Reduce* the *Distances* measured upon the *Proper Scales*, into *Semidiameters* of the *Earth*.

III. To find the *Distance* of MARS from the *Sun* and from the *Earth*, for the *Time* proposed.

Take in your *Compasses* the length of the *Aphelial Line* Ap. $\odot$, and opening the *Sector* in the Points 56.73 and 56.73. The *Sector*, as so opened, shall be the *Scale* whereby to measure the *Distances* in the *Theory* of MARS. Wherefore, if you take in your *Compasses* (out of the *Theory*) of Mars.

Fig.
LXXVI.

The Line $\left\{ \begin{array}{c} \odot \delta \\ \odot \ominus \\ \ominus \delta \end{array} \right\}$ And apply it to $\left\{ \begin{array}{c} \odot \delta \\ \odot \ominus \\ \ominus \delta \end{array} \right\}$ the Sect. so open'd, to contain $\left\{ \begin{array}{c} 55. 50 \\ 34. 75 \\ 88. 20 \end{array} \right\}$ Parts.

Which Parts multiplied by 100, will give

For $\left\{ \begin{array}{c} \odot \delta \\ \odot \ominus \\ \ominus \delta \end{array} \right\} \left\{ \begin{array}{c} 3550 \\ 3475 \\ 8820 \end{array} \right\}$ Semidiameter of the Earth.

IV. To find the *Angle of Commination*, or *Angle at the Sun* $\delta \odot \ominus$.

Lay a Ruler to $\odot$ the Centre, and the Point $\ominus$ in its Orbit; and it will cut the *Ecliptick Circle* in the Point c : Also lay a Ruler upon $\odot$ the Centre, and the Point *Mars* in its Orbit, and it will cut the *Ecliptick* in the Point d ; then will the Degrees of the *Ecliptick*, intercepted between c and d , be the quantity of the *Angle* $\delta \odot \ominus$, the *Angle at the Sun* (or of *Commination*) and will be found to be 162.00 deg.

V. To find the *Angles of Elongation* and *Parallas* of the Orb (or the *Angles at the Earth* and *Planet*) $\odot \ominus \delta$ and $\odot \ominus \delta$.

These (as I said before) may be measured upon the *Theory* by the assistance of the *Scale of Chords*: But the *Angle of Commination* and the *distance* of the *Earth* and *Mars* being known, they may more exactly be found by *Trigonometrical Calculation*: Thus;

As the Sum of the Sides $\delta \odot$ and $\ominus \odot$. 9025 — 3.954447

Is to their Difference 2075.

So is the Tang. of half the Angles at δ and $\ominus$, 9.00 d. 9.199712

To the Tangent of 2.09 deg.

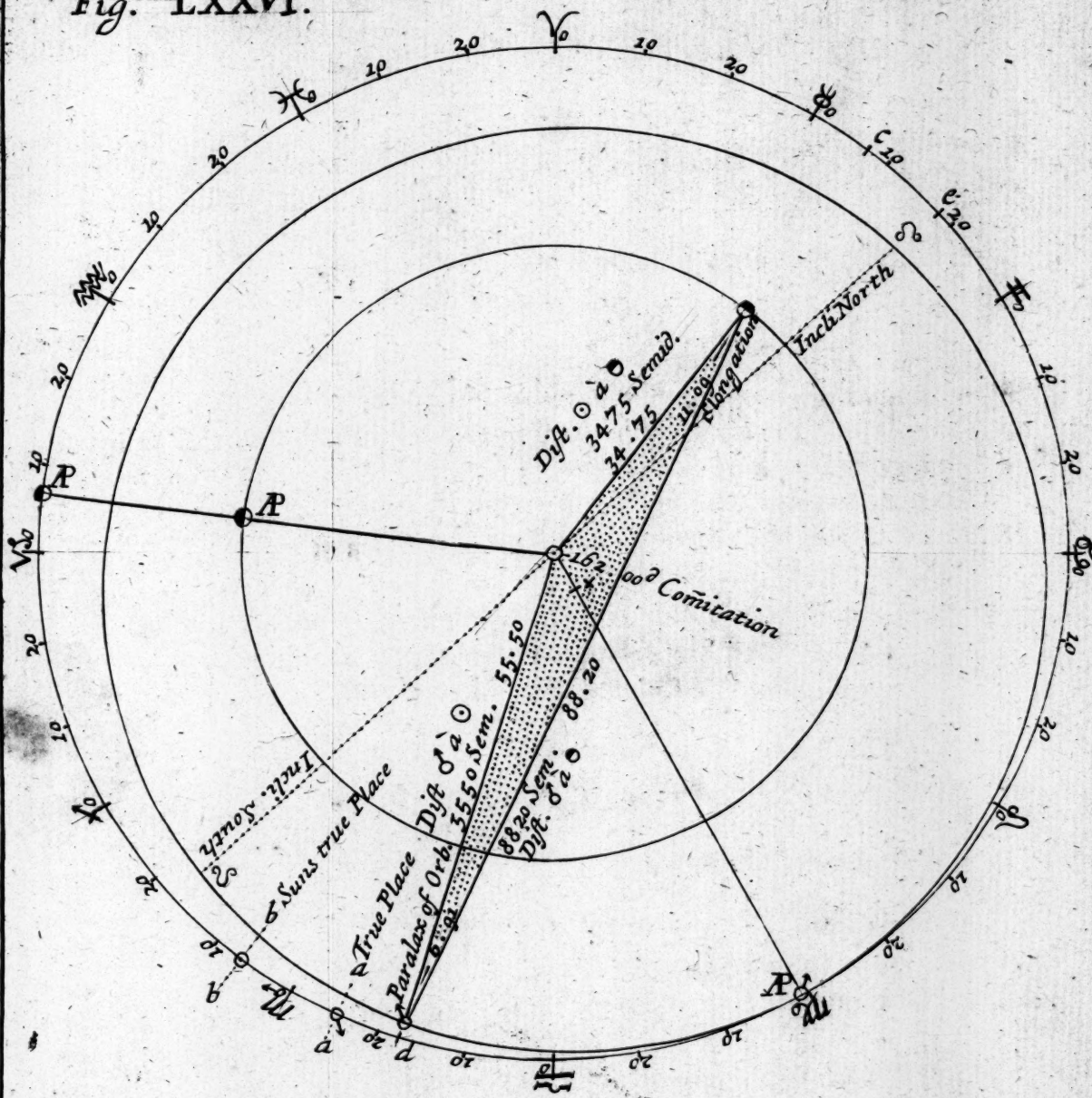
12.516730
8.562283

Which added to 9.00 deg. gives 11.09 deg. for the *Angle at the Earth* (or of *Elongation*) $\odot \ominus \delta$ — And subtracted from 9.00 deg. leaves 6.91 deg. for the *Angle at Mars* (or of the *Parallax* of the Orb.

VI. To

*The Theoric of MARS, for the
18th of October, at Noon, 1705.*

Fig. LXXVI.



*A Scale of Parts which Parts Multiplied by 100: Gives
the Distances in Semidiameters of the Earth.*

Plate XXIV. From Pag. 349. to Pag. 364.

Against P. 364.

Of Geometrical Astronomy.

365

Fig.

LXXVI.

VI. To find the Latitude of M A R S at the same time.

Before the *Latitude* can be found, the *Argument of Latitude* must be had: To the Attaining whereof, Lay a Ruler to the Centre $\odot$, and the Point δ in the *Planets Orbit*; so will the Ruler cut the *Ecliptick* in the Point e : And a Ruler laid from the Centre $\odot$ to δ , will cut the *Ecliptick* in the Point d : So that the Angle $d \odot e$ is the *Argument of Latitude*; and in this our *Example* will be found (by measuring the Degrees of the *Ecliptick Circle* between d and e) to be 149.75 deg.

The *Argument of Latitude* thus attained, the true *Latitude* may be found by the following Canon.

As the Radius, Sine 90 de.

10.

Is to the the Sine of the *Arg. of Latit.* (149.75 d.)

9.7022357

So is the Tang. of *Mars Greatest Incl.* 1.84 de.

8.5068445

To the Tangent of 0.92 deg.

8.2090802

Which 0.92 Degrees is the *Latitude* of M A R S seen at the *Sun*; and is *North*, because the *Inclination* of M A R S was *Northward*.

But to find the *Latitude* as seen at the *Earth*,

The Proportion is,

As the *Distance* $\delta \odot$, 8820

3.945468

Is to the *Distance* $\odot \delta$, 5550

3.744293

So is the Tang. of the *Latit.* seen at the *Sun* 0.92 de.

8.205702

To the Tangent of 0.58 deg.

11.949995

Which is the *Latitude* of *Mars* seen from the *Earth*.

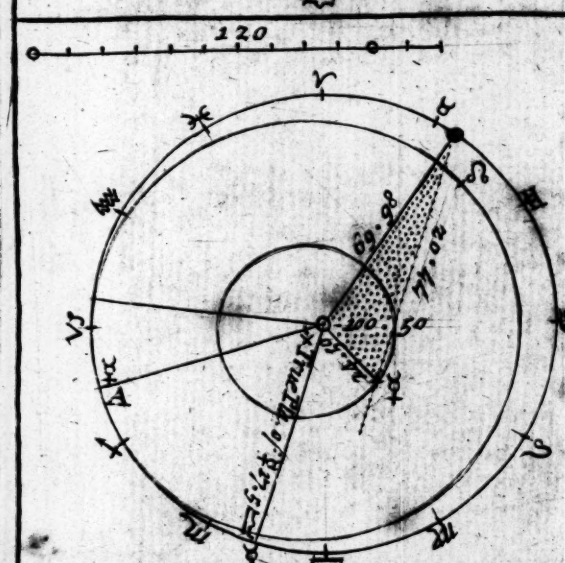
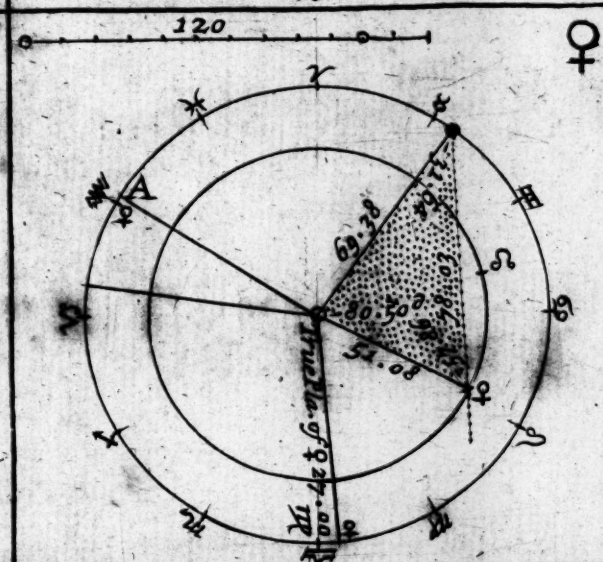
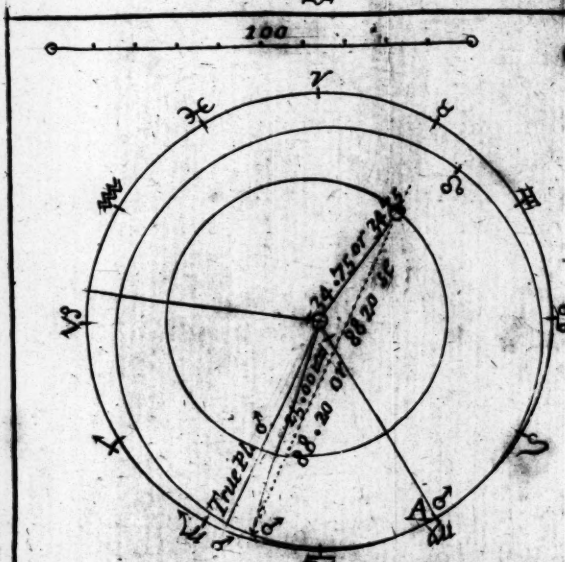
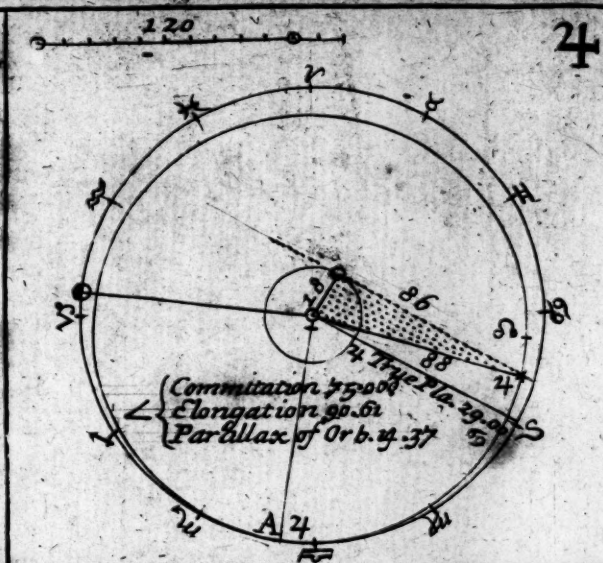
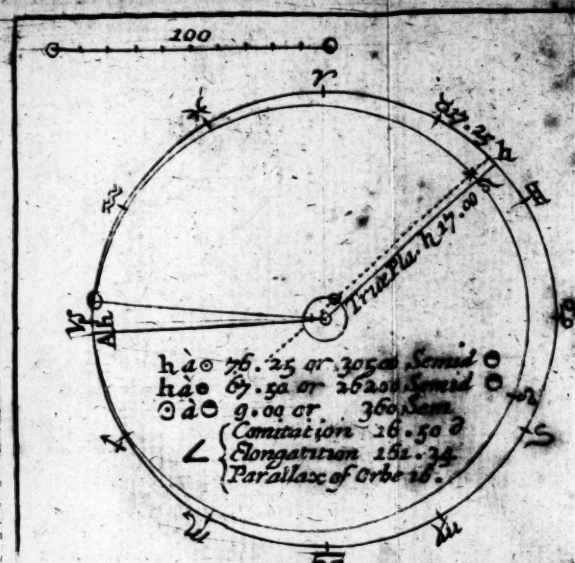
8.004527

The

The Equal Motions of the Anomalies of the Earth, and the other Five Planets, $\hbar$, ψ , δ , φ , ζ , For the 18th Day of October, at Noon, Anno 1705.

	$\odot$	$\hbar$	ψ	δ	φ	ζ
Æpocha for Years 1700	194:26	83:88	128:88	218:08	248:42	245:54
Motion in Years Compleat 4	359:96	48:86	121:40	45:59	180:69	218:86
September Compleat	269:07	9:13	22:68	143:06	77:38	37:20
Days Compleat 17	16:79	0:57	1:41	8:91	27:24	69:57
The Sums	840:08	142:44	274:37	415:64	533:73	571:21
Circles Subst.	720:			360:	360:	360:
The Equal Anomalies	120:08	142:44	274:37	55:64	173:73	211:21
Their Reduction to the Eclipt.	5:12 δ	15:19 δ	17:70 δ	17:83 δ	26:41 δ	16:10 δ
The Planets Longitudes	6:00 δ	17:00 δ	29:00 δ	25:00 δ	27:00 δ	17:50 δ
Distances from the { Sun Earth	44:30	76:25 67:51	88:00 86:00	55:50 88:20	51:08 78:03	24:50 77:02
Latitude		2:56 S	0:25 N	0:57 N	1:42 N	2:06

FINIS.



THE
THEORIES
OF
Saturn } Venus
Jupiter } Mercury
Mars }
For the 18th of
October 1705.